HIGH ENERGY LIMIT IN QCD*,**

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Some perturbative approaches to the QCD description of the deep-inelastic *ep* scattering at the small Bjorken variable x are reviewed. It is shown, that in the leading logarithmic approximation the gluon is reggeized and the pomeron is a compound state of two reggeized gluons. The relation between the Schrödinger equation for the compound state of several reggeized gluons in the multi-colour QCD and the completely integrable Heisenberg spin model is discussed. The effective action for the gluon-Reggeon interactions is constructed and applied to the problem of finding next to leading corrections to the QCD pomeron.

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1. Introduction

Recently in experiments at HERA the rapid growth of the structure functions for the *ep* scattering at small x was discovered. It is related with the corresponding increase of the parton distributions $n_i(x)$ inside the rapidly moving proton as functions of the decreasing parton momentum fraction x and the increasing photon virtuality Q^2. In the framework of the Dokshitzer-Gribov-Lipatov-Altarelly-Parisi (DGLAP) equation [1] the parton distributions grow at small x as a result of their Q^2-evolution. In the framework of the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation [2] this growth is a consequence of their x -evolution. Within the double-logarithmic accuracy these equations coincide and the increase of the structure functions

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at small x is related with the singularities of the anomalous dimensions for the corresponding twist-2 operators at non-physical values $j \to 1$ of the Lorentz spin [3]. The existing experimental data on structure functions agree with the DGLAP dynamics provided that the evolution equation in Q^2 is applied starting from rather small $Q^2 = Q_0^2$ [4]. The growth of the structure functions at small x can be also obtained with the use of the BFKL equation [5]. In this case a large uncertainty is related with the fact, that the next to leading corrections to this equation have not been calculated yet contrary to the case of the DGLAP equation where they are well known.

In this talk the approach based on the BFKL evolution equation and on its generalizations will be reviewed. In the next section the basic properties of the solution of the BFKL equation are discussed in the framework of the impact parameter representation. In the third section it will be demonstrated, that in the Regge limit of large energies $\sqrt{s}$ and fixed momentum transfers $\sqrt{-t}$ the gluon having the spin $j = 1$ at $t = 0$ lies on the Regge trajectory $j = j(t)$ and the BFKL pomeron is a compound state of two reggeized gluons. Here it is shown also, that the Bartels–Kwieciński–Praszałowicz (BKP) equations [6] for compound states of several reggeized gluons in the multi-colour QCD have remarkable properties: the two-dimensional conformal symmetry, the holomorphic factorization of their eigen functions and the existence of non-trivial integrals of motion in holomorphic and anti-holomorphic subspaces. The corresponding Hamiltonian turns out to be equivalent to the local Hamiltonian of the exactly solvable Heisenberg model with the spins being the generators of the conformal (Möbius) group. At high energies it is natural to reformulate QCD as an effective field theory for regeized gluons. In the fourth section the effective action for the interactions between the reggeized gluons and the usual quarks and gluons is constructed. The program of finding next to leading corrections to the BFKL equation is discussed in the fifth section. In the Conclusion the unsolved problems are discussed.

2. BFKL pomeron in the impact parameter space

In QCD the most important processes at large energies E ($s = E^2$) are governed by the gluon exchanges. For example, the Born amplitude for the parton-parton scattering is [2]

$$A(s,t) = 2s\, g\delta_{\lambda_a,\lambda_{a'}} T^c_{A'A} \tfrac{1}{t}\, g\delta_{\lambda_b,\lambda_{b'}} T^c_{B'B}\,, \tag{1}$$

where λ_i are helicities of the initial and final particles; A, A', B, B' are their colour indices and T^c_{ij} are colour group generators in the corresponding representation. The s-channel helicity for each colliding particle is conserved because the virtual gluon in the t-channel for small q interacts with the total colour charge Q^c commuting with space-time transformations.

Let us consider now the high energy amplitude for the colorless particle scattering described by the Feynman diagrams containing only two intermediate gluons with momenta k and $q-k$ in the t-channel. With a good accuracy we can neglect the longitudinal momenta in their propagators:

$$k^2 \simeq k_\perp^2\,, \quad (q-k)^2 \simeq (q-k)_\perp^2\,. \tag{2}$$

The polarization matrix for each gluon can be simplified at large energies $s = (p_a + p_b)^2 \gg m^2$. Namely, if its indices μ and ν belong to the blobs with incoming particles a and b correspondingly, then with a good accuracy we have

$$\delta^{\mu\nu} \to \frac{p_b^\mu p_a^\nu}{p_a p_b}\,. \tag{3}$$

By introducing the Sudakov parameters

$$\alpha = -\frac{kp_a}{p_a p_b} = -s_a/s\,, \quad \beta = \frac{kp_b}{p_a p_b} = s_b/s\,,$$

$$\vec{k} = \vec{k}_\perp\,, \quad d^4k = d^2k\,\frac{d\,s_a d\,s_b}{2\,|s|} \tag{4}$$

for the virtual gluon momenta $k\,, q-k$ one obtains for the asymptotic contribution of the diagrams with two gluon exchanges the following factorized expression:

$$A(s,t) = 2i|s|\frac{1}{2!}\int d^2k\frac{1}{\vec{k}^2}\frac{1}{(\vec{q}-\vec{k})^2}\Phi^a(\vec{k},\vec{q}-\vec{k})\Phi^b(\vec{k},\vec{q}-\vec{k})\,, \tag{5}$$

corresponding to the impact-factor representation [7]. Here the sum over the colour gluon indices is implied. The impact factors $\Phi^{a,b}$ are defined as integrals over the energy invariants $s_{a,b}$ from the photon-particle amplitudes $f_{\mu\nu}^{a,b}$:

$$\Phi^{a,b}(\vec{k},\vec{q}-\vec{k}) = \int_{-\infty}^{\infty}\frac{ds_{a,b}}{(2\pi)^2 i}\frac{p_{b,a}^\mu}{s}\frac{p_{b,a}^\nu}{s}\,f_{\mu\nu}^{a,b}(s_{a,b},\vec{k},\vec{q}-\vec{k})\,. \tag{6}$$

The impact factors describe the inner structure of colliding particles. For large $\vec{k}$ they are proportional to the number of partons N_i weighted with their colour group Casimir operators.

The impact factors are real functions of $\vec{k}$, $\vec{q}-\vec{k}$, vanishing for small $|\vec{k}\,|$ and $|\vec{q}-\vec{k}\,|$ in the case of the colorless particle scattering (*e.g.* photon-photon collisions), which is a consequence of the gauge invariance and of the absence of infrared divergencies in the integral over $s_{a,b}$ for small $|\vec{k}\,|$ and $|\vec{q}-\vec{k}\,|$.

It is convenient to present the scattering amplitude in the form of the Mellin transformation

$$A(s,t) = i\,|s| \int \frac{d\,\omega}{2\pi i}\, s^{\omega}\, f_{\omega}(q^2),\; t = -q^2 \tag{7}$$

and to pass to the impact parameter representation performing the Fourier transformation [8]:

$$f_{\omega}(q^2)\delta^2(q-q') = \int \prod_{r=1,2} \frac{d^2\rho_r d^2\rho_{r'}}{(2\pi)^4} \times \Phi^a(\vec{\rho_1},\vec{\rho_2},\vec{q})\; f_{\omega}(\vec{\rho_1},\vec{\rho_2};\vec{\rho_1'},\vec{\rho_2'})\Phi^b(\vec{\rho_1'},\vec{\rho_2'},\vec{q'})\,. \tag{8}$$

The quantity $f_{\omega}(\vec{\rho_1},\vec{\rho_2};\vec{\rho_{1'}},\vec{\rho_{2'}})$ can be considered as a four point Green function:

$$f_{\omega}(\vec{\rho_1},\vec{\rho_2};\vec{\rho_{1'}},\vec{\rho_{2'}}) = \langle 0\,|\phi(\rho_1)\phi(\rho_2)\phi(\rho_{1'})\phi(\rho_{2'})|\,0\rangle\,, \tag{9}$$

where the field $\varphi(\rho)$ describes the (reggeized) gluons. In the Born approximation $g \to 0$ with the use of the colorless property of the colliding particles it can be written as follows

$$f^0_{\omega}(\vec{\rho_1},\vec{\rho_2};\vec{\rho_{1'}},\vec{\rho_{2'}}) \to \frac{2\pi^2}{\omega} \ln\left|\frac{\rho_{11'}\rho_{22'}}{\rho_{12'}\rho_{1'2}}\right| \ln\left|\frac{\rho_{11'}\rho_{22'}}{\rho_{12}\rho_{1'2'}}\right|\,,$$

$$\vec{\rho_{ik}} \equiv \vec{\rho_i} - \vec{\rho_k}\,. \tag{10}$$

This expression is unique in comparison with all other physically equivalent expressions for f^0_{ω} because it depends only on two independent anharmonic ratios of the vectors $\vec{\rho_1}$, $\vec{\rho_2}$, $\vec{\rho_{1'}}$ and $\vec{\rho_{2'}}$ which can be chosen as follows

$$\alpha = \left|\frac{\rho_{11'}\rho_{22'}}{\rho_{12'}\rho_{1'2}}\right|\,,\; \beta = \left|\frac{\rho_{11'}\rho_{22'}}{\rho_{12}\rho_{1'2'}}\right|\,,\; \gamma = \frac{\beta}{\alpha} = \left|\frac{\rho_{12'}\rho_{1'2}}{\rho_{12}\rho_{1'2'}}\right|\,. \tag{11}$$

Therefore, f^0_{ω} is invariant under the conformal (Möbius) transformations:

$$\rho_k \to \frac{a\,\rho_k + b\,\rho_k}{c\,\rho_k + d\,\rho_k} \tag{12}$$

for arbitrary complex a, b, c and d provided that we use the complex coordinates

$$\rho_k = x_k + i\,y_k\,,\; \rho^*_k = x_k - i\,y_k \tag{13}$$

for all two-dimensional vectors $\overrightarrow{\rho_k}$ (x_k, y_k).

The solution of the BFKL equation is also Möbius invariant in LLA and can be written in the form [8]:

$$f_\omega(\overrightarrow{\rho_1}, \overrightarrow{\rho_2}; \overrightarrow{\rho_{1'}}, \overrightarrow{\rho_{2'}}) = \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{(\nu^2 + n^2/4)\ d\nu}{[\nu^2 + (n-1)^2/4]\,[\nu^2 + (n+1)^2/4]} \times \frac{G_{\nu n}(\overrightarrow{\rho_1}, \overrightarrow{\rho_2}; \overrightarrow{\rho_{1'}}, \overrightarrow{\rho_{2'}})}{\omega - \omega(\nu, n)}\,, \tag{14}$$

where for $n = \pm 1$ the integral in ν is regularized as follows:

$$\int_{-\infty}^{+\infty} d\nu\, \frac{1}{\nu^2}\, \varphi(\nu) \equiv \lim_{\varepsilon \to 0} \left(\int_{-\infty}^{+\infty} d\nu\, \frac{\theta(\nu^2 - \varepsilon^2)}{\nu^2}\, \varphi(\nu) - 2\, \frac{\varphi(0)}{|\varepsilon|} \right). \tag{15}$$

In the "energy propagator" $(\omega - \omega(\nu, n))^{-1}$ the quantity $\omega(\nu, n)$ is the eigen value of the BFKL equation [2]:

$$\omega(\nu, n) = \frac{N_c g^2}{2\pi^2} \int_0^1 \frac{dx}{1-x} \left[x^{(|n|-1)/2} \cos(\nu \ln x) - 1 \right] = -\frac{N_c g^2}{2\pi^2} \,\mathrm{Re} \left(\psi\left(\frac{1+|n|}{2} + i\nu \right) - \psi(1) \right). \tag{16}$$

The Green function $G_{\nu n}(\overrightarrow{\rho_1}, \overrightarrow{\rho_2}; \overrightarrow{\rho_{1'}}, \overrightarrow{\rho_{2'}})$ is given below:

$$G_{\nu n}(\overrightarrow{\rho_1}, \overrightarrow{\rho_2}; \overrightarrow{\rho_{1'}}, \overrightarrow{\rho_{2'}}) = \int d^2\rho_0\, E^*_{\nu n}(\rho_{1'0}, \rho_{2'0}) E_{\nu n}(\rho_{10}, \rho_{20})\,, \tag{17}$$

where

$$E_{\nu n}(\rho_{10}, \rho_{20}) = \langle 0 \mid \phi(\rho_1)\phi(\rho_2) O_{\nu n}(\rho_0) \mid 0 \rangle = \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^m \left(\frac{\rho^*_{12}}{\rho^*_{10}\rho^*_{20}}\right)^{\widetilde{m}}, \tag{18}$$

are the solutions of the homogeneous BFKL equation [8]. They are equivalent to the Polyakov three-point function for the case when the fields $\phi(\rho_i)$ describing the reggeized gluons have vanishing conformal quantum numbers ν and n. The composite field $O_{\nu n}$ describing the BFKL pomeron has the conformal weights

$$m = \frac{1}{2} + i\nu + \frac{n}{2}, \quad \widetilde{m} = \frac{1}{2} + i\nu - \frac{n}{2} \tag{19}$$

with real ν and the integer conformal spin n in accordance with the fact that they belong to the basic series of the irreducible unitary representations of the conformal group.

One can present $G_{\nu n}(\overrightarrow{\rho_1}, \overrightarrow{\rho_2}; \overrightarrow{\rho_{1'}}, \overrightarrow{\rho_{2'}})$ (17) in terms of the hypergeometric functions $F(a, b, c; x)$ taking into account its conformal invariance:

$$G_{\nu n}(\overrightarrow{\rho_1}, \overrightarrow{\rho_2}; \overrightarrow{\rho_{1'}}, \overrightarrow{\rho_{2'}}) = c_1\, x^m\, x^{*\widetilde{m}}\, F(m, m, 2m; x)\, F(\widetilde{m}, \widetilde{m}, 2\widetilde{m}; x^*) + c_2\, x^{1-m} x^{*1-\widetilde{m}} F(1-m, 1-m, 2-2m; x) \times F(1-\widetilde{m}, 1-\widetilde{m}, 2-2\widetilde{m}; x^*)\,, \tag{20}$$

where x is the complex anharmonic ratio:

$$x = \frac{\rho_{12}\, \rho_{1'2'}}{\rho_{11'}\, \rho_{22'}}\,. \tag{21}$$

The coefficients $c_{1,2}$ can be obtained from ref. [8]. The ratio c_1/c_2 is fixed from the condition, that G is a single-valued function of its arguments. With the use of the various relations among the hypergeometric functions one can derive other representations for the Green function G to continue it in the regions around the points $x = 1$ and $x = \infty$. One can use the representation in terms of the hypergeometric functions to obtain the scattering amplitude in various limiting cases with the use of the operator expansion of products of fields φ.

3. Multi-Regge processes in QCD

The most probable process at large s is the gluon production in the multi-Regge kinematics for final state particle momenta $k_0 = p_{A'}$, $k_1 = q_1 - q_2, \ldots k_n = q_n - q_{n+1}$, $k_{n+1} = p_{B'}$:

$$s \gg s_i = 2k_{i-1}k_i \gg t_i = q_i^2 = \left(p_A - \sum_{r=0}^{i-1} k_r\right)^2,$$

$$\prod_{i=1}^{n+1} s_i = s \prod_{i=1}^{n} \overrightarrow{k_i}^2\,,\ k_\perp^2 = -\overrightarrow{k}^2\,. \tag{22}$$

In LLA the production amplitude in this kinematics has the multi-Regge form [2]:

$$A_{2\to 2+n}^{\mathrm{LLA}} = A_{2\to 2+n}^{\mathrm{tree}} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}\,. \tag{23}$$

Here $s_i^{\omega(t_i)}$ are the Regge-factors appearing from the radiative corrections to the Born production amplitude $A^{tree}_{2\to 2+n}$. The gluon Regge trajectory $j = 1 + \omega(t)$ is expressed in terms of the quantity:

$$\omega(t) = -\frac{g^2 N_c}{16\pi^3} \int d^2k \frac{\vec{q}^{\,2}}{\vec{k}^{\,2}(\vec{q} - \vec{k})^2}\,, \quad t = -\vec{q}^{\,2}\,. \tag{24}$$

Infrared divergencies in the Regge factors cancel in σ_{tot} with analogous divergencies in the contributions of real gluons. The production amplitude in the tree approximation has the following factorized form [2]

$$A^{tree}_{2\to 2+n} = 2gT^{c_1}_{A'A}\Gamma_1 \frac{1}{t_1} gT^{d_1}_{c_2c_1}\Gamma^1_{2,1}\frac{1}{t_2} \ldots gT^{d_n}_{c_{n+1}c_n}\Gamma^n_{n+1,n}\frac{1}{t_{n+1}} gT^{c_{n+1}}_{B'B}\Gamma_2\,. \tag{25}$$

Here A, B and A', B', d_r $(r = 1, 2...n)$ are colour indices for initial and final gluons correspondingly. $T^c_{ab} = -if_{abc}$ are generators of the gauge group SU(N_c) and g is the Yang–Mills coupling constant. Further,

$$\Gamma_1 = \tfrac{1}{2} e^{\lambda}_{\nu} e^{\lambda' *}_{\nu'} \Gamma^{\nu\nu'}\,, \; \Gamma^r_{r+1,r} = -\tfrac{1}{2}\Gamma_\mu(q_{r+1}, q_r) e^{\lambda_r *}_{\mu}(k_r) \tag{26}$$

are the Reggeon–Particle–Particle (RPP) and Reggeon–Reggeon–Particle (RRP) vertices correspondingly. The quantities $\lambda_r = \pm 1$ are the s-channel gluon helicities in the c.m. system. They are conserved for each of two colliding particles: $\Gamma_1 = \delta_{\lambda'\lambda}$. The tensor $\Gamma^{\nu\nu'}$ can be written as the sum of two terms:

$$\Gamma^{\nu\nu'} = \gamma^{\nu\nu'+} - q^2 (n^+)^\nu \frac{1}{p_A^+}(n^+)^{\nu'}, \tag{27}$$

where we introduced the light cone vectors

$$n^- = \frac{p_A}{E},\; n^+ = \frac{p_B}{E},\; E = \sqrt{s}/2,\; n^+ n^- = 2\,, \tag{28}$$

and the light cone projections $k^\pm = k^\sigma n^\pm_\sigma$ of the Lorentz vectors k^σ. The first term is the light cone component of the Yang–Mills vertex:

$$\gamma^{\nu\nu'+} = (p_A^+ + p_{A'}^+)\delta^{\nu\nu'} - 2p_A^{\nu'}(n^+)^\nu - 2p^\nu_{A'}(n^+)\,. \tag{29}$$

The second (induced) term in (27) is a coherent contribution of the Feynman diagrams in which the pole in the t-channel is absent. Indeed, it is proportional to the factor q^2 cancelling the neighboring propagator.

Similarly the effective RRP vertex $\Gamma(q_2, q_1)$ can be presented as follows [2]

$$\Gamma^\sigma(q_2, q_1) = \gamma^{\sigma-+} - 2q_1^2 \frac{(n^-)^\sigma}{k_1^-} + 2q_2^2 \frac{(n^+)^\sigma}{k_1^+}\,, \tag{30}$$

where

$$\gamma^{\sigma+-} = 2q_2^\sigma + 2q_1^\sigma - 2(n^-)^\sigma k_1^+ + 2(n^+)^\sigma k_1^- \tag{31}$$

is the light-cone component of the Yang–Mills vertex.

Note, that Γ^σ has the important property:

$$(k_1)^\mu \Gamma_\mu(q_2, q_1) = 0, \; k_1 = q_1 - q_2 \, , \tag{32}$$

which gives us a possibility to chose an arbitrary gauge for each of the produced gluons. In the left (l) light cone gauge where $p_A e^l(k) = 0$ the polarization vector $e^l(k)$ is parameterized in terms of the two-dimensional vector $e^l_\perp$

$$e^l = e^l_\perp - \frac{k_\perp e^l_\perp}{kp_A} p_A \, , \tag{33}$$

and satisfies the Lorentz condition $k e^l = 0$. The matrix element of the reggeon-reggeon-particle vertex Γ takes an especially simple form [9]

$$\Gamma^1_{2,1} = Ce^* + C^* e, \; C = \frac{q_1^* q_2}{k_1^*}, \tag{34}$$

if we introduce the complex components

$$e = e_x + ie_y \, , \; e^* = e_x - ie_y \, ; \; k = k_x + ik_y \, , \; k^* = k_x - ik_y \tag{35}$$

for transverse vectors $\overrightarrow{e^l_\perp}, \overrightarrow{k_\perp}$. This complex representation was used in [9] to construct the effective scalar field theory for multi-Regge processes.

Using the explicit expressions for production amplitudes in the multi-Regge kinematics one can calculate the imaginary part of the elastic scattering amplitude with the vacuum quantum numbers in the crossing channel. Due to the factorized form of the production amplitudes one can write down the Bethe–Salpeter equation for the vacuum t-channel partial wave describing the pomeron as a compound state of two reggeized gluons [2]. The contribution to its integral kernel from the real gluons is proportional to the product of the effective vertices calculated in the light cone gauge (34):

$$C(p_1, p_{1'})\, C^*(p_2, p_{2'}) + \text{h.c.} = \frac{p_1^* p_2 \, p_{1'} p_{2'}^*}{| \, k \, |^2} + \text{h.c.} \, , \tag{36}$$

where p_1, p_2 and $p_{1'}, p_{2'}$ are the corresponding complex transverse components of initial and final momenta in the t-channel ($q = p_1 + p_2 = p_{1'} + p_{2'}$). In turn, the contribution related with virtual corrections to the production amplitudes is proportional to the sum of the Regge trajectories of two gluons:

$$\omega(-\overrightarrow{p_1}^2) + \omega(-\overrightarrow{p_2}^2) \sim \ln | \, p_1 \, |^2 + \ln | \, p_2 \, |^2 + c \, , \tag{37}$$

where the constant c contains the infraredly divergent terms which are cancelled with the analogous terms from the real contribution after its integration in k. The final homogeneous equation for the t-channel partial wave $f_\omega(k, q-k)$ introduced above takes the form [10]

$$E\Psi = H_{12}\Psi,\ E = -\frac{8\omega\pi^2}{g^2}\,. \tag{38}$$

Here the "Hamiltonian" H_{12} is [2, 10]

$$H_{12} = \ln \mid p_1 \mid^2 + \ln \mid p_2 \mid^2 + \frac{1}{\mid p_1 \mid^2 \mid p_2 \mid^2} \times (p_1^* p_2 \ln \mid \rho_{12} \mid^2 p_1 p_2^* + h.c.) - 4\psi(1)\,, \tag{39}$$

where $\psi(x) = \frac{d}{dx} \ln \Gamma(x)$ and $\Gamma(x)$ is the Euler Γ-function. We introduced here the complex components $\rho_k = x_k + iy_k$ for the impact parameters canonically conjugated to the momenta $p_k = i\frac{\partial}{\partial \rho_k}$ $(\rho_{ik} = \rho_i - \rho_k)$. The Hamiltonian H has the property of the holomorphic separability [10]:

$$H_{12} = h_{12} + h_{12}^*\,,\quad E = \varepsilon + \widetilde{\varepsilon}\,, \tag{40}$$

where ε and $\widetilde{\varepsilon}$ are the energies correspondingly in the holomorphic and anti-holomorphic subspaces:

$$\varepsilon\psi(\rho_1, \rho_2) = h_{12}\,\psi(\rho_1, \rho_2),\ \widetilde{\varepsilon}\widetilde{\psi}(\rho_1^*, \rho_2^*) = h_{12}^*\widetilde{\psi}(\rho_1^*, \rho_2^*),\ \Psi(\overrightarrow{\rho_1}, \overrightarrow{\rho_2}) = \psi\,\widetilde{\psi}\,. \tag{41}$$

The holomorphic hamiltonian is [10]

$$h_{12} = \frac{1}{p_1} \ln\,(\rho_{12})\, p_1 + \frac{1}{p_2} \ln\,(\rho_{12})\, p_2 + \ln(p_1 p_2) - 2\psi(1)\,. \tag{42}$$

The solutions of the homogeneous BFKL equation belong to irreducible unitary representations of the Möbius group. The generators of this group for an arbitrary number n of particles are

$$M^z = \sum_{k=1}^{n} \rho_k \partial_k\,,\ M^- = \sum_{k=1}^{n} \partial_k\,,\ M^+ = -\sum_{k=1}^{n} \rho_k^2 \partial_k\,. \tag{43}$$

Its Casimir operator is

$$M^2 = (M^z)^2 - \frac{1}{2}(M^+M^- + M^-M^+) = -\sum_{r<s} \rho_{rs}^2 \partial_r \partial_s\,. \tag{44}$$

The holomorphic factor ψ_m is an eigenfunction of the corresponding Casimir operator:

$$M^2\psi_m = m(m-1)\psi_m\,. \tag{45}$$

Simultaneously it is an eigenfunction of the BFKL equation in the holomorphic subspace:

$$h\psi_m = \varepsilon\psi_m\,,\;\; \varepsilon = \psi(m) + \psi(1-m) - 2\psi(1)\,. \tag{46}$$

The eigenvalues of the second Casimir operator M^{2*} are expressed through the conformal weight $\widetilde{m} = \frac{1}{2} + i\nu - \frac{n}{2}$.

The simple method to unitarize the scattering amplitudes obtained in LLA is related with the solution of the BKP equation [6] for compound states of n reggeized gluons:

$$E\Psi = \sum_{i<k} H_{ik}\Psi\,. \tag{47}$$

Its eigenvalue E is proportional to the position $\omega = j-1$ of the singularity of the t-channel partial wave:

$$E = -\frac{8\pi^2}{g^2N_c}\omega\,, \tag{48}$$

The simplest non-trivial example of the BKP equations is the equation for the Odderon which is a compound state of three reggeized gluons [10].

In the multi-colour QCD ($N_c \to \infty$) according to 't Hooft only planar diagrams in the colour space are important. Because the colour structure of the eigen function at large N_c is unique, the total hamiltonian H can be written as a sum of the mutually commuting holomorphic and anti-holomorphic operators [10]:

$$H = \frac{1}{2}(h + h^*)\,,\;\; [h, h^*] = 0\,, \tag{49}$$

where $\frac{1}{2}$ is a colour factor appearing for each pair of the neighboring gluons in the octet state

$$h = \sum_{i=1}^{n} h_{i,i+1}\,. \tag{50}$$

Thus, in this case the solution of the Schrödinger equation has the property of the holomorphic factorization:

$$\Psi = \sum c_k \psi_k(\rho_1, ...\rho_n)\,\widetilde{\psi}_k(\rho_1^*, ...\rho_n^*)\,. \tag{51}$$

where ψ and $\widetilde{\psi}$ are correspondingly the analytic and anti-analytic functions of their arguments and the sum is performed over the degenerate solutions of the Schrödinger equations in the homomorphic and anti-holomorphic subspaces:

$$\varepsilon\,\psi = h\,\psi,\ \varepsilon^*\,\psi^* = h^*\,\psi^*,\ E = \tfrac{1}{2}(\varepsilon + \varepsilon^*)\,. \tag{52}$$

These equations have nontrivial integrals of motion [10]:

$$t(\theta) = tr\,T(\theta)\,,\ \ [t(u), t(v)] = [t(\theta), h] = 0\,, \tag{53}$$

where θ is the spectral parameter of the transfer matrix $t(\theta)$. The monodromy matrix $T(\theta)$ is constructed from the product of the L-operators

$$T(\theta) = L_1(\theta)L_2(\theta)...L_n(\theta) \tag{54}$$

expressed in terms of the Möbius group generators:

$$L_k(\theta) = \begin{pmatrix} \theta + i\rho_k\partial_k & i\partial_k \\ -i\rho_k^2\partial_k & \theta - i\rho_k\partial_k \end{pmatrix}\,. \tag{55}$$

Thus, the solution of the Schrödinger equation is reduced to a pure algebraic problem of finding the representation of the Yang–Baxter commutation relation [11]:

$$T_{i_1 i'_1}(u)T_{i_2 i'_2}(v)(v - u + iP_{12}) = (v - u + iP_{12})T_{i_2 i'_2}(v)T_{i_1 i'_1}(u)\,, \tag{56}$$

where the operator P_{12} in its left and right hand sides transmutes correspondingly the right and the left indices of the matrices $T(u)$ and $T(v)$. Moreover, Hamiltonian (50) for the Schrödinger equation (52) coincides with the Hamiltonian for a completely integrable Heisenberg model with the spins belonging to an infinite dimensional representation of the non-compact Möbius group and all physical quantities can be expressed in terms of the Baxter function $Q(\lambda)$ satisfying the equation [11]:

$$t(\lambda)Q(\lambda) = (\lambda + i)^n Q(\lambda + i) + (\lambda - i)^n Q(\lambda - i)\,, \tag{57}$$

where $t(\lambda)$ is an eigenvalue of the transfer matrix. The solution of the Baxter equation is known for $n = 2$. In a general case $n > 2$ one can present it as a linear combination of the solutions for $n = 2$ and obtain a recurrence relation for their coefficients. But up to now the explicit solution was not obtained even for the case of the Odderon in QCD.

4. Effective action for small-x physics in QCD

At high energies the rapidity $y = \ln \frac{k^+}{k^-}$ constructed from the light-cone components $k^\pm = k^\alpha n^\pm_\alpha$ of the particle momenta is similar to the time in quantum mechanics. The corresponding Hamiltonian is determined by the interaction of gluons and quarks with a nearly equal rapidity. The gauge-invariant effective action S_{eff} local in a rapidity interval $(y_0 - \eta, y_0 + \eta)$ was constructed recently [12] and includes apart from the usual Yang–Mills action also the interaction terms:

$$S_{\text{eff}}(v, A_\pm) = -\int d^4x \; \text{tr}\left[\tfrac{1}{2}G^2_{\mu\nu}(v) + (A_-(v) - A_-)j^{\text{reg}}_+ + (A_+(v) - A_+)j^{\text{reg}}_-\right], \tag{58}$$

where the anti-hermitian SU(N_c) matrices v_σ and $A_\pm$ describe correspondingly the usual and reggeized gluons. Because the action is local in the rapidity space, we omit temporally y as an additional argument of these fields. The reggeon current $j^{\text{reg}}_\pm$ depends on $A_\pm$ in a very simple way:

$$j^{\text{reg}}_\pm = \partial^2_\sigma A_\pm , \tag{59}$$

which guarantees, that the interaction disappears on the mass shell $k^2 = 0$.

The fields $A_\pm$ are invariant

$$\delta A_\pm = 0 \tag{60}$$

under the infinitesimal gauge transformation

$$\delta v_\sigma = [D_\sigma, \chi] , \tag{61}$$

with the gauge parameter χ decreasing at $x \to \infty$, but they belong to the adjoint representation of the global SU(N_c) group and are transformed at constant χ as follows:

$$\delta A_\pm = g\,[A_\pm, \chi] . \tag{62}$$

As usual,

$$G_{\mu\nu}(v) = \tfrac{1}{g}\,[D_\mu, D_\nu] = \partial_\mu v_\nu - \partial_\nu v_\mu + g\,[v_\mu, v_\nu] , \; D_\sigma = \partial_\sigma + g\,v_\sigma . \tag{63}$$

The fields $A_\pm$ obey the additional kinematical constraints

$$\partial_+ A_- = 0 , \; \partial_- A_+ = 0 , \tag{64}$$

meaning that the Sudakov components α, β of the reggeon momentum $q = \alpha p_B + \beta p_A + k_\perp$ are negligible small in comparison with the corresponding big components α_k, β_{k-1} of the neighboring particle momenta. Such simplification takes place at the quasi-multi-Regge kinematics where the gluons in the final and intermediate states are separated in the clusters. The invariant mass of each cluster is restricted from above by a value proportional to $\exp(\eta)$ and the neighboring clusters significantly differ in their rapidities: $y_{k-1} - y_k \gg \eta$. Further, the Sudakov components α_k, β_k of their total momenta are strongly ordered: $\alpha_k \gg \alpha_{k-1}$, $\beta_k \ll \beta_{k-1}$ and the transfer momenta $k_\perp$ are restricted. The effective action describes the self-interaction of real and virtual particles inside each cluster and their coupling with neighboring reggeized gluons.

The composite reggeon field $A_\pm(v)$ is given below

$$A_\pm(v) = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + g^2 v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm - \ldots, \tag{65}$$

and can be written in the explicit form

$$A_\pm(v) = v_\pm D_\pm^{-1} \partial_\pm = -\frac{1}{g}\partial_\pm U(v_\pm)\,,\; U(v_\pm) = P\exp(-\frac{g}{2}\int_{-\infty}^{x^\pm} d\,x'^\pm v_\pm(x'))\,, \tag{66}$$

where the integral operator $D_\pm^{-1}\partial_\pm$ is implied to act at an unit constant matrix from the left hand side and the symbol P for the Wilson exponent means the ordering of the fields v in the matrix product in accordance with the increasing of their arguments $x'^\pm$. Because the interaction terms in the momentum space contain the factor $t = q^2$ killing the pole in the neighboring gluon propagator the corresponding scattering amplitudes does not have simultaneous singularities in the overlapping channels t and s. The interaction terms contain contributions of the Feynman diagrams, in which the gluons in the given rapidity interval $(y_0 - \eta, y_0 + \eta)$ are coherently emitted by the neighboring particles with essentially different rapidities.

The interaction terms of the action are gauge invariant due to the following relations

$$\left[D_\pm, j_\mp^{\mathbf{ind}}\right] = 0\,,\; j_\mp^{\mathbf{ind}} = \frac{\partial}{\partial v_\pm} tr(A_\pm(v)\,\partial_\sigma^2 A_\mp)\,, \tag{67}$$

where $j_\sigma^{\mathbf{ind}}$ is the induced current. Note, that for the particles belonging to the same cluster the parameter η plays a part of the ultraviolet cut-off in their relative rapidity.

One can verify, that the physical results do not depend on η due to cancellations between the integrals over the invariant masses of the produced clusters of particles and the integrals over their relative rapidities (for which η plays a part of the infrared cut-off). This criterion is very important for the self-consistency of the effective action.

The effective action $S_{\mathbf{eff}}$ has a nontrivial stationary point $v = \bar{v}$ satisfying the following Euler-Lagrange equations [12]:

$$[D_\sigma, G_{\sigma\pm}] = j_{\pm}^{ind}, \ \left[D_\sigma, G_{\sigma\rho}^{\perp}\right] = 0\,, \tag{68}$$

where the induced current $j_\sigma^{\mathbf{ind}}$ equals

$$j_{\pm}^{\mathbf{ind}} = \frac{1}{D_\mp}\partial_\mp\,(\partial_{\perp\sigma}^2 A_\pm)\,\partial_\mp\frac{1}{D_\mp}\,,\ j_{\perp\sigma}^{\mathbf{ind}} = 0\,, \tag{69}$$

and due to (9) satisfies the covariant conservation law:

$$\left[D_\sigma, j_\sigma^{\mathbf{ind}}\right] = 0\,. \tag{70}$$

We can construct a perturbative solution $v = \overline{v}$ of the classical equations for example in the Landau gauge. By inserting it in $S_{\mathbf{eff}}$ one can write the reggeon action in the tree approximation as follows:

$$S_{\mathbf{eff}}^{\mathbf{tree}} = -\int d^4x\,\mathrm{tr}\,\left(s_2 + g\ s_3 + g^2\ s_4 + ...\right)\,, \tag{71}$$

where

$$s_2 = \partial_\sigma^\perp A_+\ \partial_\sigma^\perp A_-\,, \tag{72}$$

$$s_3 = -\,(\partial_\sigma^2 A_-)\,A_+\partial_+^{-1}A_+ - (\partial_\sigma^2 A_+)\,A_-\partial_-^{-1}A_-\,, \tag{73}$$

$$\begin{aligned} s_4 = &-\,(\partial_\mu f_\nu^1)^2 - \tfrac{1}{4}\,[A_+, A_-]^2 \\ &+ (\partial_\sigma^2 A_-)\,A_+\partial_+^{-1}A_+\partial_+^{-1}A_+ + (\partial_\sigma^2 A_+)\,A_-\partial_-^{-1}A_-\partial_-^{-1}A_-\,, \end{aligned} \tag{74}$$

where

$$\partial_\alpha^2 f_\pm^1 = [\partial_\alpha^2, \partial_\mp^{-1}A_\mp] - \tfrac{1}{2}\,[A_\mp, \partial_\pm A_\pm]\,.$$

In a general case to cancel the infrared divergencies one should take into account apart from the classical contribution to the corresponding transition vertices also the contributions from quantum fluctuations near classical solutions. For example in LLA they are responsible for the gluon reggeization.

One can use the simple parametrization of the initial field $v_\pm$:

$$v_\pm = V_\pm + A_\pm \,. \tag{75}$$

In this case one have also $\overline{V}_\pm = 0$ at $g = 0$ in the Landau gauge $\partial_\sigma v_\sigma = 0$ and the homogeneous polynomials L^i of fields $A_\pm$ appearing in the expansion of $S_{\rm eff}$:

$$S_{\rm eff}(V, A_\pm) = -\int d^4x \; {\rm tr} \sum_{i=0}^{\infty} L^i \,, \tag{76}$$

agree with the Steinman relations. The terms L^i do not have simple gauge properties but the corresponding scattering amplitudes are invariant under the gauge transformation after using equations of motion.

In the above expansion L^i describes the interaction of physical gluons with i reggeized gluons. The corresponding Feynman vertices contain the usual Yang–Mills vertices and the induced nonlocal terms. Below we construct the gluon production amplitudes in the quasi-multi-Regge kinematics using these effective vertices. One can find in the framework of this approach also the perturbative expansion of the reggeon action $S_{\rm regg}$ defined as follows

$$\exp(-iS_{\rm regg}(A_\pm)) = \int DV \; \exp(-iS_{\rm eff}) \,, \tag{77}$$

which depends on the reggeon fields $A_\pm$.

The subsequent functional integration over $A_\pm$ corresponds to the solution of the reggeon field theory acting in the two-dimensional impact parameter subspace with the time coinciding with the rapidity. It is important, that in the above approach the t-channel dynamics of the reggeon interactions turns out to be in the agreement with the s-channel unitarity of the S-matrix in the initial Yang–Mills model. In the Hamiltonian formulation of this reggeon calculus the wave function will contain the components with an arbitrary number of reggeized gluons. Nevertheless, one can hope that at least some of the remarkable properties of the BFKL equation will remain in the general case of the non-conserving number of reggeized gluons.

Note, that to build the effective action for the multi-Regge kinematics one should take into account only two first terms of the perturbative expansion of L:

$$\begin{aligned} L_{mR} =& \tfrac{1}{2}\,(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 + \partial_\sigma^\perp A_+ \partial_\sigma^\perp A_- + g\, b_3(A_+, A_-, V)\,, \\ b_3 =& -\tfrac{1}{2}\, A_+ \partial_+^{-1} A_+ \partial_{\perp\sigma}^2 A_- - \tfrac{1}{2}\, A_- \partial_-^{-1} A_- \partial_{\perp\sigma}^2 A_+ + F_{+-}\;[A_-, A_+] \\ & - \left(\partial_+^{-1}\partial_-^{-1} F_{+-}\right)[\partial_\sigma A_-, \partial_\sigma A_+] + \left(\partial_-^{-1} F_{-\sigma}\right)[A_-, \partial_\sigma A_+] \\ & + \left(\partial_+^{-1} F_{+\sigma}\right)[A_+, \partial_\sigma A_-] - A_+\left[F_{-\sigma}, \partial_-^{-1} F_{-\sigma}\right] \\ & - A_-\left[F_{+\sigma}, \partial_+^{-1} F_{+\sigma}\right]\,, \end{aligned} \tag{78}$$

where we introduced the abelian strength tensor:

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu , \tag{79}$$

and omitted in the last line some terms containing the factors $\partial_\sigma V_\sigma$ and $\partial_\sigma^2 V_\mu$ and vanishing for the real gluons. The Feynman vertices of this theory coincide on mass shell with the effective reggeon-particle vertices of the leading logarithmic approximation.

5. Next-to-leading corrections to the BFKL equation

The imaginary part of the elastic scattering amplitude calculated with the use of the s-channel unitarity condition through the squared production amplitude in a quasi-multi-Regge kinematics contains the infrared divergences at small $k_{i\perp}^2$ and κ. To avoid such divergencies the dimensional regularization is used in the gauge theories. It is important, that in the D-dimensional space the gluon has $D-2$ degrees of freedom.

The generalized BFKL equation for the virtual gluon cross-section can be written in the integral form as follows

$$\sigma(\overrightarrow{q_1}, q_1^+) = \sigma_0(\overrightarrow{q_1}, q_1^+) + \int \frac{dq_2^+}{q_2^+} \mu^{4-D} \int d^{D-2}\overrightarrow{q_2}\, K_\delta(\overrightarrow{q_1}, \overrightarrow{q_2}) \sigma(\overrightarrow{q_2}, q_2^+) , \tag{80}$$

where the integration region for the longitudinal momentum q_2^+ is restricted from above by the value proportional to q_1^+:

$$q_2^+ < \delta\, q_1^+ . \tag{81}$$

The intermediate infinitesimal parameter $\delta > 0$ is introduced instead of the above parameter η to arrange the particles in the groups with strongly different rapidities. The integral kernel $K_\delta(\overrightarrow{q_1}, \overrightarrow{q_2})$ takes into account the interaction among the particles inside each group where δ plays role of the ultraviolet cut-off in their relative rapidities. The kernel K_δ can be calculated in the perturbation theory:

$$K_\delta(\overrightarrow{q_1}, \overrightarrow{q_2}) = \sum_{r=1}^{\infty} \left(\frac{g^2}{2(2\pi)^{D-1}} \right)^r K_\delta^{(r)}(\overrightarrow{q_1}, \overrightarrow{q_2}) . \tag{82}$$

The next-to-leading term in K_δ related with the two gluon production is given below [13]

$$K^{(2)}_{\mathbf{gluons}} = \frac{16\, N_c^2}{2\overrightarrow{q_1}^2\, \overrightarrow{q_2}^2} \int_\delta^{1-\delta} \frac{dx}{x(1-x)} \int \frac{d^{D-2}\overrightarrow{k_1}}{\mu^{D-4}} R . \tag{83}$$

For the physical value $D = 4$ of the space-time dimension one can express R as the sum of two terms:

$$R = R(+-) + R(++)\,, \tag{84}$$

where $R(+-)$ and $R(++)$ are the contributions from the production of the gluons with the same and opposite helicity correspondingly.

$$R(+-) =$$
$$\frac{1}{2}\left(\mid c^{+-}(k_1,k_2)\mid^2 + \mid c^{+-}(k_2,k_1)\mid^2 + Re\, c^{+-}(k_1,k_2)\, c^{-+}(k_2,k_1)\frac{k_1^*\, k_2}{k_1\, k_2^*}\right),$$
$$R(++) =$$
$$\frac{1}{2}\left(\mid c^{++}(k_1,k_2)\mid^2 + \mid c^{++}(k_2,k_1)\mid^2 + Re\, c^{++}(k_1,k_2)\, c^{++}(k_2,k_1)\frac{k_1^*\, k_2^*}{k_1\, k_2}\right). \tag{85}$$

Here the complex functions $c^{+-}(k_1, k_2)$ and $c^{++}(k_1, k_2)$ describing the production of two gluons with the same and opposite helicity correspondingly are given below:

$$c^{+-}(k_1,k_2) \equiv c^{11} + ic^{21} - ic^{12} + c^{22} = \overline{c^{-+}}(k_1,k_2) = -x\,\frac{q_2\, q_1^*}{(k_1 - x\,\Delta)\, k_1^*}\,,$$
$$c^{++}(k_1,k_2) = c^{11} + ic^{21} + ic^{12} - c^{22} = \overline{c^{--}}(k_1,k_2)$$
$$= -\frac{x\,(Q_1)^2}{\left((\vec{k_1} - x\vec{q_1})^2 + x(1-x)\vec{q_1}^{\,2}\right)} + \frac{x\,\vec{q_1}^{\,2}\,(k_2)^2}{\vec{\Delta}^2\left((\vec{k_1} - x\vec{\Delta})^2 + x(1-x)\vec{\Delta}^2\right)}$$
$$-\frac{x(1-x)q_1k_2q_2^*}{\Delta^*\,(k_1 - x\Delta)k_1^*} - \frac{xq_1^*k_1q_2}{\vec{\Delta}^2\,(k_1^* - x\Delta^*)} + \frac{xq_2^*Q_1}{\Delta^*\,k_1^*}\,. \tag{86}$$

These expressions were obtained independently also in Ref. [14].

All divergencies were extracted in an explicit form from the gluon and quark production using the dimensional regularization [13]. The infrared divergencies should cancel with the corresponding virtual contributions. The one-loop corrections to the particle-particle-reggeon vertex and two-loop corrections to the gluon Regge trajectory were calculated earlier [15]. The total one-loop correction to the BFKL equation will be calculated soon in an explicit form in terms of the dilogarithm integrals [13].

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