BFKL POMERON IN THE IMPACT PARAMETER PICTURE*

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The dipole picture of high-energy deep inelastic collisions is described. Nucleon structure function and diffractive dissociation in the triple-pomeron limit are discussed in some detail.

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1. The subject of the present note is the so-called "small-x physics" which is being vigorously studied at HERA. More precisely, I shall discuss deep inelastic lepton scattering in the Lipatov limit: $\nu \to \infty$ and Q^2 fixed at a large value. Lipatov was the first one to realize that in QCD this limit has two very interesting features. On the one hand, since we are in the region of large Q^2, the scattering amplitudes are, at least in principle, calculable by perturbative methods. On the other hand, the standard features of the high-energy scattering are expected to show up. In particular, one may hope to see the regge behaviour which thus becomes calculable from "the first principles" *i.e.* from Quantum Chromodynamics. In a series of papers, Lipatov and his collaborators showed that this is indeed the case [1].

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One of the important results from the work of Lipatov *et al.* is that the exchange of multi-gluon ladders between two colliding point-like particles gives the elastic amplitude which increases as a power of the center-of-mass energy of the collision. The power corresponds to the "pomeron intercept"

$$\alpha_{\mathcal{P}} = 1 + \Delta_{\mathcal{P}} = 1 + \frac{4\alpha N_c}{\pi} \log 2 \tag{1}$$

which is larger than 1 and thus gives a total cross-section increasing as a power ($\Delta_{\mathcal{P}}$) of the energy. Although such a behaviour contradicts unitarity (and thus corrections are necessary to restore the Froissart bound), the observation of the power law increase of the structure function F_2 at small x_{Bj} [16, 17] indicates that in the kinematic range of the present experiments the "Born" formula of Lipatov *et al.* may be sufficient to describe the data. This is surely far from trivial and the question stirred a hot debate (see *e.g.* [2]).

Recently, a new approach to the problem of high-energy scattering at large Q^2 was developed independently by Mueller [5–7] and by Nikolaev [8]. They observed two points:

(i) For collisions at high energy it is advantageous to work in impact parameter space rather than in transverse momentum space. The reason is clear: at high collision energies, impact parameter is a good quantum number, conserved in the collision process. This fact significantly simplifies the form of the scattering amplitudes, as was shown already long time ago, in a somewhat different context [9, 10].

(ii) Since we are always dealing with the collisions of composite colourless objects, it seems reasonable to describe them in terms of the simplest colourless structures, *i.e.* the $q\bar{q}$ colour dipoles, rather than in terms of elementary (but coloured) objects, *i.e.* quarks and gluons. This idea is not new [11], but it becomes particularly natural at high energies: because of Lorentz contraction, the longitudinal positions of the members of the $q\bar{q}$ pair are close to each other (provided the ratio of their light-cone momenta remains finite). Therefore, such a pair can indeed be imagined as a well-defined object. A practical advantage of this description is a substantial simplification of the colour structure of the system, in particular of the colour flow between its different parts. The disadvantage is that the equivalence between the two descriptions can only be proven in the large N_c limit and it is not clear if this restriction can be avoided.

2. In the following we shall discuss the formulation proposed by Mueller [5–7]. He follows the idea of Balitsky and Lipatov [12] and considers "onium-onium" scattering, *i.e.* scattering of two $q\bar{q}$ bound states whose transverse size is concentrated at sufficiently small values to justify application of perturbative QCD for the calculation of their internal structure. For a real $q\bar{q}$

pair such situation occurs, *e.g.* for J/ψ and Υ. But it can also be realized in case of a virtual photon of sufficiently large Q^2.

The internal structure of such an "onium", when calculated in the leading-log order of perturbative QCD, is dominated by a cascade of gluons. In Ref. [7] the impact parameter representation of this perturbative gluon cascade in terms of $q\bar{q}$ dipoles was explicitly constructed (as we already mentioned, this was possible only in the large N_c limit). Thus the two colliding "onia" can be represented by two "clouds" of the $q\bar{q}$ dipoles whose distribution in transverse size and position in the impact parameter space is explicitly calculable. It was furthermore shown [6] that the result (1) of Lipatov *et al.* for the total onium-onium cross-secton is obtained if the dipole-dipole elastic amplitude is approximated by the exchange of two gluons. At this point it is worth to emphasize another advantage of the dipole representation: the dipole-dipole cross-section is *finite* (in contrast to the gluon-gluon one). In the two-gluon-exchange approximation the total cross-section for the collision of two dipoles of the transverse sizes r_a and r_b ($r_a < r_b$) is given by

$$\sigma(r_a, r_b) = 2\pi\alpha^2 r_a^2 \left(1 + \log\left(\frac{r_b}{r_a}\right)\right) . \tag{2}$$

Note that this cross-section is *energy-independent.*

From all this work one obtains a fairly simple and intuitive picture of the scattering of two strongly interacting objects in the Lipatov (or light-cone) limit: It can be considered as a collision of two bunches of $q\bar{q}$ dipoles. Each dipole from one bunch interacts with each dipole from another bunch by exchange of two gluons. The power law increase of the cross-section with increasing energy is obtained because the number of dipoles in each bunch increases as a power of the incident energy. This is a direct consequence of the cascade nature of the dipole emission: since the length of the cascade is proportional to the available rapidity interval Y, the number of emitted dipoles is expected to be proportional to $\exp(cY)$. Explicit calculations [5] show that the constant c is equal to $\Delta_{\mathcal{P}}$ given by (1).

Using this idea, the formula for the total cross-section in onium-onium scattering is written as

$$\sigma_{\mathrm{tot}} = \int \frac{d\rho_a}{\rho_a}\frac{d\rho_b}{\rho_b} n_a(\rho_a, y_a) n_b(\rho_b, y_b) \sigma(\rho_a, \rho_b) , \tag{3}$$

where $n_a(\rho_a, y_a)$ is the density of dipoles in the incident onium (ρ_a is the transverse size of the dipole and y_a is the rapidity difference between the fastest and slowest (anti)quarks involved). Graphical representation of this formula is shown in Fig. 1.

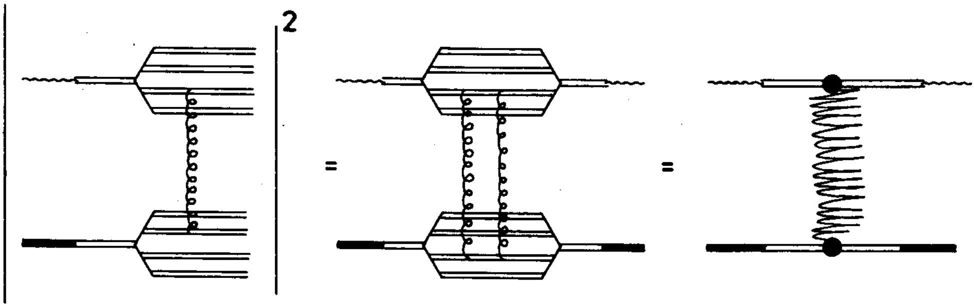

Fig. 1. Graphical representation of the formula for total cross-section.

The dipole density in an "onium" is a convolution of *(i)* the square of the (modulus of) the wave function of the onium and *(ii)* the distribution $n(r_0, r, y)$ of the $q\bar{q}$ dipoles of transverse size r emitted by the original dipole of size r_0 in a cascade of length y

$$n_a(\rho, y) = \int d^2\rho_o dz \mid \psi_a(\rho_o, z) \mid^2 n(\rho_o, \rho, y)\,, \tag{4}$$

where z is the light-cone momentum fraction carried by one of the partons forming the onium. The density $n(r_0, r, y)$ can be determined from an equation describing the evolution of the QCD cascade in the light-cone limit. The equation was discussed at length in [5–7] where also its solution was derived.

3. To apply these results to deep inelastic lepton-nucleon scattering one makes an (admittedly rather crude) assumption that the target proton can be adequately represented by an "onium"[1]. Once this is accepted, it turns out that the distribution $n_p(\rho_p, y_p)$ contains only one effective parameter r_p which is the average transverse size of the "onium" representing the target proton:

$$r_p = \int dzd^2r \mid \psi_p(z, r) \mid^2 r\,, \tag{5}$$

where ψ_p is the wave function of the "onium" in question and z is the light-cone momentum fraction carried by one of the partons forming the onium.

The wave function of the incident virtual photon (*i.e.* the probability amplitude to find a $q\bar{q}$ pair inside the virtual photon) is known since some time [9, 10]. For transverse photons, the corresponding probability distribution summed over quark and antiquark polarizations reads [8]

$$\mid \Psi_\gamma(r, z; Q^2) \mid^2 = \frac{2N_c\alpha_{em}}{(2\pi)^2} e_f^2 (z^2 + (1-z)^2)\hat{Q}^2 K_1^2(\hat{Q}r) \tag{6}$$

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[1] It appears that this assumption can be relaxed without major changes in the results, except for the overall normalization [13].

with $\hat{Q}^2 = z(1-z)Q^2$ (quark masses are neglected for simplicity). e_f^2 is the sum of the squares of the quark charges and K_1 stands for the Bessel function of the second kind. One sees from this formula that the end-points in longitudinal momentum ($z \approx 0$, $z \approx 1$) give a non-vanishing contribution. One sees also that in this region of z the transverse size of the $q\bar{q}$ pair can take arbitrarily large values. Therefore doubts were expressed if the perturbative approach can be applied in this region (and if the onium picture is adequate at all) [14, 15]. While accepting these objections, I feel that a good way to evaluate their quantitative importance is to complete the perturbative calculations and compare them with the data. The difference, then, would signal the non-perturbative contribution. It seems to me a good enough reason to pursue the perturbative calculations.

Using (3) and (6) and the formula for $n(r_0, r, y)$ derived in [5, 6] one arrives at the following expression for the total cross-secton in virtual photon-proton scattering

$$\sigma_{\mathrm{tot}} = \frac{9\pi^3}{32} N_c \alpha^2 \alpha_{em} e_f^2 x_{Bj}^{-\Delta_{\mathcal{P}}} \left(\frac{2a(x_{Bj})}{\pi}\right)^{1/2} \frac{r_p}{Q} \times \exp\left(-\frac{a(x_{Bj})}{2} \log^2\left(\frac{r_p Q}{2}\right)\right), \tag{7}$$

where

$$a(x) = \frac{\pi}{7\alpha N_c \zeta(3) \log(1/x)}. \tag{8}$$

The prediction for the proton structure function F_2 is obtained from the relation

$$F_2 = \frac{Q^2}{4\pi^2 \alpha_{em}} \sigma_{\mathrm{tot}}. \tag{9}$$

An important consequence of (7) is that the structure function does not factorize: x_{Bj} and Q^2 dependencies are interconnected. This effect is illustrated quantitatively in Fig. 2 where the "effective" pomeron intercept $\Delta_{\mathcal{P}}^{\mathrm{eff}} = -d(\log F_2)/d(\log x_{Bj})$ is plotted versus x_{Bj} for two values of Q^2. One sees that Eq. (7) predicts Q^2-dependent deviations from the simple power law. However, on the average, the value of $\Delta_{\mathcal{P}}^{\mathrm{eff}}$ is not very different from the "true" value of $\Delta_{\mathcal{P}}$ obtained from theory.

These ideas were applied and confronted with experimental data obtained by ZEUS and H1 collaborations [16, 17] by Navelet, Peschanski and Royon [18]. It was shown that the Q^2 and x_{Bj} dependence given by (7) is consistent with the data and even the normalization is not too far apart (they used a different form of normalization than that in (7), however, so

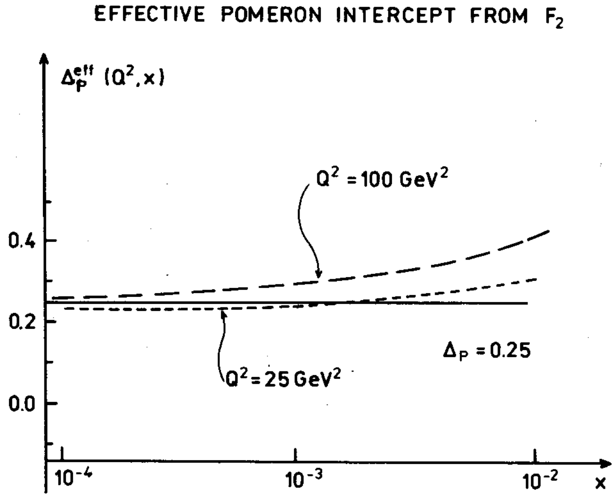

Fig. 2. Effective pomeron intercept from the structure function F_2.

this point requires more discussion[2]).

4. This unquestionable success of the model invites one to extend the investigations to other processes. Diffractive dissociation is an obvious next candidate. It was discussed already by several authors in the general framework of perturbative QCD and/or pomeron exchange [19]. Recently, together with Robi Peschanski, we have been trying to complete a calculation of diffractive production in the triple-pomeron limit started some time ago by Mueller and Patel [6]. I am now going to describe results of this work.

The graphical representation of the formula for the cross-section of diffraction dissociation is shown in Fig. 3. One sees that it consists of two terms:

(a) The "triple-pomeron" term giving the dominant contribution at large masses of the excited system. It is the sum of all dipole-dipole interactions.

(b) the "elastic" term which represents the elastic scattering of the onium on the target proton.

Mueller and Patel [6] gave an explicit formula for the "triple-pomeron" term of Fig. 3(a). When adapted to virtual photon-proton collision, it reads

$$\frac{\beta d\sigma_D}{d\beta} = \int d^2\bar{r} d\bar{z} \mid \Psi_\gamma(\bar{r},\bar{z};Q^2) \mid^2 \frac{\beta d\sigma_d(\bar{r},\beta,x_{\mathcal{P}})}{d\beta}, \tag{10}$$

[2] It may be noted at this point that the normalization of the cross-section is sensitive to the value of the dipole–dipole cross-section (2). It will change, *e.g.*, if the assumption of two-gluon-exchange is replaced by another one (*cf.* also footnote 1).

where $\beta = Q^2/(Q^2+M^2)$ and where the single diffractive cross-section in dipole-proton scattering is given by

$$\frac{\beta d\sigma_d}{d\beta} = \int \frac{dr}{r} d^2\bar{s}_1 n_p(b+\bar{s}_1, r, \xi) \int \frac{dr'}{r'} d^2\bar{s}_2 n_p(b+\bar{s}_2, r', \xi) \times \int \frac{d\bar{r}_1}{\bar{r}_1}\frac{d\bar{r}_2}{\bar{r}_2} n_2\left(\bar{r}; \bar{s}_1, \bar{r}_1; \bar{s}_2, \bar{r}_2; \frac{x_{Bj}}{\xi}, \frac{x_{\mathcal{P}}}{\xi}\right) T(r,\bar{r}_1)\, T(r',\bar{r}_2)\, d^2b. \tag{11}$$

$n_p(b, r, \xi)$ is the single dipole density in the proton at the transverse distance b from its center within the rapidity interval $y = -\log\xi$. n_2 is the double dipole density in the colliding dipole of transverse size $\bar{r}$ [6]. $\bar{s}_1, \bar{r}_1; \bar{s}_2, \bar{r}_2$ are transverse positions and sizes of the two dipoles; $\log(\xi/x_{Bj})$ is the rapidity difference between the slowest and fastest parton in the photon and $\log(x_{\mathcal{P}}/x_{Bj}) = -\log\beta$ is the rapidity range defining the mass of the diffractively excited system. Finally, $T(r,\bar{r}) = (1/2)\sigma(r,\bar{r})$ where σ is the dipole-dipole cross-section given by (2).

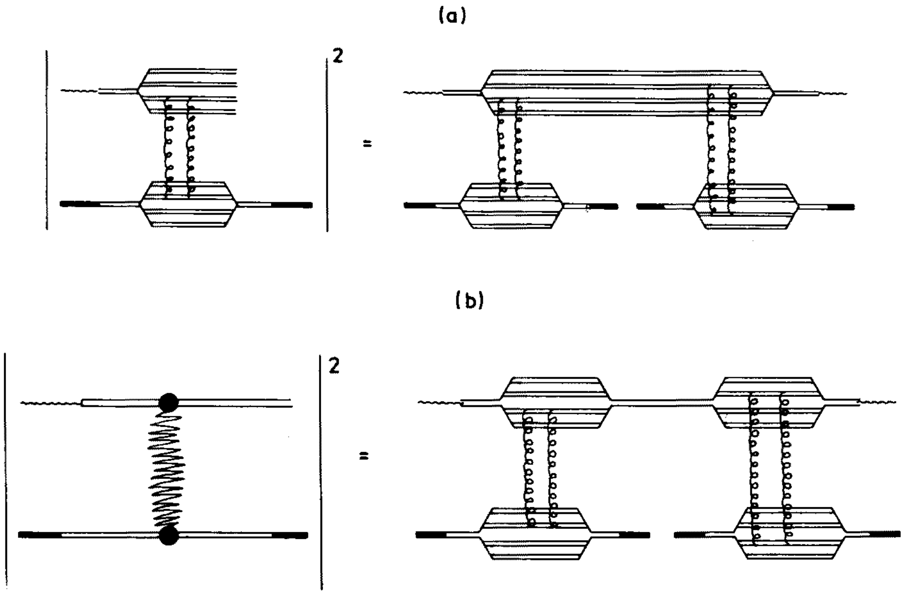

Fig. 3. Graphical representation of the formula for cross-section of diffractive dissociation.

To carry out the integrations in (11) it was necessary to find an analytic formula for n_2. This was obtained from the evolution equation derived in [6], using the technique developed in [20]. Other elements of the Eq. (11) were already given in [6, 7]. With this input it turned out possible to derive an explicit formula for $\beta d\sigma/d\beta$. Since the results are already published [21], I shall only quote the final result and discuss its physical content.

We calculated the "diffractive structure function" of the proton following from (11), *i.e.* integrated over the momentum transfer to the target. It is related to $\beta d\sigma_D/d\beta$ by

$$F_2^{D(3)}(Q^2, x_{\mathcal{P}}, \beta) = \frac{Q^2}{4\pi^2\alpha_{em}} x_{\mathcal{P}}^{-1} \frac{\beta d\sigma_D}{d\beta}, \tag{12}$$

where σ_D stands for the cross-section for diffractive dissociation of the virtual photon. In the "triple-pomeron limit" $\beta \to 0$, $F_2^{D(3)}$ can be written in a factorized form

$$F_2^{D(3)} = \Phi_{\mathcal{P}}(x_{\mathcal{P}}) F_{\mathcal{P}}(Q^2, \beta) \tag{13}$$

with

$$\Phi_{\mathcal{P}} = x_{\mathcal{P}}^{-1-2\Delta_{\mathcal{P}}} \left(\frac{2a(x_{\mathcal{P}})}{\pi}\right)^3 \tag{14}$$

and

$$F_{\mathcal{P}}(Q^2, \beta) = \frac{9G\pi}{8} e_f^2 \alpha^5 N_c^2 \frac{r_P Q}{2} \beta^{-\Delta_{\mathcal{P}}} \left(\frac{2a(\beta)}{\pi}\right)^{1/2} \times \exp\left[-\frac{a(\beta)}{2} \log^2\left(\frac{r_P Q}{2}\right)\right]. \tag{15}$$

where $G = 0.915\ldots$ is Catalan's constant[3]. The first factor in (13) is readily identified (up to a multiplicative constant) as the "Pomeron flux factor" inside the proton. The second factor is the "Pomeron structure function".

The first observation one can make is that the pomeron flux factor given by (14) differs substantially from the one usually assumed (*i.e.* from a simple power of $x_{\mathcal{P}}$) by a logarithmic factor $(2a(x_{\mathcal{P}})/\pi)^3$. This has an important consequence for the phenomenology of diffraction. Indeed, Eq. (14) predicts that the $x_{\mathcal{P}}$ dependence of the diffractive structure function should differ from the simple power law (at least for small values of β). It will be of course very interesting to test this prediction with the coming data. To quantify this result, we show in the Fig. 4 the "effective pomeron intercept" calculated from (14)

$$\Delta_{\mathcal{P}}^{\text{eff}} = -\tfrac{1}{2}\left(1 + \frac{d(\log F_2^D)}{d(\log x_{\mathcal{P}})}\right) = \Delta_{\mathcal{P}} - \frac{3}{2\log(1/x_{\mathcal{P}})}. \tag{16}$$

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[3] Note the extra factor 8π which was missing in the Eq. (11) of [21].

One sees that not only $\Delta_{\mathcal{P}}^{\text{eff}}$ is not independent of $x_{\mathcal{P}}$ (as it would be the case for a simple power law) but, moreover, it is substantially smaller than its "true" value (*i.e.* $\Delta_{\mathcal{P}}$) in the whole range of $x_{\mathcal{P}}$ presently available. When confronted with the results shown in Fig. 2, we thus conclude that (a) the "effective" pomeron intercept determined from the power-law fit to the diffractive structure function is expected to be substantially smaller than the one determined from the structure function F_2, and (b) that its value should depend on the region of the fit. This seems to be indeed observed in the HERA data [22, 23].

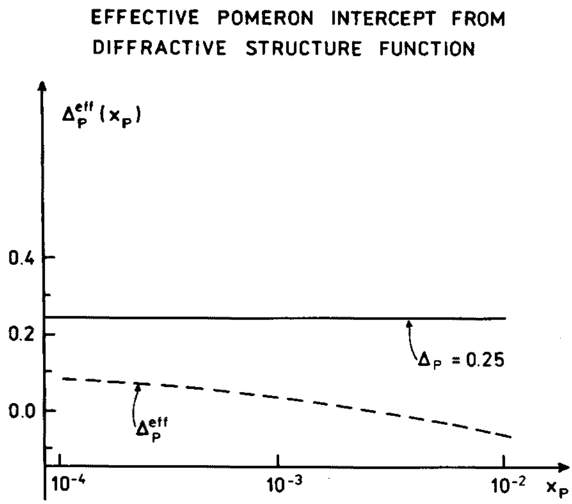

Fig. 4. Effective pomeron intercept from the diffractive structure function.

Let us turn now to the pomeron structure function (15). When compared with Eqs (7), (9) one sees that the form of $F_{\mathcal{P}}$ is identical to that of F_2 (with the obvious substitution $\beta \leftrightarrow x_{Bj}$). This result may be naturally understood in the Ingelman-Schlein picture of the pomeron [24], as can be seen from the following argument. In the Ingelman-Schlein picture, the pomeron is a part of the proton structure which is responsible for diffractive interactions. In the leading-log approximation we are considering here, the dipole (or gluon) content of the proton is described by a process of scale-invariant cascade [5]. This implies a fractal structure of the system. Since any part of a fractal has the same structure as the whole fractal, it is natural that we recover the same dipole distribution in the Pomeron as in the parent proton[4]. It would clearly be of great interest to verify this prediction with the future data.

5. As should be clear from what was already said, more work is needed to obtain a complete perturbative QCD description of diffractive dissocia-

[4] This explanation was suggested to me by J. Turnau.

tion in deep inelastic scattering. Let me conclude this report by listing the most obvious points.

(a) The limit ($\beta \to 0$) which we have discussed, covers only a very small region of the available phase-space. Moreover, it is not easy to access experimentally, particularly if one wants at the same time to keep x_{Bj} and $x_{\mathcal{P}}$ reasonably small. Therefore the extension of the formulae (13)–(15) to finite β is necessary although, presumably, the triple pomeron term is not very important in this region. This seems feasible, as shown in [21].

(b) The "elastic" contribution shown in Fig. 3(b) is expected to dominate at $\beta > 0$ [8, 26]. Therefore precise evaluation of this term is crucial if one wants to describe diffraction dissociation at finite β. We are presently working on this problem within the dipole approach [25].

(c) The momentum transfer dependence of the process is of great interest, because it may give information on the effective slope of the pomeron trajectory.

(d) Double diffraction dissociation must be included for precise description of the existing data (present detectors cannot identify the proton in the final state).

Finally, let me add that, as is well known since long time [27, 28], diffractive dissociation of the virtual photon plays an important role in nuclear shadowing effects. To exploit fully this relation, however, one needs to know $d\sigma_D/d\beta$ in the full range of β at a fixed x_{Bj}. So it is necessary to extend the calculation of diffractive dissociation to the region of fairly large $x_{\mathcal{P}}$. It is not clear if this is feasible in the present framework.

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