THE ODDERON AND INVARIANTS OF ELLIPTIC CURVES[*,†]

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In this talk we present some links of the theory of the odderon with elliptic curves. These results were obtained in an earlier work [19]. The natural degrees of freedom of the odderon turn out to coincide with conformal invariants of elliptic curves with a fixed 'sign'. This leads to a formulation of the odderon which is modular invariant with respect to Γ^2 — the unique normal subgroup of $SL(2, \mathbb{Z})$ of index 2.

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1. Introduction

Recently the study of the small Bjorken x region of Deep Inelastic Scattering has attracted much attention. The behavior of the structure functions in this limit turns out to be governed by the exchange of Regge poles. The one most important for the F_2 structure function is the $C = +1$ pole with vacuum quantum numbers — the BFKL pomeron, described in the framework of perturbative QCD as the exchange of two 'reggeized' gluons. Already in the late 70's this amplitude was calculated, and the formula for the intercept of the renowned BFKL pomeron was derived (see [1, 2]). Lipatov's solution depends in a crucial way on the global conformal symmetry of the problem. The next step led to the derivation of the Bartels-Kwieciński-Praszałowicz equation describing the exchange of three reggeized gluons ([3, 4]). The corresponding $C = -1$ pole, called the odderon is thought to have influence on the F_3 structure function. Later this approach was extended to the case of an arbitrary number of reggeons in the form of the

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Generalized Leading Logarithm Approximation (GLLA) [4]. It is important to emphasize that the odderon should not be treated just as a correction to the pomeron, but that it has it's own distinct physical signature.

The theory of the odderon turned out to be much more difficult to solve than the BFKL pomeron case, and a wide variety of approaches have been attempted. Among them is the very surprising connection with exactly solvable lattice models, namely the Heisenberg XXX s=0 spin chain ([6, 5, 8, 9]). Within this framework variants of the Bethe ansatz have been tried ([8, 7, 9, 10]), quasiclassical approximation [11], but still the explicit value of the odderon intercept is unknown, apart from some variational bounds ([12]).

In this talk we will present the results obtained earlier in [19], that the theory of the odderon possesses modular invariance, well known from conformal field theory and string theory. This observation follows from an intriguing link of the odderon with invariants of elliptic curves. First we will recall the theory of the odderon, then, following Lipatov, the consequences of global $SL(2, \mathbb{C})$ invariance. The new results in [19] were the analysis of the role of cyclic symmetry in this framework and the link with modular invariance through the theory of elliptic curves.

2. The odderon

The Regge limit of QCD is defined as the kinematical region

$$s \gg -t \approx M^2 \,, \tag{1}$$

where M is the hadron mass scale, or, in the case of Deep Inelastic Scattering, as the small $x = Q^2/s$ limit. Here we sketch the equivalence between the Regge intercept of amplitudes and energy levels of a two-body Hamiltonian within the GLLA approximation.

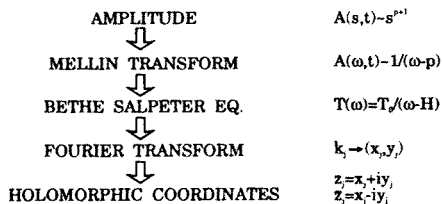| AMPLITUDE | $A(s,t) \sim s^{p+1}$ |
⇓	
MELLIN TRANSFORM	$A(\omega,t) \sim 1/(\omega-p)$
⇓	
BETHE SALPETER EQ.	$T(\omega) = T_o/(\omega-H)$
⇓	
FOURIER TRANSFORM	$k_i \rightarrow (x_i, y_i)$
⇓	
HOLOMORPHIC COORDINATES	$z_i = x_i + iy_i$
	$\bar{z}_i = x_i - iy_i$

Fig. 1. The main steps leading to the equivalence of the Regge intercept and energy levels of a certain hamiltonian.

The main steps leading to this equivalence are briefly summarized in Fig. 1. The aim is to find the Regge behavior of the amplitude $A(s,t) \sim s^{\omega_0+1}$. This amplitude is described in the framework of perturbative QCD as a sum of graphs corresponding to the exchange of N reggeized gluons with all possible gluonic interactions between them. The BFKL pomeron corresponds to graphs with $N = 2$, while the odderon is the $N = 3$ contribution (apart from higher N corrections). The Regge behavior of the amplitudes can be directly translated, in the case of DIS, into the x-dependence of the appropriate proton structure functions at low x.

In the first step the power behavior of the amplitude is translated into singularities of the Mellin transform

$$A(s,t) = is \int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{M^2}\right)^{\omega} A(\omega,t) . \tag{2}$$

Rewriting this amplitude as the convolution of "hadron" wave functions $\Phi_{A,B}$ and a kernel $T(\{k_i\}, \{k'_j\}, \omega)$ we get:

$$A(\omega,t) = \int d^2k_i \int d^2k'_j \Phi_A(\{k_i\}) T(\{k_i\}, \{k'_j\}, \omega) \Phi_B(\{k'_j\}) , \tag{3}$$

where $\{k_i\}$ and $\{k'_j\}$ are the transverse momenta of the N exchanged reggeons (in the case of odderon $N = 3$). The rest of this work deals with the singularity structure of the kernel $T(\{k_i\}, \{k'_j\}, \omega)$. The next step amounts to writing the Bethe–Salpeter equations for the kernel T:

$$\omega T(\omega) = T_0 + \mathcal{H}T(\omega) \tag{4}$$

which corresponds to the iteration of gluon interactions between the reggeons. Here T_0 is the free propagator and $\mathcal{H}$ is the operator corresponding to the insertion of single gluonic interactions between all pairs of reggeons. This equation can be formally solved:

$$T(\omega) = \frac{T_0}{\omega - \mathcal{H}} . \tag{5}$$

It is clear now that in order to find the Regge intercept it suffices to find the eigenvalues of the Hamiltonian operator $\mathcal{H}$. The last step is to go to transverse 'impact parameter' space and introduce holomorphic coordinates. After performing Fourier transformation ($k_i \longrightarrow b_i$) and using the complex notation $z_j := x_j + iy_j$, the Hamiltonian splits into a sum of a holomorphic part and an antiholomorphic part. In the large N_c limit the two commute. Due to this so-called holomorphic separability we may seek eigenfunctions of the Hamiltonian as a product of eigenfunctions of the holomorphic and antiholomorphic operators. Before we discuss the odderon case let us recall the description of the BFKL pomeron in this framework.

2.1. The BFKL pomeron

The BFKL pomeron corresponds to the exchange of 2 reggeons. The holomorphic and antiholomorphic hamiltonians are given by:

$$H(z_1, z_2) = \sum_{l=0}^{\infty} \frac{2l+1}{l(l+1) - L_{12}^2} - \frac{2}{l+1}, \tag{6}$$

$$H(\bar{z}_1, \bar{z}_2) = \sum_{l=0}^{\infty} \frac{2l+1}{l(l+1) - \bar{L}_{12}^2} - \frac{2}{l+1}, \tag{7}$$

where

$$L_{12}^2 := -z_{12}^2 \frac{d}{dz_1} \frac{d}{dz_2}, \tag{8}$$

$$\bar{L}_{12}^2 := -\frac{d}{d\bar{z}_1} \frac{d}{d\bar{z}_2} \bar{z}_{12}^2 \tag{9}$$

being the holomorphic and antiholomorphic Casimir operators of the group $SL(2, \mathbb{C})$. Although the problem seems at first glance to be quite intractable, it is in fact quite easy to solve. The crucial ingredient is the $SL(2, \mathbb{C})$ invariance of the system. This enables us to consider wavefunctions in a definite unitary representation of $SL(2, \mathbb{C})$, and simply insert the eigenvalues of the Casimir operators into (6) to obtain the energy.

The celebrated BFKL solution ($N = 2$ case) corresponds in this language to finding the maximal eigenvalue of the equations

$$H(z_1, z_2)\Psi(z_1, z_2) = E\Psi(z_1, z_2) \quad \bar{H}(\bar{z}_1, \bar{z}_2)\Psi(\bar{z}_1, \bar{z}_2) = E\Psi(\bar{z}_1, \bar{z}_2), \tag{10}$$

and has the known solution

$$E = -[\psi(m) + \psi(1-m) - 2\psi(1)] \quad \bar{E} = -[\psi(\bar{m}) + \psi(1-\bar{m}) - 2\psi(1)], \tag{11}$$

where ψ is the derivative of the logarithm of the Euler Γ function and m is a conformal weight. The maximum of (11) is achieved at $m = 1/2$ and reproduces the BFKL slope

$$\omega_0^{\mathrm{BFKL}} = \frac{2\alpha_s N_c}{4\pi}(E + \bar{E}) = \frac{\alpha_s N_c}{\pi} 4\ln 2. \tag{12}$$

2.2. The odderon

In the case of the odderon the problem looks deceptively similar. Now the (holomorphic) hamiltonian is given by the sum of three terms, each identical to the ordinary BFKL hamiltonian, namely:

$$(H(z_1, z_2) + H(z_2, z_3) + H(z_3, z_1))\Psi(z_1, z_2, z_3) = E\Psi(z_1, z_2, z_3), \quad (13)$$

where $H(z_i, z_j)$ is given by the same expression as (6). The eigenvalue E of the holomorphic hamiltonian and the corresponding eigenvalue $\bar{E}$ of the antiholomorphic one are related to the Regge intercept by the formula:

$$\omega_0 = \frac{\alpha_s N_c}{4\pi}(E + \bar{E}). \quad (14)$$

The reason why this makes the problem at least an order of magnitude more difficult is the fact that the three terms in (13) do not commute. Furthermore there is no natural small parameter in which one might try to perform a kind of perturbative expansion.

2.3. SL(2, ℂ) invariance

Since the global $SL(2, \mathbb{C})$ invariance has proved to be so powerful as to solve the BFKL pomeron, it is natural to try to use it to simplify the problem also in the case of the odderon. This analysis has been done by Lipatov [14].

The Hamiltonian H is invariant with respect to the action of $SL(2, \mathbb{C})$ on holomorphic functions given by:

$$(g \cdot \Psi)(z_1, z_2, z_3) = \Psi\left(\frac{az_1 + b}{cz_1 + d}, \frac{az_2 + b}{cz_2 + d}, \frac{az_3 + b}{cz_3 + d}\right)$$

$$\text{for } g = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \in SL(2, \mathbb{C}). \quad (15)$$

Therefore, it commutes with the holomorphic Casimir operator for this representation:

$$\hat{q}_2 := -z_{12}^2 \frac{d}{dz_1}\frac{d}{dz_2} - z_{23}^2 \frac{d}{dz_2}\frac{d}{dz_3} - z_{31}^2 \frac{d}{dz_3}\frac{d}{dz_1}, \quad (16)$$

where $z_{ij} = z_i - z_j$. This enables us to consider functions transforming under the unitary representations of $SL(2, \mathbb{C})$ labelled by $n \in \mathbb{N}$ and $\nu \in \mathbb{R}$. In this case the eigenvalue q_2 is $((1 + n)/2 + i\nu)((-1 + n)/2 + i\nu)$.

Lipatov [14] has chosen an ansatz, which automatically diagonalizes $\hat{q}_2$:

$$\Psi_{z_0}(z_1, z_2, z_3) = \left(\frac{z_{12}z_{23}z_{31}}{z_{10}^2 z_{20}^2 z_{30}^2}\right)^{m/3} \varphi(\lambda) \, , \qquad (17)$$

where $m = 1/2 + i\nu + n/2$, n is an integer and ν is a real number. Here, $z_0 \in \mathbb{C}$ is just a parameter and λ is the anharmonic ratio:

$$\lambda = \frac{z_{12}z_{30}}{z_{13}z_{20}} \, . \qquad (18)$$

A breakthrough occurred when Lipatov [13] established the existence of another integral of motion — an operator $\hat{q}_3$:

$$\hat{q}_3 = z_{12}z_{23}z_{31}\partial_1\partial_2\partial_3 \, , \qquad (19)$$

which commutes with the hamiltonian H.

Lipatov further derived the form of the operator $\hat{q}_3$ within this ansatz. Inserting $\Psi_{z_0}(z_1, z_2, z_3)$ into the equation

$$\hat{q}_3\Psi_{z_0}(z_1, z_2, z_3) = q_3 \cdot \Psi_{z_0}(z_1, z_2, z_3) \, , \qquad (20)$$

and canceling the factor $(\ldots)^{m/3}$ he obtained:

$$\nabla_1 \frac{1}{\lambda(1-\lambda)} \nabla_2 \nabla_3 \varphi(\lambda) = q_3 \varphi(\lambda) \, , \qquad (21)$$

where

$$\nabla_1 = \frac{m}{3}(1 - 2\lambda) + \lambda(1-\lambda)\partial \, , \qquad (22)$$

$$\nabla_2 = \frac{m}{3}(1 + \lambda) + \lambda(1-\lambda)\partial \, , \qquad (23)$$

$$\nabla_3 = -\frac{m}{3}(2 - \lambda) + \lambda(1-\lambda)\partial \, . \qquad (24)$$

One of the strategies for solving the odderon problem, proposed by Lipatov [13], was to diagonalize the conservation laws $\hat{q}_2$ and $\hat{q}_3$ and to substitute the solution into the Schroedinger equation in order to find the energy eigenvalue.

Unfortunately no one has succeeded in doing this. So it is quite natural to seek for a new symmetry which might be powerful enough to obtain some progress.

3. Cyclic invariance

It is easy to see that both the Hamiltonian H and $\hat{q}_3$ are invariant under cyclic permutations of the gluonic coordinates z_1, z_2, z_3. We show now how this symmetry manifests itself in the formalism of the preceding section. Under the permutation $z_1 \longrightarrow z_2 \longrightarrow z_3$ the anharmonic ratio transforms as follows:

$$\lambda \longrightarrow 1 - \frac{1}{\lambda} \longrightarrow \frac{1}{1 - \lambda} \, . \tag{25}$$

As we are interested mainly in obtaining the leading behavior of the odderon amplitudes, which corresponds to finding the energy of the ground state of the system, it is natural to postulate that the relevant eigenfunction is symmetric under this transformation and so

$$\varphi(\lambda) = f(s_1, s_2, s_3, \tilde{j}) \, , \tag{26}$$

where s_i are the symmetric polynomials in $x_1 = \lambda, x_2 = 1 - 1/\lambda$ and $x_3 = 1/(1 - \lambda)$, and $\tilde{j}$ is the Vandermonde determinant. Namely

$$s_1 \;=\; x_1 + x_2 + x_3 = \frac{\lambda^3 - 3\lambda + 1}{\lambda(\lambda - 1)} \, , \tag{27}$$

$$s_2 \;=\; x_1 x_2 + x_2 x_3 + x_3 x_1 = \frac{\lambda^3 - 3\lambda^2 + 1}{\lambda(\lambda - 1)} \, , \tag{28}$$

$$s_3 \;=\; x_1 x_2 x_3 = -1 \, , \tag{29}$$

$$\tilde{j} \;=\; (x_1 - x_2)(x_2 - x_3)(x_3 - x_1) = \frac{(\lambda^2 - \lambda + 1)^3}{\lambda^2(\lambda - 1)^2} \, . \tag{30}$$

It turns out that the only independent quantity is

$$A = s_1 + s_2 = \frac{(\lambda + 1)(2\lambda - 1)(\lambda - 2)}{\lambda(\lambda - 1)}$$

related to $\tilde{j}$ by the equation $4\tilde{j} = A^2 + 27$. It is convenient to introduce the notation:

$$B \;:=\; 8A = \sqrt{j - 1728} \, , \tag{31}$$

$$j \;:=\; 256\tilde{j} = 2^8 \frac{(\lambda^2 - \lambda + 1)^3}{\lambda^2(\lambda - 1)^2} \, . \tag{32}$$

At this moment we make a refinement of Lipatov's ansatz, namely

$$
\begin{aligned}
\Psi_{z_0}(z_1, z_2, z_3) &= \left(\frac{z_{12}z_{23}z_{31}}{z_{10}^2 z_{20}^2 z_{30}^2}\right)^{m/3} f(B) \\
&= \left(\frac{z_{12}z_{23}z_{31}}{z_{10}^2 z_{20}^2 z_{30}^2}\right)^{m/3} f\left(8\frac{(\lambda+1)(2\lambda-1)(\lambda-2)}{\lambda(\lambda-1)}\right), \quad (33)
\end{aligned}
$$

where $m = 1/2 + i\nu + n/2$, n is an integer and ν is a real number. λ is the anharmonic ratio:

$$
\lambda = \frac{z_{12}z_{30}}{z_{13}z_{20}}. \tag{34}
$$

Now we insert the function $\varphi(\lambda) = f(B)$ into the conservation law (21). After reexpressing the result in terms of j and $B = \sqrt{j - 1728}$ we get:

$$
\left\{\frac{j^2}{2}\frac{d^3}{dB^3} + 2Bj\frac{d^2}{dB^2} + \left(j\left(1 + \frac{m(1-m)}{6}\right) - 3\cdot 2^8\right)\frac{d}{dB} \right.
$$
$$
\left. + \frac{(m-3)m^2}{27}B - 8q_3\right\}f(B) = 0. \tag{35}
$$

The advantage of considering this equation is that all the discrete symmetries present in the form of the nonlinear transformation (25) act trivially on this equation. The variable j (or really $\sqrt{j - 1728}$) seems to be the true physical variable of the theory. There are no additional residual symmetries which one could take into account. In the next section we give a geometrical interpretation of the j variable in terms of elliptic curves. This will enable us to rephrase the theory of the odderon in a modular invariant way.

4. Modular invariance

According to one of the many possible definitions (see *e.g.* [15]), an elliptic curve is a complex curve of genus one (see Fig. 2). There are two alternative descriptions of these objects.

The first one, very common in the physics literature dealing with CFT and string theory, is the description of elliptic curves as complex tori $\mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau)$ parametrized by $\tau \in \mathbb{C}$ in the upper half-plane. Tori are obtained by identifying opposite edges in the parallelogram bounded by 0, 1, τ and $1+\tau$.

Obviously for some values of the parameters, say τ_1 and τ_2, we may obtain indistinguishable tori. For example if we cut through a non-contractible loop, rotate one edge through 2π and glue back the edges, we obtain an equivalent torus. Such an operation is called a Dehn twist. The notion of isomorphic tori makes this more precise.
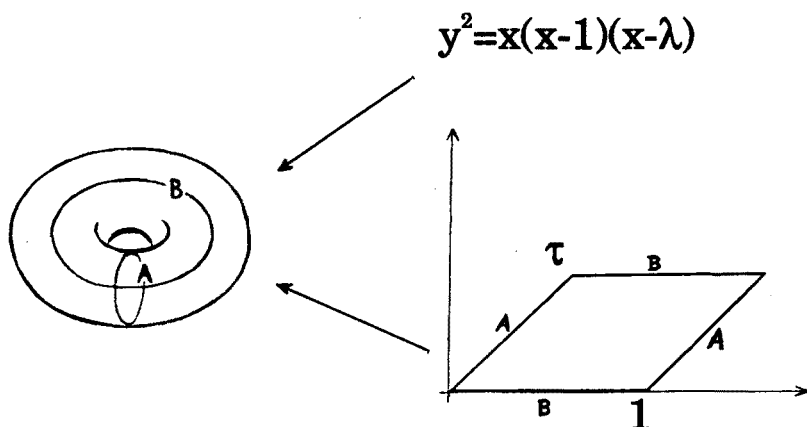
$$y^2 = x(x-1)(x-\lambda)$$

Fig. 2. Elliptic curves or complex tori.

Two complex curves are isomorphic (*i.e.* can be considered as indistinguishable) when there is a one to one holomorphic mapping between them. It is natural to look for some parameter which is identical for isomorphic tori, but enables us to distinguish between distinct ones. Such a parameter is called a modulus. In fact one can associate with each elliptic curve a complex number — its j-invariant, which possesses precisely those properties. Two elliptic curves are isomorphic *if and only if* their j-invariants coincide. In this description the j-invariant is a well known transcendental function of τ. Moreover, the symmetry which leaves j invariant corresponds in this description to modular invariance in the τ-plane *i.e.*

$$j(\tau) = j(\tau') \iff \tau' = \left(\frac{a\tau + b}{c\tau + d}\right) \quad \text{for} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{Z}) \; . \quad (36)$$

The geometrical meaning of this symmetry is as follows: all modular transformations are generated by 'Dehn twists' (see *e.g.* [17]) — these are isomorphisms obtained by cutting the torus along one loop, then twisting one edge through 2π and gluing it back.

At this point we see that if we could express all the operators in our theory in terms of the j-invariant, we could reformulate everything in terms of τ and obtain a modular invariant theory.

The reason why this might be interesting is that modular invariance has proven to be a very strong constraint in QFT, allowing for example for the classification of the partition functions of minimal models in CFT. Recently another application which could be relevant in our case was the work of S.-T. Yau and B.H.Lian on solution of 3rd order Fuchsian ordinary differential equations in terms of modular forms [18].

Before we make the connection with the odderon, consider functions of the form $\sqrt{j(\tau) - c}$. It turns out that the only possible value of the constant c for which this function is globally well defined is 1728. Such an expression can be seen as parametrising tori with a 'sign'. We treat two tori as equivalent if one can obtained from the other by an *even* number of Dehn twists. Using the above mentioned correspondence between Dehn twists and modular transformations one can find that the invariance group is now an infinite group generated by the transformations $\tau \longrightarrow \tau + 2$ and $\tau \longrightarrow -1/\tau$. This group is the unique normal subgroup of $\mathrm{SL}(2, \mathbb{Z})$ of index 2 and is denoted by Γ^2.

4.1. Modular invariance of the odderon

To apply the preceding concepts to the odderon, we must use an alternative but equivalent description of elliptic curves. This is the Weierstrass parameterization which labels each elliptic curve by a complex number $\lambda \in \mathbb{C}$. The curve given by λ is given by the equation

$$y^2 = x(x-1)(x-\lambda), \tag{37}$$

where x and y are complex coordinates. This is also a complex torus but presented in a different way. In fact the link between those descriptions is given by the correspondence [16]:

$$\lambda(\tau) = \left(\frac{\Theta_2(0; \tau)}{\Theta_3(0; \tau)} \right)^4, \tag{38}$$

where $\Theta_2(0; \tau)$ and $\Theta_3(0; \tau)$ are the Jacobi theta functions.

The j-invariant considered earlier can be expressed in terms of the parameter λ labeling the elliptic curves. It is now given by the formula:

$$j = 2^8 \cdot \frac{(\lambda^2 - \lambda + 1)^3}{\lambda^2(\lambda - 1)^2}. \tag{39}$$

Note that this expression is identical to the Vandermonde determinant considered before (32), which is (up to the square root) the correct physical variable of the odderon. We see that our physical variables (31) and (32) are just conformal invariants of elliptic curves, and using the more common description in terms of the τ parameter we obtain a modular invariant theory with respect to the group Γ^2. The benefit is that we may use an altogether different set of tools which has proven to be very useful in a variety of applications.

5. Conclusions

In this talk we have presented the derivation of the modular invariance of the odderon. We have shown that the global $SL(2, \mathbb{C})$ invariance and cyclic symmetry lead to the introduction of certain natural variables in terms of which the theory can be formulated. Furthermore these quantities can be interpreted as conformal invariants of elliptic curves with a 'sign' — the parity of the number of 'Dehn' twists. This at once leads to the modular symmetry of the theory.

Since the modular symmetry is a very strong constraint in QFT, we hope that one can obtain some progress in solving the odderon, especially as this symmetry opens up a new set of tools to attack the problem. Furthermore it is very intriguing that such geometrical interpretation of the odderon symmetries exists. It is challenging to exploit this fact to obtain some relation with an effective string theory.

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