F_L AT LOW x AND LOW Q^2 *

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A model for the longitudinal structure function F_L in the region of low x and low Q^2 is discussed. It is constructed using the k_T factorization theorem and a photon-gluon fusion mechanism suitably extrapolated to the region of low Q^2. A phenomenological model for higher twist is presented, which is based on the assumption that the contribution of quarks having limited transverse momenta is dominated by the soft pomeron exchange mechanism.

PACS numbers: 12.38.Qk, 12.39.Mk

1. Introduction

The longitudinal structure function F_L corresponds to the interaction of longitudinally polarized photons in the one–photon–exchange approximation of deep inelastic lepton-nucleon scattering. The experimental determination of F_L is difficult since it usually requires measurement of the energy dependence of this process for fixed x and Q^2. The structure function F_L vanishes within the naive parton model [1] and also in the leading logarithmic approximation of QCD. In the next–to–leading order approximation the structure function F_L acquires a leading twist contribution proportional to $\alpha_s(Q^2)$. The main process contributing to F_L at low x is photon–gluon fusion, $g\gamma \to q\bar{q}$. The longitudinal structure function F_L is therefore mainly sensitive to the gluon distribution.

In practice it is the ratio $R(x, Q^2) = F_L(x, Q^2)/F_T(x, Q^2)$ which is determined experimentally (where $F_T(x, Q^2)$ is transverse structure function). Current measurements of $R(x, Q^2)$ (SLAC, NMC at CERN [2]) are very

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scarce, especially at low x. In the region of large Q^2 the structure function F_L is fairly well described by perturbative QCD, while at lower values of Q^2 it is not well understood mainly because of the non–perturbative contributions. A better understanding of F_L and R in this region is important in order to implement the radiative corrections needed for the extraction of the $F_2(F_3)$ structure functions from deep inelastic scattering measurements. It is also crucial for the reliable extraction of the spin dependent structure function $g_1(x, Q^2)$ [3] from experimental data, since the uncertainties in R are one of the main sources of systematic errors.

In this paper we present a model for the structure function F_L which provides extrapolation into the region of low Q^2. We also propose a model for the higher twist term. It is based on the assumption that the contribution of quarks having limited transverse momenta is dominated by soft pomeron exchange. We introduce an effective coupling which can be determined from the background contribution to the structure function F_T [4]. Calculations for $R(x, Q^2)$ in the region of low Q^2 are also presented. This paper is based on the results presented in [5].

2. k_T factorization vs collinear factorization

The off–shell approximation is closely connected with the k_T (or high energy) factorization theorem [6, 7] which can be presented in the following way,

$$F_L(x, Q^2) = \int\limits_x^1 \frac{dx'}{x'} \int \frac{dk_T^2}{k_T^2} F_L^{\gamma g}\left(\frac{x}{x'}, k_T^2, Q^2\right) f(x', k_T^2) \,. \tag{1}$$

The dominant diagram for the F_L is shown on Fig. 1, where x', k_T are the longitudinal fraction of proton's momentum and the transverse momentum of the gluon g respectively. The function $F_L^{\gamma g}\left(\frac{x}{x'}, k_T^2, Q^2\right)$ can be regarded as the longitudinal structure function of the off–shell gluon with virtuality k_T^2, and $f(x, k_T^2)$ is the unintegrated gluon distribution which in principle should be obtained from the BFKL equation [8, 9]. In formula (1) there is double convolution over the variables x' and k_T, and therefore it is called the off–shell approach. On the other hand the well known renormalization group analysis is basically connected with collinear factorization formula,

$$F(x, Q^2) = \sum_i \int\limits_x^1 \frac{dx'}{x'} C_i(x/x', \alpha_s, Q^2/\mu^2) x' h_i(x', \mu^2) \,. \tag{2}$$

The $h_i(x', \mu^2)$ are the integrated parton distribution functions which satisfy conventional Altarelli-Parisi equations [10] and C_i are the coefficient functions which can be calculated perturbatively. The summation is performed

over different species of partons and μ^2 is a factorization scale which usually is taken to be Q^2 in case of deep inelastic scattering.
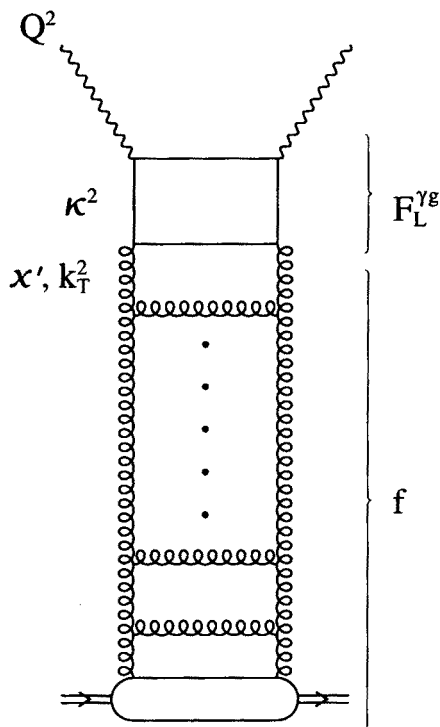

Fig. 1. The diagramatic representation of the k_T factorization formula for F_L.

It has been shown in [6, 11] that the leading twist part of the k_T factorization formula can be recast into the collinear form (2) where all small x' (x/x') effects are resummed to all orders in $\alpha_s \ln(x')$ ($\alpha_s \ln(x/x')$) in the splitting (and coefficient) functions. For instance the moments of the coefficient functions $\bar{C}_L(\omega, \alpha_s)$

$$\bar{C}_L(\omega, Q^2, \alpha_s(Q^2)) \equiv \int\limits_0^1 \frac{dz}{z} z^\omega C_L(z, Q^2, \alpha_s(Q^2)) \tag{3}$$

are related to the double moment function $\tilde{F}_L^{\gamma g}(\omega, \gamma)$ by

$$\bar{C}_L(\omega, Q^2, \alpha_s(Q^2)) = \gamma_{gg}(\bar{\alpha}_s/\omega)\tilde{F}_L^{\gamma g}(\omega, \gamma = \gamma_{gg}(\bar{\alpha}_s/\omega)) \tag{4}$$

where $\bar{\alpha}_s = N_c \alpha_s / \pi$, and

$$
\begin{aligned}
\bar{F}_L^{\gamma g}(\omega, Q^2) &= \frac{1}{2\pi i} \int\limits_{1/2-i\infty}^{1/2+i\infty} d\gamma \tilde{F}_L^{\gamma g}(\omega, \gamma) \left(\frac{Q^2}{k^2}\right)^\gamma , \\
\bar{F}_L^{\gamma g}(\omega, Q^2) &= \int\limits_0^1 \frac{dx}{x} x^\omega F_L^{\gamma g}(x, Q^2) .
\end{aligned} \tag{5}
$$

The quantity $\gamma_{gg}(\bar{\alpha}_s/\omega)$ is the gluon anomalous dimension related to the appropriate splitting function by

$$
\gamma_{gg}(\bar{\alpha}_s/\omega) = \int\limits_0^1 dz z^\omega P_{gg}(z, \alpha_s) . \tag{6}
$$

Knowing the expansion of $\tilde{F}_L^{\gamma g}$ in terms of the anomalous dimension and the expansion of γ_{gg} in powers of $\bar{\alpha}_s/\omega$, one can obtain the following formula for the coefficient function $C_L(z, \alpha_s)$ (in case of F_L),

$$
C_L(z, \alpha_s) = \alpha_s \left[C_L^{(0)}(z) + \alpha_s C_L^{(1)}(z) + \alpha_s^2 \ln(\tfrac{1}{z}) C_L^{(2)}(z) + \alpha_s^3 \ln^2(\tfrac{1}{z}) C_L^{(3)}(z) + \ldots \right], \tag{7}
$$

where as $z \to 0$, $C_L^{(0)}(z) \to 0$ $\qquad C_L^{(i)}(z) \to \text{const} \neq 0$, $\quad i \geq 1$.

If instead of the BFKL solution in (1), we take

$$
f(x, k_T^2) = \left. \frac{\partial x g(x, Q^2)}{\partial \ln Q^2} \right|_{Q^2 = k_T^2} , \tag{8}
$$

where $x g(x, Q^2)$ is a solution of the Altarelli–Parisi equations without small x corrections in P_{gg} but preserving the original phase space (no $\mathbf{k_T}$ ordering), then only the first three non-trivial corrections in small x are included in the expansion for C_L. This approximation seems to be reliable, but not of course at very low values of x where the BFKL effects could play an important role. In our calculations we consider $x > 10^{-4}$.

3. The photon–gluon fusion model for F_L

Here we show how after an expanding the integrand in the off-shell formula (1) around $k_T^2 = 0$ one can obtain the on–shell approximation [12] with additional power corrections. If we use the $\mathbf{k_T}$ factorization theorem

to calculate F_L, then we obtain the following form for F_L at low x and large Q^2:

$$F_L(x, Q^2) = 2\frac{Q^4}{\pi^2}\sum_q e_q^2 I_q(x, Q^2)\,, \tag{9}$$

where

$$I_q(x, Q^2) = \int \frac{dk_T^2}{k_T^4} \int_0^1 d\beta \int d^2\kappa_T' \alpha_s(Q^2)\beta^2(1-\beta)^2\frac{1}{2}\left(\frac{1}{D_{1q}} - \frac{1}{D_{2q}}\right)^2 f(x', k_T^2) \tag{10}$$

and

$$\begin{aligned}
D_{1q} &= \kappa_T^2 + \beta(1-\beta)Q^2 + m_q^2\,, \\
D_{2q} &= (\boldsymbol{\kappa_T} - \boldsymbol{k_T})^2 + \beta(1-\beta)Q^2 + m_q^2\,.
\end{aligned} \tag{11}$$

The transverse momentum $\boldsymbol{\kappa_T'}$ and the variable x' are defined as follows:

$$\boldsymbol{\kappa_T'} = \boldsymbol{\kappa_T} - (1-\beta)\boldsymbol{k_T}\,, \tag{12}$$

$$x' = x\left(1 + \frac{\kappa_T'^2 + m_q^2}{\beta(1-\beta)Q^2} + \frac{k_T^2}{Q^2}\right)\,. \tag{13}$$

The integration over x' is implicit in (10). One can make it explicit by changing the variables from $\kappa_T'^2$ to x' using the above formula (13). The integration limits in (10) are additionally constrained by the condition:

$$x' < 1\,. \tag{14}$$

The condition $x' > x$ demanded by the integration limit is automatically satisfied, see (13). In the on–shell approximation of the formulae (9), (10) one expands integrands of the corresponding integrals around $k_T = 0$ and retains the leading term. The integrals I_q can then be expressed in terms of the conventional integrated gluon distribution $g(y, Q^2)$

$$I_q(x, Q^2) = \pi \int_0^1 d\beta \int d\kappa_T'^2 \alpha_s(Q^2)\beta^2(1-\beta)^2\frac{\kappa_T'^2}{D_q^4}yg(y, Q^2)\,, \tag{15}$$

where now

$$y = x\left(1 + \frac{\kappa_T'^2 + m_q^2}{\beta(1-\beta)Q^2}\right) \tag{16}$$

and

$$D_q = \kappa_T'^2 + \beta(1-\beta)Q^2 + m_q^2\,. \tag{17}$$

In order to extrapolate formula (9) (with I_q given by (10) or (15)) to the low Q^2 region we have to freeze the evolution of $g(y, Q^2)$ and the argument of $\alpha_s(Q^2)$. To do this we substitute $Q^2 + 4m_q^2$ instead of Q^2 as the argument of α_s and of $yg(y, Q^2)$.

This substitution may be justified by analyticity arguments *i.e.* we want I_q to have threshold singularities for $Q^2 < -4m_q^2$. Moreover, for heavy quarks and for moderately large values of Q^2, it is the heavy mass squared and not Q^2 which should be taken as the scale of α_s and of the parton distributions. It should be noted that after these modifications the structure function F_L can be continued down to $Q^2 = 0$, respecting the kinematical constraint $F_L \to Q^4$ as $Q^2 \to 0$. The formula (15) can be rearranged to give the Altarelli-Martinelli [13] integral over y but with the modified integration limits and additional power corrections. First we use y and β as the integration variables instead of $\kappa_T'^2$ and β. From (17) we find

$$\kappa_T'^2 = \beta(1 - \beta)Q^2\left(\frac{y}{x} - 1\right) - m_q^2 , \tag{18}$$

$$D_q = \beta(1 - \beta)Q^2\frac{y}{x} . \tag{19}$$

From Eq. (18) we can deduce various integration limits since

$$\beta(1 - \beta)Q^2\left(\frac{y}{x} - 1\right) - m_q^2 > 0 \tag{20}$$

and since $1 > \beta > 0$,

$$\frac{1}{4} > \beta(1 - \beta) > \frac{m_q^2}{Q^2(\frac{y}{x} - 1)} . \tag{21}$$

From inequality (21) we obtain the lower limit for integration over y

$$y > x\left(1 + \frac{4m_q^2}{Q^2}\right) \tag{22}$$

and of course,

$$\beta(1 - \beta) > \frac{m_q^2}{Q^2(\frac{y}{x} - 1)} . \tag{23}$$

It is convenient to make another change of the integration variables:

$$\beta = \frac{1}{2} + \lambda . \tag{24}$$

The inequality (21) gives the following limits for the variable λ

$$-\sqrt{\frac{1}{4} - \frac{m_q^2}{Q^2(\frac{y}{x} - 1)}} < \lambda < \sqrt{\frac{1}{4} - \frac{m_q^2}{Q^2(\frac{y}{x} - 1)}} . \tag{25}$$

Finally we get the following representation for F_L:

$$F_L = 2 \sum_q e_q^2 (J_q^{(1)} - 2\frac{m_q^2}{Q^2} J_q^{(2)}) \,, \tag{26}$$

where

$$J_q^{(1)} = \frac{\alpha_s}{\pi} \int\limits_{\bar{x}_q}^{1} \frac{dy}{y} \left(\frac{x}{y}\right)^2 \left(1 - \frac{x}{y}\right) \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}} \, yg(y, Q^2) \tag{27}$$

$$J_q^{(2)} = \frac{\alpha_s}{\pi} \int\limits_{\bar{x}_q}^{1} \frac{dy}{y} \left(\frac{x}{y}\right)^3 \ln\left(\frac{1 + \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}}}{1 - \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}}}\right) yg(y, Q^2) \tag{28}$$

and

$$\bar{x}_q \equiv x \left(1 + \frac{4m_q^2}{Q^2}\right) \,. \tag{29}$$

It should be noted that expressions (26), (27), (28) reduce to the following formula

$$F_L(x, Q^2) = \frac{\alpha_s(Q^2)}{\pi} \int\limits_{x}^{1} \frac{dy}{y} \left(\frac{x}{y}\right)^2 \left(1 - \frac{x}{y}\right) yg(y, Q^2) \tag{30}$$

given by [13] in the $Q^2 \to \infty$ limit. The expression $\frac{m_q^2}{Q^2} J_q^{(2)}$ represents the power correction enhanced by the logarithmic factor $\ln Q^2/m_q^2$. The modification of the lower limit becomes obvious, if we consider the energy–momentum conservation constraint for the subprocess $\gamma g \to q\bar{q}$.

4. A phenomenological model for the higher–twist contribution to F_L

The kinematical constraint restricts the behaviour of F_L in the region of small values of Q^2. Analysing the longitudinal cross section for deep inelastic scattering one concludes that in order to remove the possible singularities of this observable at $Q^2 \to 0$, (*i.e.* $\sigma_L \sim F_L/Q^2$), F_L should behave as Q^4. This implies that $\sigma_L \sim Q^2$, so it vanishes in the real photoproduction limit as expected. Phenomenological analysis of the quantity R in the region of moderately large Q^2 ($Q^2 > 1$ GeV^2) implies that the higher twist effects (*i.e.* terms that behave like $1/Q^2$ as $Q^2 \to \infty$) should play an important role [14].

In this paragraph we introduce a phenomenological model for the higher twist term which satisfies the constraint $F_L \sim Q^4$ as $Q^2 \to 0$. The idea is to treat the contributions from low and high quark momenta in a different way. We divide the integration over $\kappa_T'^2$ (in (10) for the off–shell model (or in (27), (28) for the on–shell model) into two parts – the region $0 < \kappa_T'^2 < \kappa_{0T}'^2$, and $\kappa_T'^2 > \kappa_{0T}'^2$ – where $\kappa_{0T}'^2$ is an arbitrary cut–off, chosen to be of order $1\mathrm{GeV}^2$. The region of high momenta is treated in the usual way (using off–shell (9), (10) or on–shell (26), (27), (28) formula) whereas for low momenta we assume the existence of an effective coupling constant:

$$\alpha_s(Q^2) zg(z, Q^2) \to A \ . \tag{31}$$

The constant A is not a free parameter. We can estimate it from F_2 assuming that the non-perturbative contribution to F_2 in the scaling region also comes from the region of low values of $\kappa_T'^2$ and is controlled by the same parameter. The contribution to F_L from the region $0 < \kappa_T'^2 < \kappa_{0T}'^2$ is calculated using on–shell formula, making the substitution (31) and integrating over $\kappa_T'^2$,

$$\Delta F_L(Q^2) = A \frac{2}{\pi} Q^4 \sum_q e_q^2 \int_0^1 d\beta \beta^2 (1-\beta)^2 \left\{ \frac{1}{6} \frac{1}{[\beta(1-\beta)Q^2 + m_q^2]^2} \right.$$
$$\left. - \frac{1}{6} \frac{\beta(1-\beta)Q^2 + m_q^2 + 3\kappa_{0T}'^2}{[\beta(1-\beta)Q^2 + m_q^2 + \kappa_{0T}'^2]^3} \right\} \ . \tag{32}$$

This formula vanishes like $1/Q^2$ for large values of Q^2, therefore it has the form of a higher twist term. It also obeys the kinematical constraint for the structure function F_L , *i.e.* vanishes like Q^4 for $Q^2 \to 0$.

To evaluate the constant A we consider the corresponding formula for F_T,

$$F_T(x, Q^2) = 2 \sum_q e_q^2 \frac{Q^2}{4\pi^2} \int \frac{dk_T^2}{k_T^4} \int_0^1 d\beta \int d^2\kappa_T' \alpha_s(\kappa_T'^2 + m_0^2)$$
$$\times \left\{ [\beta^2 + (1-\beta)^2] \left[\frac{\kappa_T^2}{D_{1q}^2} - \frac{\kappa_T \cdot (\kappa_T - k_T)}{D_{1q}D_{2q}} \right] + \frac{m_q^2}{D_{1q}^2} - \frac{m_q^2}{D_{1q}D_{2q}} \right\} f(x', k_T^2) \ , \tag{33}$$

where κ_T', D_{1q} and D_{2q} are defined by Eqs. (11), (12). If we rewrite the integrands to be symmetric in variables κ_T and $\kappa_T - k_T$ we obtain,

$$\left[\frac{\kappa_T^2}{D_{1q}^2} - \frac{\kappa_T}{\cdot}(\kappa_T - k_T)D_{1q}D_{2q} \right] = \frac{1}{2} \left[\frac{\kappa_T}{D_{1q}} - \frac{\kappa_T - k_T}{D_{2q}} \right]^2 \ ,$$
$$\frac{m_q^2}{D_{1q}^2} - \frac{m_q^2}{D_{1q}D_{2q}} = \frac{m_q^2}{2} \left[\frac{1}{D_{1q}} - \frac{1}{D_{2q}} \right]^2 \ . \tag{34}$$

If we now expand the right hand side of the Eq. (34) in powers of k_T^2/Q^2 (on–shell approximation) we obtain the following formula,

$$
\left[\frac{\boldsymbol{\kappa}_T}{D_{1q}} - \frac{\boldsymbol{\kappa}_T - \boldsymbol{k}_T}{D_{2q}}\right]^2 \simeq \left[\frac{\boldsymbol{k}_T}{D_q} - \frac{2\boldsymbol{\kappa}'_T(\boldsymbol{\kappa}'_T \cdot \boldsymbol{k}_T)}{D_q}\right]^2
$$

$$
= k_T^2\left(\frac{1}{D_q^2} - \frac{4cos^2\phi\kappa_T'^2}{D_q^3} + \frac{4cos^2\phi\kappa_T'^4}{D_q^4}\right) , \quad (35)
$$

where we have dropped terms containing odd powers of $cos\phi$ which give vanishing contribution after performing integration over $d\phi$. Then F_T takes the form

$$
F_T(x,Q^2) = 2A\sum_q e_q^2 \frac{Q^2}{4\pi} \int\limits_0^1 d\beta \int\limits_0^{\kappa_{0T}'^2} d\kappa_T'^2
$$

$$
\times \left\{\frac{1}{2}[\beta^2+(1-\beta)^2]\left(\frac{1}{D_q^2} - \frac{2\kappa_T'^2}{D_q^3} + \frac{2\kappa_T'^4}{D_q^4}\right) + \frac{m_q^2\kappa_T'^2}{D_q^4}\right\} . \quad (36)
$$

Performing the integration over $\kappa_T'^2$, substituting (31) and taking the limit $Q^2 \to \infty$, we get the following form for the formula for the structure function F_L,

$$
A = \frac{2\pi F_T^{bg}}{\sum_q e_q^2\left\{-\frac{2}{3}\ln\frac{m_q^2}{m_q^2+\kappa_{0T}'^2} + \frac{1}{3}\frac{\kappa_{0T}'^2}{m_q^2+\kappa_{0T}'^2}\right\}} . \quad (37)
$$

The value for F_T^{bg} is taken from the phenomenological model for F_T [4] where F_T is assumed to have the form,

$$
F_2 \simeq F_T = F_T^{bg} + F_T^{\text{BFKL}} , \quad (38)
$$

where F_T^{bg} is the background originating from the non-perturbative region and its value is fixed by the data to be around 0.4. Assuming $\kappa_{0T}'^2 = 1$ GeV^2 one obtains $A \simeq 1.96$.

5. Numerical results

In this section we present the numerical analysis for the structure function F_L. We shall also determine the ratio R and compare it with SLAC parametrisation [15]. We include u, d, s, c quark contributions, with quark masses $0.35, 0.35, 0.5, 1.5$ GeV, respectively. We have used both GRV [16] and MRS(A) [17] parton distributions. In GRV the LO gluons and coupling
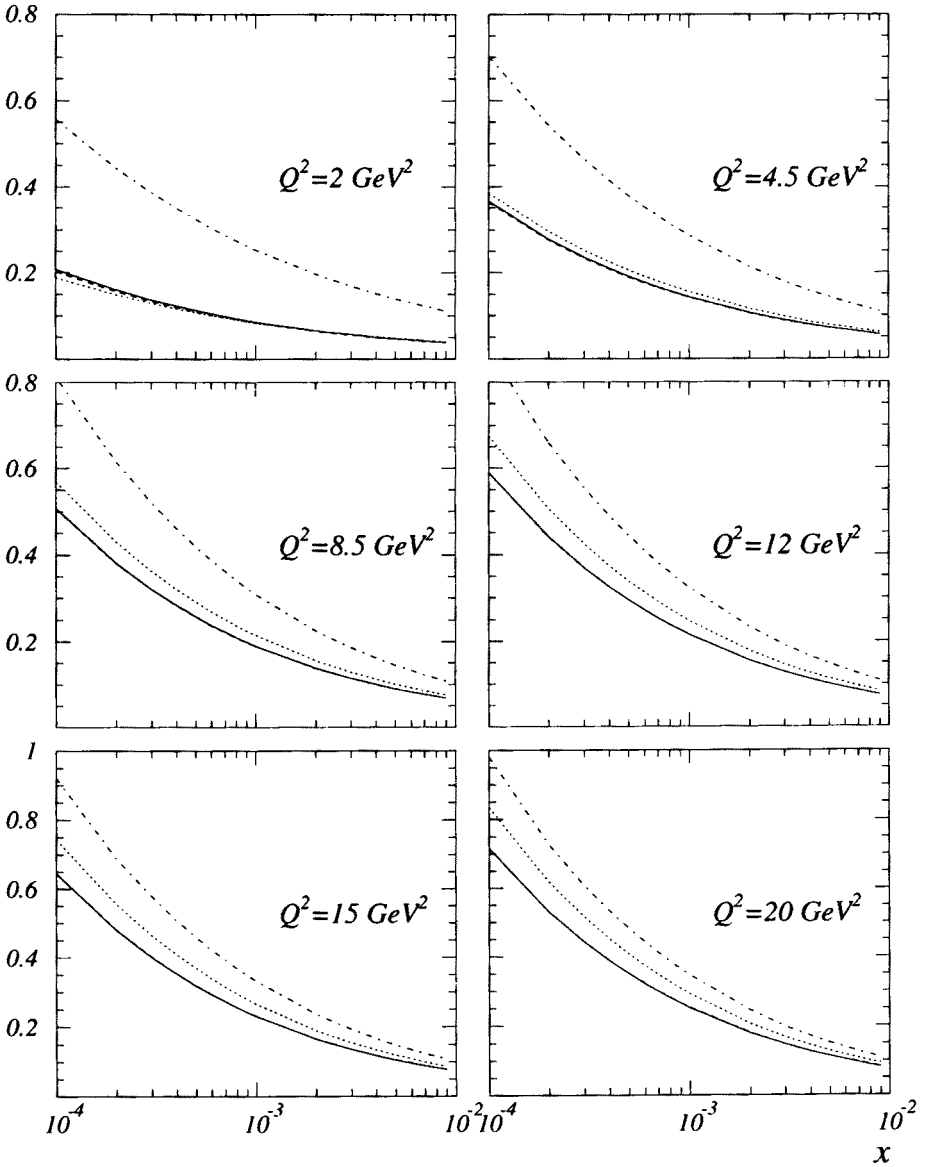
Fig. 2. F_L plotted as a function of x for different values of Q^2. We use GRV partons [16]. Solid line: off–shell approximation with $k_{0T}^2 = 0.23$ GeV^2, dashed line: off–shell approximation with $k_{0T}^2 = 1.0$ GeV^2, dotted line: on–shell approximation, dashed-dotted line on–shell approximation with massless quarks u, d, s.

constant were used. The LO coupling constant has the following form,

$$\frac{\alpha_s(Q^2)}{2\pi} = \frac{2}{\beta_0 \ln \frac{Q^2}{\Lambda^2}},$$ (39)

where $\beta_0 = 11 - 2N_f/3$, N_f is the number of flavours, and the value of the constant Λ depends on number of active flavours

$$\Lambda_{LO}^{(3,4)} = 0.232, 0.200 \text{ GeV}.$$ (40)

We include the charm quark production threshold *i.e.* if $Q^2 > 4m_c^2$ then $N_f = 4$, otherwise $N_f = 3$.

In order to extrapolate the structure function F_L to low Q^2 we use the method of freezing the argument of the coupling and of the gluon distributions as explained before. For the off–shell formula we have used the prescription proposed in [18]. In order to regularise the integration over k_T^2 we have introduced the cut-off k_{0T}^2 . The contribution of integration from 0 to k_{0T}^2 is calculated using the approximate formula (26), (27), (28) with the substitution $yg(y, k_{0T}^2)$. The magnitude of the structure function F_L turns out to depend rather weakly on the chosen cut-off k_{0T}^2. The dominant contribution to F_L is given by the photon-gluon fusion subprocess $\gamma g \to q\bar{q}$ (so the gluon distribution is the most important at low x) but the contributions from quarks have also been included with the modification $x \to x(1 + 4m_q^2/Q^2)$ for the lower limit of the integration over y.

$$\Delta F_L(x, Q^2) = \sum_i e_i^2 \frac{\alpha_s(Q^2 + 4m_i^2)}{\pi} \frac{4}{3} \int_{\tilde{x}_q}^{1} \frac{dy}{y} (\frac{x}{y})^2$$
$$\times [q_i(y, Q^2 + 4m_i^2) + \bar{q}_i(y, Q^2 + 4m_i^2)],$$ (41)

where $\bar{x}_q$ is given by (29). In Fig. 2. we compare our results for massive and massless quarks. As can be observed the quark masses play an important role in the region of low Q^2. But even at moderate values of Q^2 the difference is visible. This is because the on–shell model (26), (27), (28) with masses contains the power correction term multiplied by the logarithmic factor, $\sim m^2/Q^2 \ln(Q^2/m^2)$. Note also that the difference between the on–shell and off–shell calculations is very small.

In Fig. 3 we present F_L as a function of x, with and without higher twist corrections. We have chosen the cut-off parameter $\kappa_{0T}'^2$ to be 1.0 GeV^2, but there is no significant dependence on this quantity (within the range of $0.8 < \kappa_{0T}'^2 < 2.0$ GeV^2). Higher twist gives higher values for F_L, especially in the region where x is not very small ($x > 0.001$). On the other hand it gives
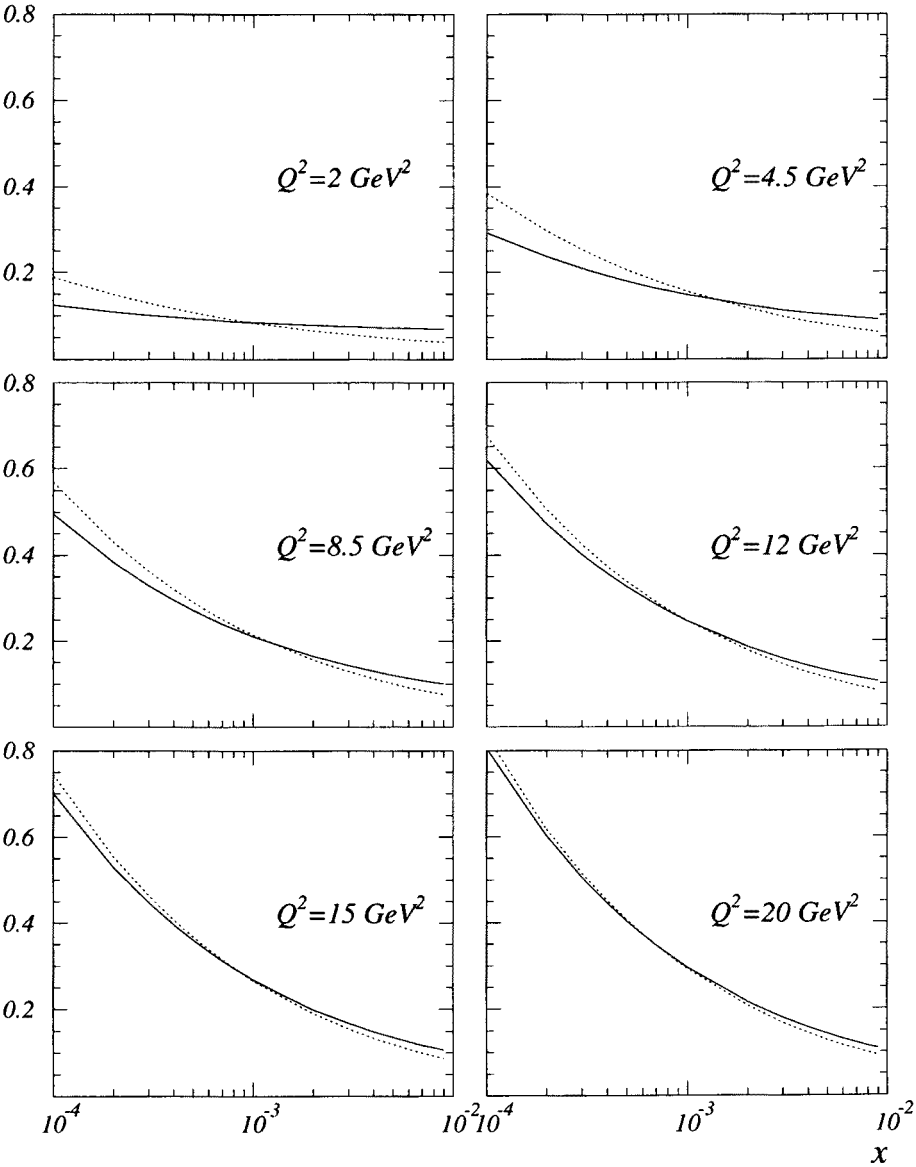
Fig. 3. F_L as a function of x for different values of Q^2 calculated in the on–shell approximation: comparison of higher twist effect, partons distributions are obtained using the GRV parametrisation. Solid and dotted lines correspond to the results with and without the higher twist term, respectively.
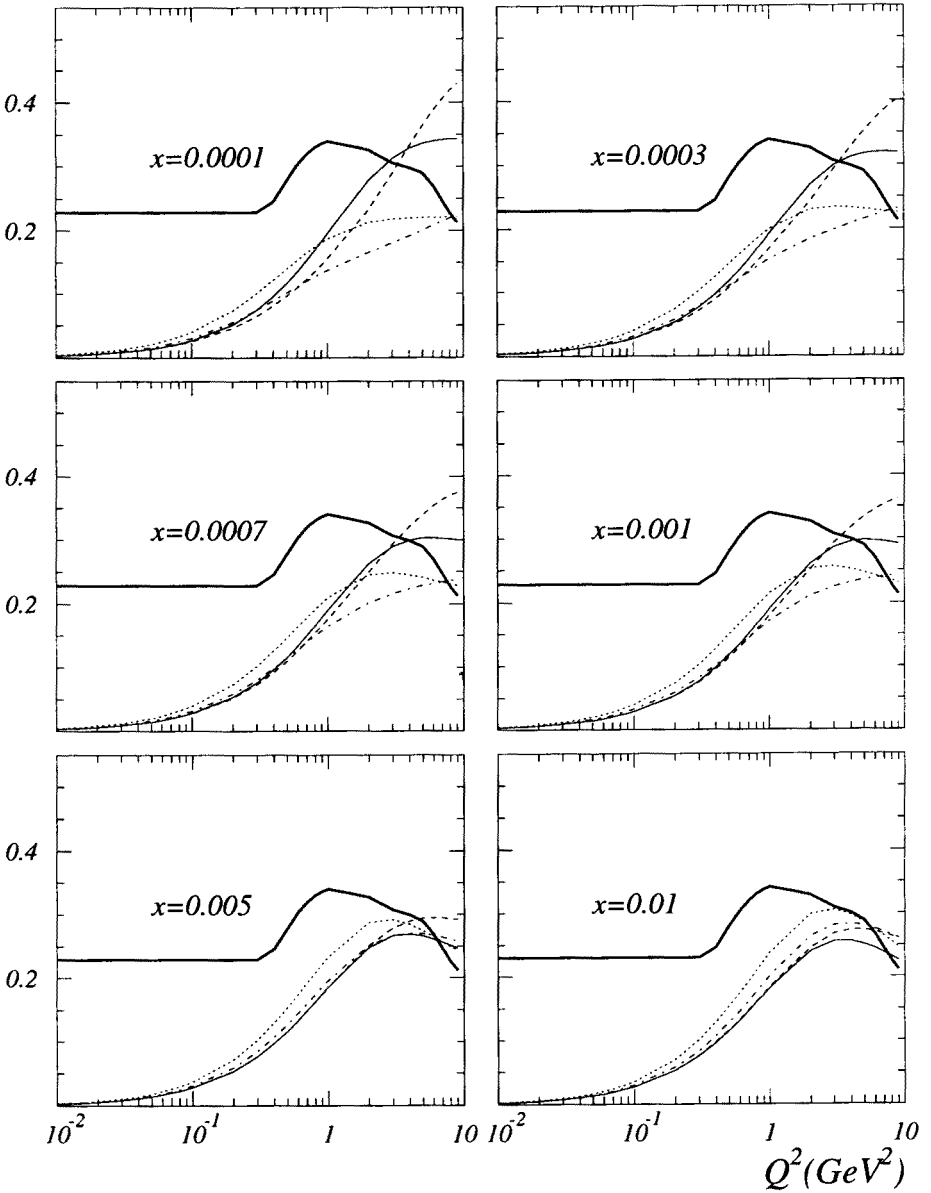
Fig. 4. The ratio R plotted as a function of Q^2 for different values of x with the higher twist effect included and compared with the SLAC parametrisation (thick solid lines). Solid and dashed lines correspond to the off–shell and on–shell approximation with the GRV parametrisation of the parton distributions [16]. The dotted and dashed-dotted lines correspond to off–shell and on–shell approximation calculated using MRS(A) partons [17].

lower values for small x than the model without higher twist. This is due to
the fact that the contribution coming from low values of the quark momenta
is now described by the soft pomeron exchange. The difference between
these calculations decreases as Q^2 increases.

In Fig. 4 we present the results for the ratio $R(x, Q^2) = F_L(x, Q^2)/$
$(F_2(x, Q^2) - F_L(x, Q^2))$ together with SLAC [15] parametrisation. For both
parton sets (GRV and MRS(A)) the structure function F_2 has been calcu-
lated from the VMD model [19]. We also observe the same effect as for the
longitudinal structure function: the model with higher twist predicts bigger
values for the ratio R at high x.

For intermediate values of Q^2 this model coincides with SLAC parametri-
sation, whereas for lower values of Q^2 it vanishes quickly as $Q^2 \to 0$. It
should be noted that the difference between the parametrisations in the low
Q^2 region are not very large, they both decrease very fast with decreasing
Q^2. This suggests that the ratio R and the structure function F_L are not
very sensitive to the differences in parton parametrisations in this region.
Because the SLAC model is not applicable in the low Q^2 region, using it as
an estimate of the ratio R may cause significant errors.

6. Summary

We have analysed the proton structure function F_L in the low Q^2 and
low x region. We have studied the off–shell formula which originates from
the high energy factorization theorem and we have proved that in the limit
when $k_T^2 \to 0$ it reduces to the on–shell approximation. The numerical
differences between these two approaches are not very significant. We have
also studied the structure function F_L in the very low Q^2 region, where
we have proposed the method of continuing the formulae for the structure
functions down to $Q^2 = 0$. The non-zero values of quark masses have strong
consequences in the structure function behaviour. Both the off–shell and on–
shell formulae contain power corrections connected with quark masses (*i.e.*
terms like $m^2/Q^2 \ln(Q^2/m^2)$). We have also proposed a model for the higher
twist term which is based on the assumption that the contribution of quarks
having limited transverse momenta is dominated by soft pomeron exchange.
This results in change of the x dependence of the structure function F_L. We
have also performed calculations for R. The model for the ratio R enables us
to estimate this quantity in the low Q^2 region. It is particularly important
because the uncertainty in the ratio R is the main source of systematic
errors when extracting other structure functions from experimental data.

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