ON DYNAMICALLY GENERATED PARTON DISTRIBUTION FUNCTIONS AND THEIR PROPERTIES*

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The idea of "dynamically" generated parton distribution functions, based on regular initial conditions at low momentum scale, is reanalyzed with particular emphasize paid to its compatibility with the factorization mechanism. Basic consequences of this approach are discussed and compared to those of the conventional approach, employing singular initial distribution functions.

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1. Introduction

One of the highlights [1, 2] of the recent International Conference on High Energy Physics in Brussels has been the remarkable success of "dynamically" generated parton distribution functions (DGPD) advocated by Glück, Reya and Vogt (GRV) [3–8], in predicting the rapid rise of proton structure function $F_2^{ep}(x,Q)$ at low x, observed at HERA [9, 10]. [1]

The GRV group is one of three main groups (the other two being the Durham (MRS) [11–13] and the CTEQ [14, 15] ones), which systematically analyze hard scattering data within the framework of perturbative QCD. What distinguishes GRV approach from those of the other two groups is their claim that the DGPD are more than just parametrizations of our inability to compute structure functions directly from first principles. GRV argue that by imposing certain condition on the initial parton distributions at low momentum scale, one obtains more predictive results. Without

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[1] Throughout the paper the term "distribution" stands for distribution function.

this additional theoretical input the conventional parametrizations, using a moderate initial scale $Q_0 \approx 2$ GeV, are unstable when extrapolated to low x region. For that reason both the MRS and CTEQ groups usually present several sets of such parametrizations, differing just in low x region.

The idea of DGPD is intuitively appealing and actually almost as old as QCD itself [3]. Confronted with growing amount and variety of data, it has, however, undergone significant modifications [4, 5, 7] and in the process lost most of its original appeal. As the GRV approach relies on very low initial scale in the range $0.5 - 0.6$ GeV, it has been met with reservations and scepticism [16–18]. In response to this criticism and in order to bring further arguments in favour of their approach, GRV have included in their recent paper [7] an extensive discussion of several of these points.

To relate physics of short distances, the true realm of perturbative QCD, to that of distances comparable to the proton size would certainly represent a major achievement. The purpose of this paper is to discuss whether this can really be done in the way suggested in [3–8]. Throughout this paper I shall concentrate on the analysis of the basic assumptions and consequences of the GRV approach, with only occasional reference to comparison with experimental data.

The paper is organized as follows. In the next section I shall briefly recall the development of the idea of DGPD, from its inception [3] up to the present status [7]. In Section 3 the applicability of perturbative QCD at distances as large as 0.4 fm will be discussed. In particular I shall comment on the implications and interpretation of recent lattice calculations [19], quoted in [7]. The indispensable role of power corrections in going from short distances (where partons live) to distances comparable to the proton size (where the appropriate degrees of freedom are the constituent quarks) is emphasized in Section 4. In Section 5 the compatibility of the DGPD with the factorization mechanism is discussed in detail. In particular it is shown why it is very difficult for gluons and sea quarks to be valence-like. This discussion also shows how the conventional parametrizations, based on singular input distributions, avoid this problem. In Section 6 results of the conventional approach in the small x region are briefly reviewed and cast into a simple form suitable for the comparison with the DGPD. This comparison, carried out in Section 7, identifies two basic signatures of the DGPD. Throughout the paper I adopt the notation in which the QCD coupling $a = \alpha_s/\pi$ satisfies the usual RG equation

$$\frac{da(M,\mathrm{RS})}{d\ln M} = -ba^2(M,\mathrm{RS})\left(1 + ca(M,\mathrm{RS}) + c_2a^2(M,\mathrm{RS}) + \cdots\right), \quad (1)$$

where b, c are the first two, universal, coefficients [2]

$$b = \frac{33 - 2n_f}{6}, \quad c = \frac{153 - 19n_f}{66 - 4n_f}, \tag{2}$$

while all the higher order coefficients c_k, $k \geq 2$, in (1) are free parameters, defining the so called renormalization convention (RC) [20]. Together with the specification of the initial condition on the solution of (1) they define the renormalization scheme (RS).

2. The evolution of the idea of dynamically generated partons

The original idea of [3] was to generate parton distributions at large momentum scales, where experimental data are available, by means of the DGLAP leading order [3], leading twist evolution equations, starting at some small momentum scale [4] $\mu \approx 0.55$ GeV from purely valence-like quark distributions, with vanishing light sea and gluon ones [5]

$$G(x,\mu) = \overline{u}(x,\mu) = \overline{d}(x,\mu) = \overline{s}(x,\mu) = s(x,\mu) = 0\,. \tag{3}$$

The quark distributions at the initial scale μ, obtained by backward evolution from measured structure function $F_2^{eN}(x,Q)$ at $Q_0^2 = 3$ GeV^2, were constrained to satisfy quark number sum rule

$$\int_0^1 dx\,[u(x,\mu) + d(x,\mu)] = 3\,, \tag{4}$$

which provides a fundamental bridge between the parton model of Feynman and the old nonrelativistic "quasinuclear colored model" of Gell-Mann, Zweig, Greenberg, Lipkin and others. The scale μ was fixed by imposing the momentum sum rule

$$\int_0^1 dx\,x\,[u(x,\mu) + d(x,\mu)] = 1\,. \tag{5}$$

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[2] Assuming QCD with three colors and n_f massless quark flavors.

[3] In later GRV papers also the NLO DGLAP evolution equations were used.

[4] In the rest of this paper Q_0 is used for the general initial scale, while the symbol μ is reserved for the initial scale within the GRV approach, *i.e.* the one at which the parton distributions become valence-like.

[5] In GRV approach heavy quarks c, b and t are not considered as intrinsic partons in the nucleon, but are produced from intrinsic gluons via the boson–gluon fusion mechanism [7].

In 1976 there were too few data on hard scattering processes to test the DGPD thoroughly. With more and increasingly accurate data becoming available in late eighties, the GRV group was forced to modify their original idea by allowing nonvanishing valence-like gluon distribution $G(x,\mu)$ as well [4]. Moreover, $G(x,\mu)$ was related to the input valence-like quark distributions as follows:

$$G(x,\mu) = \frac{n_G}{3}\left[u(x,\mu) + d(x,\mu)\right], \quad \overline{q}(x,\mu) = 0\,. \tag{6}$$

In [4] μ and n_G were fixed by means of the momentum sum rule at the initial scale μ, now including also the gluon contribution, together with the comparison of theoretical predictions with data on direct photon production. About the same μ as in [3] and $n_G = 2$, was obtained.

Confronted with still more data GRV had finally to include in their initial parton distributions also the valence-like nonstrange sea [5] so that the momentum sum rule now reads

$$\int_0^1 dx\, x\left[u_v(x,\mu) + d_v(x,\mu) + 2\overline{u}(x,\mu) + 2\overline{d}(x,\mu) + G(x,\mu)\right] = 1\,. \tag{7}$$

In one of their latest NLO global analysis, published in [7], $\mu = 0.58$ GeV and

$$xu_v(x,\mu) = 0.988x^{0.543}\left(1 + 1.58\sqrt{x} + 2.58x + 18.1x^{3/2}\right)(1-x)^{3.38}, \tag{8}$$

$$xd_v(x,\mu) = 0.182x^{0.316}\left(1 + 2.51\sqrt{x} + 25.0x + 11.4x^{3/2}\right)(1-x)^{4.113}, \tag{9}$$

$$x(\overline{u} + \overline{d})(x,\mu) = 1.09x^{0.3}(1 + 2.65x)(1-x)^{8.33}, \tag{10}$$

$$xG(x,\mu) = 26.2x^{1.9}(1-x)^{4.0}, \tag{11}$$

$$xs(x,\mu) = x\overline{s}(x,\mu) = 0\,. \tag{12}$$

3. Does perturbative QCD make sense at 0.4 fermi?

The initial scale $\mu \doteq 0.58$ GeV corresponds to a distance 0.37 fermi. As pointed out by a number of authors [16–18] such distances are probably too large for a meaningful purely perturbative treatment. It is fair to say that GRV do not trust their results at such low momentum scales but claim they are good approximations only above somewhat higher scale $\mu_{\mathbf{pert}} \doteq 0.75$

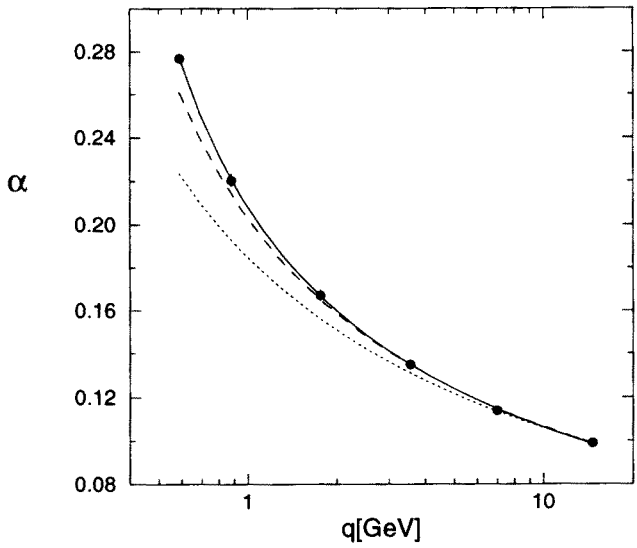

Fig. 1. Results of nonperturbative evaluation of particularly defined QCD running coupling $\alpha_s^{\rm latt}(\mu)$ on the lattice. Taken from Ref. [19].

GeV [7]. However, even the latter value seems too low for the applicability of leading twist, low order (LO or NLO) perturbative QCD. The following paragraphs are intended to throw some light on this problem.

In this context let me first comment on the results of recent lattice calculation [19], quoted in [7]. According to [7] these results "confirm the perturbative NLO (2 loop) predictions for $\alpha_s(Q)$ down to $Q = 0.55$ GeV." This claim relies on Fig. 1, taken from [19], where the results of nonperturbative lattice evaluation [6] of the QCD running coupling $\alpha_s^{\rm latt}(q)$ is plotted as a function of q in the region of $q \in (0.5, 14)$ GeV. The agreement between $\alpha_s^{\rm latt}(q)$ and the curve corresponding to 2-loop perturbative β-function, *i.e.* the solution of (1) with only the first two universal terms on its r.h.s., down to 0.5 GeV is indeed remarkable. However, what is demonstrated by Fig. 1 is merely the fact that a particularly defined lattice coupling $\alpha_s^{\rm latt}$ coincides with the coupling defined in the so called 't Hooft RC [7] down to $q = 0.55$ GeV. It is worth emphasizing that even on the lattice there is no unique "nonperturbative" β-function and, consequently, no unique nonperturba-

[6] The definition of $\alpha_s^{\rm latt}$ used in [19] is rather involved and is therefore not mentioned here.

[7] In this RC all nonunique β-function coefficients $c_j, j \geq 2$, are set to zero by definition.

tive coupling $\alpha_s^{\mathbf{latt}}$! While asymptotic freedom of QCD guarantees that at short distances couplings in different RS coincide, they may be arbitrarily far apart at large ones. Fig. 1 contains an interesting evidence for the closeness of two (out of an infinite number) definitions of the coupling, but tells us nothing about the applicability of perturbation theory in any of them. The authors of [19] are well aware of this limitation and on page 495 of [19] therefore write: "We would like to emphasize, however, that our results do not prove that perturbation theory provides a good approximation to all quantities of interest up to couplings as large as 3.48. Such a general statement is bound to be false and the running coupling in our scheme may very well turn to be an exceptional case." [8]

To illustrate the importance of nonperturbative effects at distances 0.3–0.4 fermi, let us consider the magnitude $F(r)$ of the force between two static quarks at a distance r. This quantity has been extensively studied on the lattice and is usually written as the sum

$$F(r) = \frac{4}{3}\frac{\alpha_s^{q\bar{q}}(r)}{r^2} + \kappa = F_p(r) + F_{np}\,, \tag{13}$$

where the first term, dominant at short distances, comes purely from perturbation theory while the second term describes the nonperturbative, long range confining force with κ denoting the string tension. The coupling $\alpha_s^{q\bar{q}}$ in the numerator of (13) is related to $\alpha_s^{\mathbf{latt}}$, mentioned above, as follows [19]

$$\alpha_s^{q\bar{q}}(M) = \alpha_s^{\mathbf{latt}}(M)\left(1 + k_1\alpha_s^{\mathbf{latt}}(M) + \cdots\right)\,, \quad k_1 = 1.33776\,. \tag{14}$$

Evaluating both terms in (13) for $\kappa = (0.48\ \mathrm{GeV})^2$ and two values of M ($M = \mu = 0.55$ GeV, corresponding to $r = 0.37$ fm and $M = \mu_{\mathbf{pert}} = 0.75$ GeV, corresponding to $r_{\mathbf{pert}} = 0.27$ fm), using the values of $\alpha_s^{\mathbf{latt}}$ from [19], we find

$$F_p(0.37\ \mathrm{fm}) \doteq 0.17\ \mathrm{GeV}^2,\ \ \mathrm{F_p}(0.27\ \mathrm{fm}) \doteq 0.27\ \mathrm{GeV}^2,\ \ \mathrm{F_{np}} \doteq 0.24\ \mathrm{GeV}^2. \tag{15}$$

This is the kind of comparison which really tells us how important is the perturbative contribution to a particular physical quantity, in this case the interquark force (13). For this quantity the nonperturbative contribution clearly dominates over the perturbative one at the distance $r = 0.37$ fm and is roughly equal to it at $r = 0.27$ fm, the distance at which dynamical perturbative predictions should, according to [7], "become reliable and experimentally relevant". The above example is merely an illustration and

[8] The value 3.48 quoted above corresponds to g^2.

the relative importance of perturbative and nonperturbative parts may well depend on the physical quantity in question, but it gives at least some indication that at $\mu_{\mathbf{pert}} \doteq 0.75$ GeV perturbation contributions can hardly be expected to be a good approximation to the full results.

One of the basic features of perturbation theory at low momentum scales (large distances) is the increasing sensitivity of finite order perturbative approximants to the choice of the renormalization and factorization schemes and renormalization and factorization scales [9], which makes perturbative predictions progressively more ambiguous in this region. To illustrate the crucial importance at large distances of higher order terms in purely perturbative expansions let me briefly recall the essence of Ref. [21]. There the familiar R-ratio in e^+e^- annihilation into hadrons at the center of mass energy Q

$$R_{e^+e^-}(Q) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\left(\sum_{i=1}^{n_f} e_i^2\right)(1 + r(Q))\,, \tag{16}$$

where in perturbative QCD

$$r(Q) = a(M, \mathrm{RS}) \times \left[1 + r_1\left(\frac{Q}{M}, \mathrm{RS}\right) a(M, \mathrm{RS}) + r_2\left(\frac{Q}{M}, \mathrm{RS}\right) a^2(M, \mathrm{RS}) + \cdots\right] \tag{17}$$

is investigated in the infrared region. This analysis has two important ingredients

- the use of the NNLO approximation to (17), with c_2 chosen by means of the PMS [20],
- smearing of $R_{e^+e^-}(Q)$ over some interval Δ of Q:

$$\overline{R}_{e^+e^-}(Q, \Delta) \equiv \frac{\Delta}{\pi} \int_0^\infty ds \frac{R_{e^+e^-}(\sqrt{s})}{(s - Q^2)^2 + \Delta^2}\,. \tag{18}$$

The second ingredient of this procedure is vital as the detailed structure of $R_{e^+e^-}(Q)$ in the resonance regions is clearly beyond the reach of perturbative QCD. Nevertheless the fact that after the smearing and for not too small values of Δ, the agreement between (18) and the data is quite good, as documented by Fig. 6 of [21], is remarkable. This agreement depends crucially on the fact that c_2, as chosen by the PMS, is negative since for positive

[9] In this note these two in principle different scales will be identified and the dependence on the choice of factorization scheme disregarded.

c_2 the NNLO perturbative expansion for (18) blows up in the IR region, as does the NLO one. Also the magnitude of c_2 is important as it determines the magnitude of $\overline{R}$ in the IR region. The success of such a procedure might look suspicious as in the QCD sum rule approach resonance parameters, and thus also their contribution to the smeared spectra, are dual to power corrections. The observed agreement between data and NNLO perturbative approximation in the PMS approach can be interpreted as a signal that by an appropriate choice of c_2 (or in general of the RC), it may be possible to include in some sense also the effects of these power corrections. Such an interplay of perturbative and nonperturbative contributions is quite plausible, as they actually coexist within OPE. I have mentioned the analysis of [21] merely to emphasize that at large distances the inclusion of NNLO perturbative terms is probably indispensable for meaningful and reasonably complete description of physical quantities.

The message of the previous paragraph may also have some relevance for "perturbative stability" observed within the GRV approach [7] in the comparisons of the LO and NLO approximations to the leading twist DGLAP equations. Since the LO approximation cannot be associated with any well-defined RS, and the importance of higher order terms depends sensitively on the RS, such a comparison makes little quantitative sense. Only by comparing the NLO and NNLO approximation can such information be obtained. Unfortunately, there are only a few simple quantities for which the NNLO calculations are available. The perturbative part (17) of the ratio (16) is one of them . Evaluating just for illustration the first three known terms of (17) at $\mu_{\mathbf{pert}} \doteq 0.75$ GeV, for three flavors and in the conventional $\overline{\mathrm{MS}}$ RS, we find that they are roughly in the ratio $1 : 0.22 : 0.33$! Not only there is no sign of perturbative stability for this quantity, but at such low scales the complicated problem of the presumable divergence of perturbation expansions in fixed RS becomes of utmost phenomenological importance.

4. The interpretation of input parton distributions

Let us now turn to the interpretation of parton distributions at the initial scale μ. While in [3] they were considered to correspond to three *constituent* quarks, according to the latest paper [7] the initial valence-like quark and gluon distributions "should rather be identified with *current* quark content of hadrons." This slight but crucial shift of interpretation should justify why the sea and gluon distributions do not vanish at the initial scale μ, as would seem appropriate for real constituents of the proton. Such an extension would still be reasonable if the additional sea quark and gluon valence-like initial distributions were small admixtures to the basically three valence quarks component of the nucleon.

In [4] the fact that two valence "constituent" gluons at $\mu \approx 0.5$ GeV were required by the data was considered "fine"since "they may combine to give color and spin singlets as it is required for the nucleon.". However, the success of the conventional SU(6) quark model relies on the fact that all color singlet combinations of three constituent quarks do exist in the nature, not only some of them! But in the color singlet state of three quarks and two gluons the latter do not have to couple to a color singlet. The system of three quarks and two gluons would have much richer spectrum of low-lying states than the state of mere three quarks. It would be a kind of "hybrid" states, suggested in the early eighties, but never found. In order to avoid these problems arguments would have to be invented to show why these two constituent gluons must (and not only may) couple to a color and spin singlet. I am not aware of any such argument.

Similar problems arise for the initial distributions in [7], summarized at the end of Section 2. Integrating over the initial distributions without the prefactor x to get the probabilities, we find [10]

$$\int_0^1 G(x,\mu)dx \doteq 1\,, \quad \int_0^1 \left(\overline{u}(x,\mu)+\overline{d}(x,\mu)\right) dx \doteq 1.6\,. \tag{19}$$

The more accurate and copious data used in [7] lead to the result that the initial parton distributions describe a system composed of 2.72 u quarks, 1.88 d quarks, 0.72 $\overline{\mathrm{u}}$ antiquark, 0.88 $\overline{d}$ antiquark and about one valence gluon. So again valence sea (anti)quarks and gluons are by no means a small admixture but on the contrary provide a dominant component of the initial parton distributions.

According to GRV this is no cause for concern as perturbation theory is not expected to hold at the initial scale $\mu \approx 0.55$ GeV, but only above $\mu_{\mathbf{pert}} \doteq 0.75$ GeV. If, however, initial parton distributions do not describe at least approximately physics at the scale μ, what justifies then the adjective "dynamical"? In particular why to impose the fundamental sum rules (4) and (5), or (7), upon which the GRV approach is based? For instance, if the initial parton distributions $q(x,\mu), \overline{q}(x,\mu)$ and $G(x,\mu)$ are irrelevant for physics, why should they satisfy the momentum sum rule (7), which expresses the fact that quarks, antiquarks and gluons carry together the whole momentum of the proton? And why should there be just two u

[10] There is an misleading claim in [7] that their input distribution functions imply that "proton consists dominantly of valence quarks and valence-like gluons, with only 10% $\mathrm{q\overline{q}}$ excitations (sea quarks)." The mentioned 10% concerns the momentum fraction carried by sea quarks and antiquarks, not the probabilities themselves.

and one d quarks at the scale μ? The answer which GRV offer exploits the invariance of these sum rules under the LO DGLAP evolution equations and the fact their they do hold at short distances. This reasoning goes, however, against the very spirit of the DGPD, which I see in the possibility to use some known features of physics at long distances in order to predict physics at short ones. Imposing restrictions on the initial distributions in the situation when the latter have no physical meaning is basically a mathematical game with a little physical content. Nevertheless, such a game can have nontrivial consequences, which, if confirmed by data, would signal some interesting physics behind the GRV approach and would justify it *a posteriori*.

In my view the relation between constituent quarks and partons cannot be described by the leading twist DGLAP evolution equations, even if these were taken to all orders [11]. As pointed out in Section 2, the distance ≈ 0.4 fm, which corresponds to $\mu \approx 0.55$, is not much smaller that the approximate size of three constituent quarks in the proton. The whole point of introducing the concept of constituent quarks is that it represents the *effective* degree of freedom appropriate for describing the proton at distances comparable to its size. Contrary to the current quarks, which are associated with quark fields entering directly the QCD Lagrangian, constituent quarks have no such firm basis and are merely an intuitively introduced concept, a kind of quasiparticle, which is reasonably well-defined only in the nonrelativistic quark model! The phenomenological success of this model suggests that in low momentum transfer processes the proton behaves approximately as composed of three constituent quarks, each with a mass of about 300 MeV. Constructing the fields corresponding to constituent quarks from those of the current quarks could probably be compared to the Bogolubov transformation between electrons and fermionic quasiparticles in the BCS theory of superconductivity. We expect the transition from short distances, where current partons are the right degrees of freedom, to large ones, where constituent quarks are the appropriate effective degrees of freedom, to be smooth, but involve complicated multiparton effects. In the framework of OPE such effects are described by multiparton distributions, which naturally appear as a part of power corrections [23]. We cannot hope to get constituent quarks at low scale from quarks, antiquarks and gluons at large scales merely by means of the leading twist DGLAP evolution equations.

5. Valence-like initial distributions and factorization

In this Section the compatibility of the GRV approach with the mechanism of factorization of parallel singularities will be discussed. In order to

[11] For related discussion see [22].

make the discussion as clear as possible and to concentrate on the essence of the problem, a number of simplifications will be made.

First, we shall be primarily interested in the low x domain, roughly $x \leq 10^{-2}$. Secondly, all the considerations will be done within the LO DGLAP equations. The inclusion of the NLO corrections is not essential for any of the points discussed below. In the LO approximation the basic quantity of interest, proton structure function $F_2^{\mathrm{ep}}(x,Q^2)$, can be expressed in terms of elementary quark distributions as follows:

$$F_2^{\mathrm{ep}}(x,Q^2) = x\Big[\frac{4}{9}\left(u(x,Q^2)+\overline{u}(x,Q^2)\right) + \frac{1}{9}\left(d(x,Q^2)+\overline{d}(x,Q^2)+s(x,Q^2)+\overline{s}(x,Q^2)\right)\Big]. \quad (20)$$

Assuming SU(3) symmetry of the proton sea the r.h.s. of (20) can be written as a combination of the valence u_v, d_v and the common sea $D \equiv u_{\mathrm{sea}} = d_{\mathrm{sea}} = s_{\mathrm{sea}}$ distributions

$$F_2^{\mathrm{ep}}(x,Q^2) = \frac{4}{9}xu_v(x,Q^2) + \frac{1}{9}xd_v(x,Q^2) + \frac{4}{3}xD(x,Q^2), \quad (21)$$

or, alternatively, as a sum of separate contributions from u and d quarks

$$F_2^{\mathrm{ep}}(x,Q^2) = \left(\frac{4}{9}xu_v(x,Q^2) + \frac{4}{3}xD^{(u)}(x,Q^2)\right) + \left(\frac{1}{9}xd_v(x,Q^2) + \frac{4}{3}xD^{(d)}(x,Q^2)\right). \quad (22)$$

In terms of conventional moments of various functions (distribution, branching, *etc.*)

$$f(n,Q) \equiv \int_0^1 x^n f(x,Q)dx \quad (23)$$

the LO DGLAP evolution equation for the nonsinglet quark distribution reads

$$\frac{dq_{\mathrm{NS}}(n,Q)}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}P_{qq}^{(0)}(n)q_{\mathrm{NS}}(n,Q), \quad (24)$$

while for the quark singlet and gluon distributions we have a system of coupled equations

$$\frac{dG(n,Q)}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\left[P_{GG}^{(0)}(n)G(n,Q) + P_{Gq}^{(0)}(n)(q(n,Q)+\overline{q}(n,Q))\right], \quad (25)$$

$$\frac{d(q(n,Q)+\overline{q}(n,Q))}{d\ln Q} = \frac{\alpha_s(Q)}{\pi} \times \left[2n_f P_{qG}^{(0)}(n)G(n,Q) + P_{qq}^{(0)}(n)(q(n,Q)+\overline{q}(n,Q))\right]. \quad (26)$$

As in the small x region the evolution of the gluon distribution is driven by the branching $G \to G + G$, we shall furthermore drop the second term on the r.h.s. of (25). Moments of the gluon distribution satisfy then the same kind of differential equation as quark nonsinglet distribution:

$$\frac{dG(n,Q)}{d\ln Q} = \frac{\alpha_s(Q)}{\pi} P^{(0)}_{GG}(n) G(n,Q) \tag{27}$$

and therefore also the corresponding solutions have the same form

$$q_{\rm NS}(n,Q) = A_{\rm NS}(n) \left[\frac{ca(Q)}{1+ca(Q)}\right]^{-P^{(0)}_{qq}(n)/b}, \tag{28}$$

$$G(n,Q) = A_G(n) \left[\frac{ca(Q)}{1+ca(Q)}\right]^{-P^{(0)}_{GG}(n)/b}, \tag{29}$$

where $A_{\rm NS}(n), A_G(n)$ are unique finite constants, determining the asymptotic behaviour of the moments $q_{\rm NS}(n,Q), G(n,Q)$ as $Q \to \infty$. According to the factorization mechanism [24] these constants contain all the information on long range properties of the nucleon, incalculable in perturbation theory. They represent one way of specifying the boundary conditions on the solution of evolution equations (24) and (27). Another and almost, but not entirely, equivalent way follows from taking the ratio of (28) and (29) at two different scales. In this case boundary conditions are specified at finite initial Q_0 and we have

$$q_{\rm NS}(n,Q) = q_{\rm NS}(n,Q_0) \left[\frac{a(Q_0)}{a(Q)} \frac{1+ca(Q)}{1+ca(Q_0)}\right]^{P^{(0)}_{qq}(n)/b}, \tag{30}$$

$$G(n,Q) = G(n,Q_0) \left[\frac{a(Q_0)}{a(Q)} \frac{1+ca(Q)}{1+ca(Q_0)}\right]^{P^{(0)}_{GG}(n)/b}. \tag{31}$$

Let me first discuss the compatibility of the initial valence-like distributions with the factorization on the simpler case of the gluon density $G(x,Q)$. Note that although they are of the same form, there is a profound difference between the solutions (28) and (29) for $n = 0$ (the moment giving the integral over the parton distributions). This is due to the fact that while $P^{(0)}_{qq}(0) = 0$, we have $P^{(0)}_{GG}(0) = +\infty$! The former is a consequence of the quark number conservation in the $q \to q + G$ branching, while the latter comes from the $1/x$ spectrum of soft gluons. Eq. (28) implies that the integral over the nonsinglet quark density, $q_{\rm NS}(0,Q) = A_{\rm NS}$ is independent of Q and provided it is finite at some Q it stays so everywhere. For the gluons there are two possibilities:

a) $A_G(0) > 0$ and then $G(x,Q)$ cannot be valence-like at any Q for which $a > 0$ since

$$G(0,Q) = A_G(0)\left[\frac{1+ca(Q)}{ca(Q)}\right]^{+\infty} = A_G(0)(+\infty) = \infty, \tag{32}$$

b) $A_G(0) = 0$, in which case (32) is ill-defined, but (31) applied to $n = 0$ can still be used

$$G(0,Q) = G(0,Q_0)\left[\frac{a(Q_0)}{a(Q)}\frac{1+ca(Q)}{1+ca(Q_0)}\right]^{+\infty}. \tag{33}$$

If we now impose valence-like behavior on the gluon distribution at the initial $Q_0 = \mu$, *i.e.* assume finite $G(0,\mu)$, (33) implies, due to monotonous behavior of the square bracket as a function of Q, that $G(0,Q)$ is a discontinuous function of the factorization scale Q at $Q = \mu$:

$$\begin{aligned} &G(0,Q) = +\infty, \quad \forall Q > \mu \\ &G(0,\mu) = \text{const.} > 0 \\ &G(0,Q) = 0 \qquad \forall Q < \mu\,. \end{aligned} \tag{34}$$

Similar discontinuity at the initial scale μ appears also for sea quarks and antiquarks.

In the case b), realized in the GRV approach, it is difficult to understand why the divergence of $P^{(0)}_{GG}(0)$, which is a purely perturbative phenomenon, should be accompanied by the vanishing of nonperturbative quantity $A_G(0)$. For instance, in $4-\epsilon$ dimensions $P^{(0)}_{GG}(n,\epsilon)$ is finite and there is no obvious reason to expect $A_G(0,\epsilon) = 0$. Sending $\epsilon \to 0$ we understand why $P^{(0)}_{GG}(0,\epsilon) \to \infty$, but why should simultaneously $A_G(0,\epsilon) \to 0$? Although the lowest twist contribution may not provide a reliable description of the proton at the initial scale μ, we are not free to impose arbitrary constraints on its properties. Despite these reservations, the case b) is certainly mathematically interesting option and I shall therefore in the rest of this section analyze some if its consequences, in particular for the behavior of $F_2^{ep}(x,Q)$ as a function of Q.

In the small x region, to which we restrict our attention, the LO expression for the distribution of sea quarks within a single quark which at the scale μ is described by the initial distribution $\delta(1-x)$, has the form [25]

$$D_0(x,Q) = \frac{1}{x}\frac{32}{3}C_2\zeta^2 e^{-a\zeta}\frac{I_2(v)}{v^2}, \tag{35}$$

where for three colors and n_f flavors

$$a \equiv 11 + \frac{2n_f}{27}, \quad \zeta \equiv \frac{1}{2b} \ln \frac{a(\mu)}{a(Q)}, \quad v \equiv \sqrt{48\zeta \ln\left(\frac{1}{x}\right)}, \quad C_2 = \frac{4}{3} \tag{36}$$

and $I_2(v)$ is the modified Bessel function. For general initial distribution $q(x,\mu)$ we have

$$\begin{aligned} D(x,Q) &= \int_x^1 \frac{dy}{y} q(y,\mu) D_0(x/y,Q) \\ &= \frac{1}{x}\frac{32}{3} C_2 \zeta^2 \mathrm{e}^{-a\zeta} \int_x^1 dy q(y,\mu) \frac{I_2(w)}{w^2}, \end{aligned} \tag{37}$$

where now $w \equiv \sqrt{48\zeta \ln(y/x)}$. Note that for fixed x and $\zeta \to 0$, corresponding to $Q \to \mu^+$, $v \to 0$ and the function $D_0(x,Q)$ behaves as

$$D_0(x,Q) = \frac{1}{x}\frac{4}{3} C_2 \zeta^2, \tag{38}$$

vanishing for $\zeta = 0$, *i.e.* at $Q = \mu$. However, the physically relevant case of fixed $Q > \mu$ and $x \to 0$ corresponds to slightly more complicated limit $v \to \infty$. Eq. (35) then implies

$$D_0(x,Q^2) = \frac{1}{x}\frac{32}{3} C_2 \zeta^2 \mathrm{e}^{-a\zeta + v(x)} \frac{1}{\sqrt{2\pi v(x)}} \frac{1}{v^2(x)}, \tag{39}$$

where the terms depending on $v(x)$ induce additional dependence on x. This modifies slightly the behavior of $D_0(x,Q)$ at small x, but does not change its nonintegrability due to the dominant $1/x$ factor. Consequently, $D_0(x,Q)$, considered as a function of Q does not vanish uniformly in the whole interval of $x \in (0,1)$ when $Q \to \mu$! The same holds for $D(x,Q)$. This nonuniform convergence means that for Q arbitrarily close to the input $Q_0 = \mu$, there is always a region of x close to $x = 0$, where approximately $D(x,Q) \propto 1/x$. And it is this region which causes the divergence of the integral

$$\int_0^1 dx D(x,Q) = \infty, \tag{40}$$

for any $Q > \mu$. In the conventional approach with singular, *i.e.* nonintegrable, initial distributions, this discontinuity is absent. Using Eq. (37)

and assuming for small x quark initial distribution in the form $q(x,\mu) = Ax^{-\lambda}$, $\lambda > 1$, we find that for small x and $Q \to \mu^+$ the integral

$$\int_x^1 dy q(y,Q) \frac{I_2(w)}{w^2} \tag{41}$$

behaves differently than for $\lambda < 1$. For $\lambda > 1$ the integrand of (41) is a nonintegrable function of x in the interval $(0,1)$, finite lower integration bound is therefore crucial and we get for any $Q > \mu$

$$\int_x^1 dy q(y,Q) \propto \frac{A}{\lambda - 1} x^{1-\lambda} \implies D(x,Q) \propto \frac{1}{x} \int_x^1 dq(y,\mu) \propto Ax^{-\lambda}. \tag{42}$$

The singular initial distribution overrides the radiation pattern characterizing the emissions from individual quarks and the radiated sea quark distribution $D(x,Q)$ is therefore of the same form as the initial $q(x,\mu)$. This well-known feature of singular initial distributions implies that for low x the form of the x-dependence is essentially independent of Q and the initial scale plays no exceptional role.

6. Conventional partons in the small x region

In this section results of the conventional analysis based on singular initial distributions will be cast into a simple form suitable for the comparison with the GRV results. In order to check the very essence of the GRV approach only the results based on the original initial conditions (3) with vanishing antiquark and gluon initial distributions will be discussed. Moreover, I shall concentrate on the small x region, where the approximations of the previous section are expected to hold [25]. Neglecting the second term in (25) the function $H(x,M) \equiv xG(x,M)$ satisfies the equation

$$\frac{dH(x,M)}{d\ln M} = a(M) \int_x^1 \frac{dz}{z} \left(z P_{GG}^{(0)}(z) \right) H\left(\frac{x}{z}, M \right), \tag{43}$$

where

$$P_{GG}^{(0)}(z) \equiv 6 \left(\left[\frac{z}{1-z} \right]_+ + \frac{1-z}{x} + z(1-z) + \left(\frac{33 - 2n_f}{36} - 1 \right) \delta(1-z) \right). \tag{44}$$

Assuming $H(x,M)$ in a singular factorizable form

$$H(x,M) = x^{-\lambda}F(M,\lambda), \quad \lambda > 0 \tag{45}$$

and substituting (45) into (43) we get

$$\frac{dF(M,\lambda)}{d\ln M} = a(M)F(M,\lambda)\left(C(\lambda) - \int_0^x dz z^{\lambda} P_{GG}^{(0)}(z)\right), \tag{46}$$

where

$$\gamma^{(0)}(\lambda) \equiv \int_0^1 dz z^{\lambda} P_{GG}^{(0)}(z)$$

$$= 6\frac{\lambda(\lambda+1)+(\lambda+2)(\lambda+3)}{\lambda(\lambda+1)(\lambda+2)(\lambda+3)} - \frac{3}{2} - 6\lambda\sum_{k=0}^{\infty}\frac{1}{(2+k)(\lambda+2+k)}. \tag{47}$$

extends the definition of the LO anomalous dimension $\gamma^{(0)}(n)$ to noninteger positive values of λ. Note that $\gamma^{(0)}(\lambda)$ is a decreasing function of its argument and negative for λ above $\lambda_0 \approx 0.85$. The second term in the brackets of (46) is proportional to x^{λ} and can therefore be neglected in the small x region. $F(M,\lambda)$ then satisfies the equation

$$\frac{dF(M,\lambda)}{d\ln M} = a(M)\gamma^{(0)}(\lambda)F(M,\lambda), \tag{48}$$

which has a simple solution

$$F(M,\lambda) = A\,(a(M))^{-\gamma^{(0)}(\lambda)/b}, \tag{49}$$

where A is an arbitrary overall normalization constant.

In the small x region the comparison of the GRV and conventional results can be made particularly transparent by translating (49) into the corresponding expression for the proton structure function $F_2^{ep}(x,Q^2)$ by means of the LO Prytz's relation [26]

$$\frac{dF_2^{ep}(\frac{x}{2},Q)}{d\ln Q} = \kappa a(Q)H(x,Q), \qquad \kappa = \frac{20}{27}. \tag{50}$$

This formula was shown to be a good approximation for F_2 in the low x region and should be sufficient for our purposes. Alternatively, we could solve the DGLAP evolution equations for the coupled quark singlet and

gluon distributions exactly, and then approximate these solutions by the power-like behavior, but the procedure based on the combination of (49) and (50) is much simpler. Anticipating also F_2^{ep} in the factorizable form

$$F_2^{ep}(x,Q) = x^{-\lambda} F_2^{ep}(Q,\lambda)\,, \tag{51}$$

and using (50) in combination with the explicit result (49) for $F(M,\lambda)$ we get

$$\frac{dF_2^{ep}(M,\lambda)}{d\ln M} = A\kappa \left(\frac{1}{2}\right)^{\lambda} (a(M))^{-\gamma^{(0)}(\lambda)/b+1}\,, \tag{52}$$

from where we obtain

$$F_2^{ep}(M,\lambda) = \frac{\kappa}{2\gamma^{(0)}(\lambda)} F(M,\lambda) = \frac{A\kappa}{\gamma^{(0)}(\lambda)} \left(\frac{1}{2}\right)^{\lambda} (a(M))^{-\gamma^{(0)}(\lambda)/b}\,. \tag{53}$$

The positivity of $F_2^{ep}(M,\lambda)$ requires positive $C(\lambda)$, which in turn implies $\lambda < \lambda_0$. The measured behavior of F_2^{ep} in the small x region is well within this limit.

7. DGPD vs. conventional partons — numerical comparison

Can DGPD be distinguished from the conventional parton parametrizations at all? In order to identify reasonably unambiguous signatures of DGPD I shall concentrate on its "orthodox" version.

In the first kind of comparisons, the GRV results for $F_2^{ep}(x,Q^2)$ were obtained via (22) from u and d quark valence-like initial distributions (8)–(9) [12]. The valence distributions were evolved by means of the standard DGLAP evolution equations using the method of Jacobi polynomials [27, 28]. For the sea parts (37) was used. Three light quarks were taken into account in generating the sea. In order to facilitate the absolute comparison between the two approaches without reference to experimental data, the constants A and λ in (49) were related in such a way that the conventional results coincide with the GRV ones for $x = 10^{-3}$ and $Q^2 = 5$ GeV^2. As a result A becomes a function of λ. The choice of the normalization point is, of course, to a large extent arbitrary, but none of the basic messages of this section depends on it. In Fig. 2a results of the GRV approach are plotted for 10 values of $Q^2 = 0.35$, 0.4, 0.6, 1, 2, 5, 10, 20, 50, 100 GeV^2, *i.e.* starting very close to the initial $\mu^2 = 0.34$ GeV^2. The thick solid curve corresponds to the initial $F_2^{ep}(x,\mu)$. In Fig. 2b the corresponding sea

[12] No essential features of the following comparisons depend on this particular choice of the initial distributions.

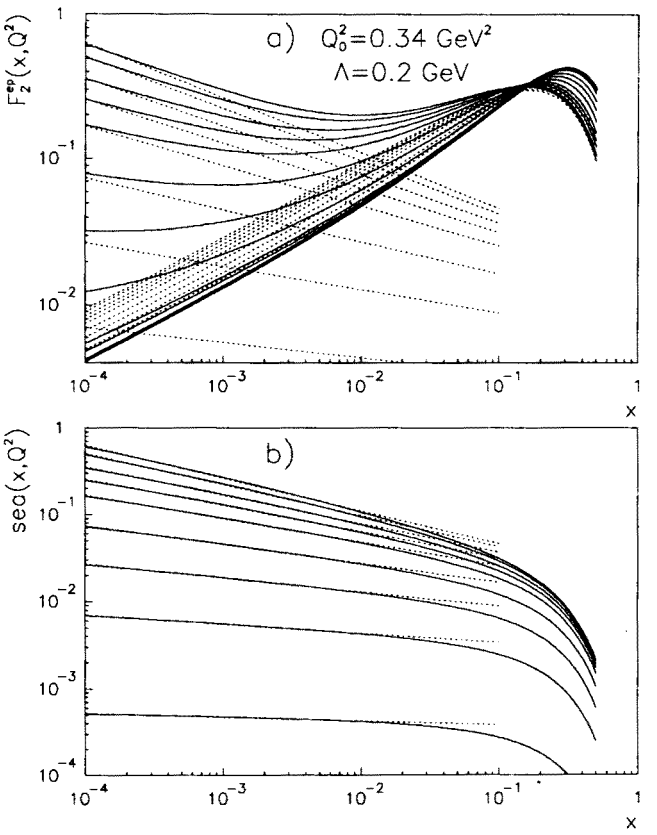

Fig. 2. a) $F_2^{ep}(x,Q^2)$ as a function of x for the initial distribution (8)–(9) and 10 values of Q^2 defined in the text. In the low x region the curves are in ascending order in Q^2 starting from below. The dotted lines correspond to the power-like fits described in the text and the thick solid curve describes the initial $F_2^{ep}(x,\mu)$. b) The same as in a), but for the total sea component only.

distributions, which in the low x region are well approximated by the power-like behavior of the form $x^{-\lambda}$, are plotted. Figs 2a, 2b nicely illustrate the way $F_2^{ep}(x,Q)$ approaches the input function $F_2^{ep}(x,\mu)$ as $Q \to \mu^+$. In particular the nonuniform convergence, discussed in Section 5, is clearly visible. As $Q \to \mu^+$ the region of increasing F_2^{ep} moves steadily to the left, but regardless of how close Q is to μ, such a region eventually appears, leading to the discontinuity of the integral over quark distributions at the initial scale μ.

Fitting for each Q^2 the total GRV sea contribution in the interval of $x \in (10^{-4}, 10^{-2})$ to the form $A(Q)x^{-\lambda(Q)}$ leads to characteristic dependence of the parameters $A(Q)$ and $\lambda(Q)$ on Q, displayed in Figs 3a, 3b by thick solid curves. The corresponding power-like fits are shown as dotted curves in Figs 2a, 2b. As $Q \to \mu^+$ the interval where they provide a good approximation to the full GRV results shifts systematically to smaller values of x, reflecting the shift to the left of the relative importance of the sea component.

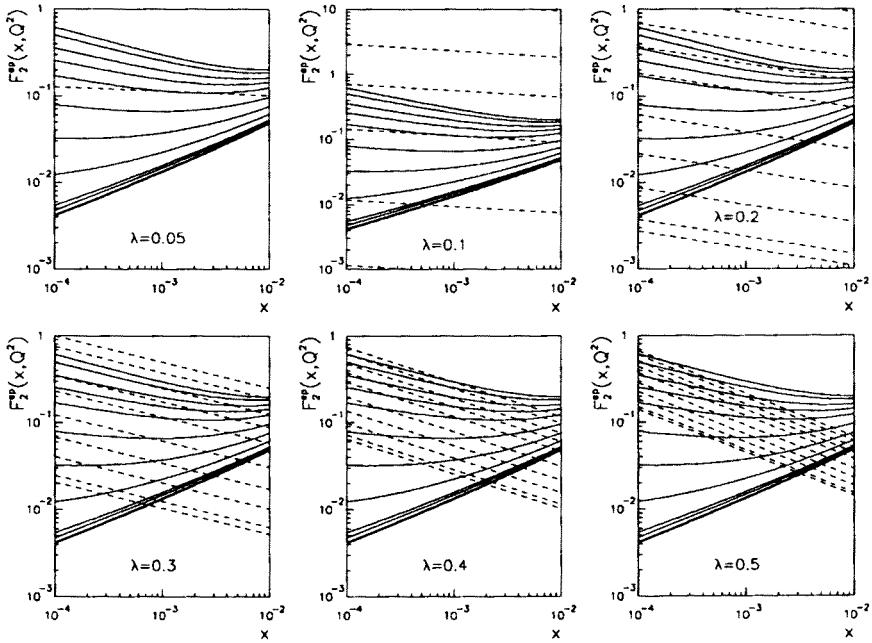

Fig. 3. Q^2-dependence of the normalization factor $A(Q)$ (a) and the exponent $\lambda(Q)$ (b). In a) the thick solid curve is given by the power-like fit of the GRV results, while the dashed lines, describing $F_2^{ep}(Q, \lambda)$ of the conventional approach, correspond to six values of $\lambda = 0.05$, 0.1, 0.2, 0.3, 0.4 and 0.5. In b) the thick solid curve describes again the GRV result for the initial distributions (8)–(9), while the other curves correspond (from above in decreasing order of α) to five initial distributions (54) with $\beta = 3$ and $\alpha = 0.1$, 0.3, 0.5, 0.7, 0.9. In b) similar curves for $\beta = 4$ would be essentially indistinguishable from those in the figure.

In Fig. 4 GRV results are compared with those of the conventional approach, shown as dashed lines, for six values of λ and the same values of Q^2 as in Fig. 2. The lowest, thick solid curves of the GRV approach

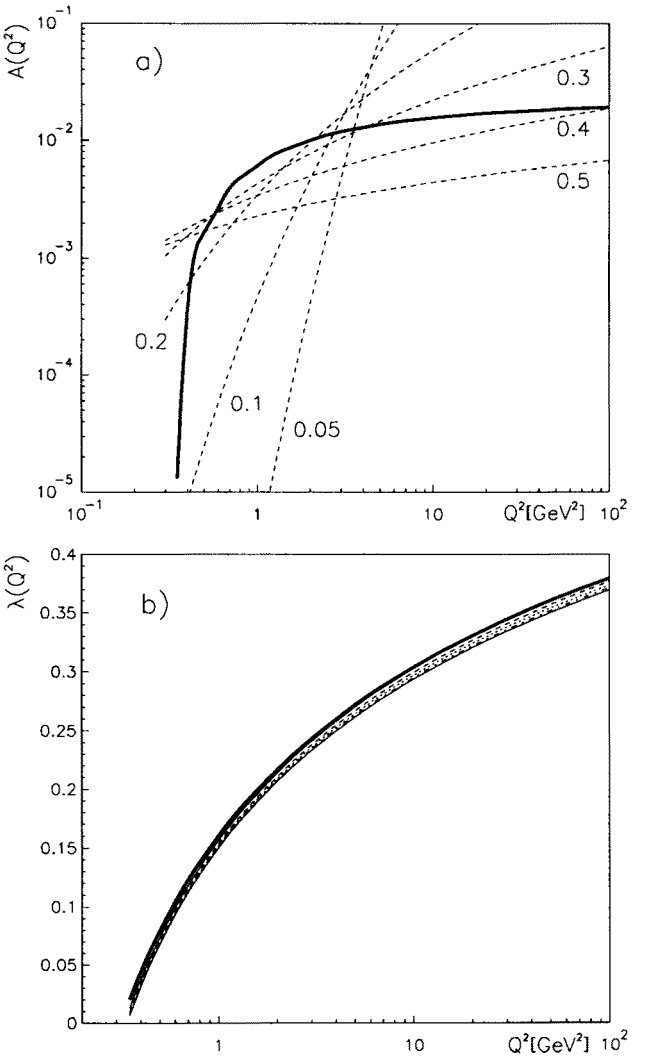

Fig. 4. The comparison of the GRV (solid curves) and conventional (dashed curves) results for $F_2^{\mathrm{ep}}(x, Q^2)$ separately for six values of λ in the latter approach.

correspond again to the initial distributions (8)–(9) at $\mu^2 = 0.34$ GeV^2 and have no analog among the dashed curves. Note that for smaller values of λ not all curves of the conventional approach fit into the frame of the plots. To summarize the message of the Fig. 4, $F_2^{\mathrm{ep}}(Q, \lambda)$, defined in (51), is plotted in Fig. 3a as a function of Q^2 for six values of λ together with the corresponding function $A(Q)$ of the GRV approach.

In order to investigate the sensitivity of the parameters $\lambda(Q)$ and $A(Q)$ in the GRV approach to the choice of the initial distributions, I have repeated the calculations presented in Fig. 2a for the set of u and d quark initial distributions of the form

$$xq_v(x,\mu) = Ax^{\alpha}(1-x)^{\beta}, \tag{54}$$

for two values of $\beta = 3, 4$ and five values of $\alpha = 0.1, 0.3, 0.5, 0.7, 0.9$. In all cases the overall normalization factor A was fixed to give two u and one d quark at the initial scale μ. Results corresponding to $\beta = 3$ and the five mentioned values of α are displayed in Figs 5a–5e. The general pattern remains the same as in Fig. 2a, but there is a marked difference between the dependence of the exponent $\lambda(Q)$ (curves included in Fig. 3b) and the normalization factor $A(Q)$ (see Fig. 5f) on α and β (not shown). While the exponent $\lambda(Q)$ depends on the values of both α, β very weakly, $A(Q)$ depends on α strongly and β moderately. The insensitivity of the exponent $\lambda(Q)$ to details of the initial distributions can be understood by closer inspection of the formula (37). The figures 3–5 suggest two distinct features of the GRV results:

- The characteristic Q^2 dependence of the exponent $\lambda(Q)$. In the conventional approach λ is arbitrary Q^2-independent number, while in the GRV approach it is an almost unique function of Q, which starts rapidly from zero at the initial scale μ, but then progressively slows down. The most sensitive region is clearly that close to the initial μ, but even at large Q^2 the characteristic Q^2 dependence of GRV results persists. In the region of Q^2 between 2 and 100 GeV^2, where leading twist can perhaps be trusted and data from HERA are available, $\lambda(Q^2)$ varies slowly in the range $(0.25, 0.38)$. Unfortunately the current experimental error on λ at HERA is too large for a reliable discrimination of this dependence from a constant one, characterizing the conventional approach. The pinning down the Q^2 dependence expected in the GRV approach reliably would require lowering the experimental error on $\lambda(Q)$ to about 0.03 in a broad range of Q^2.
- The Q^2 dependence of the normalization factors $A(Q)$ and $F_2^{ep}(Q,\lambda)$ respectively. Here the differences are much more pronounced and increase as we go to both small and large values of Q^2. They depend also much more on the choice of the initial distributions. In GRV approach we cannot go below the initial scale, while in the conventional approach there is no such strict limitation in the small x region. Fig. 3a indicates that the region close to the initial scale is particularly sensitive to the differences between the two approaches.

The above comparisons show that the "orthodox" version of GRV partons could be easily distinguished from the conventional parametrizations,

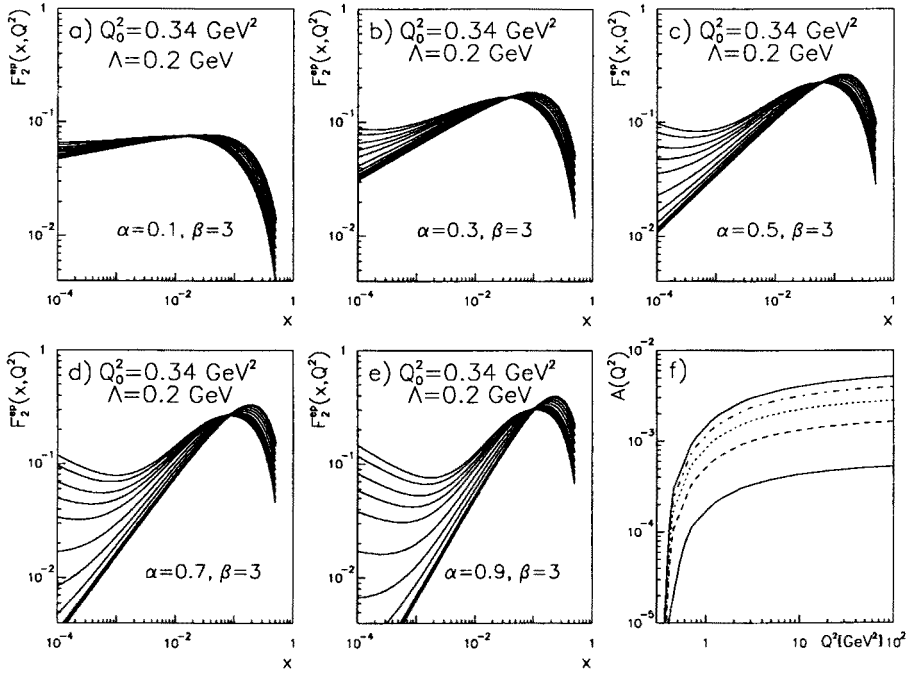

Fig. 5. The same as in Fig. 2a (a–e) and Fig. 3a (f), but for $\beta = 3$ and five values of α in (54). In f) the curves are ordered from above in order of increasing α.

based on the singular initial distributions. However, once valence-like initial gluons and antiquarks are added, the difference in the behavior of normalization factors $A(Q)$ and $F_2^{ep}(Q,\lambda)$ largely disappears. This is documented by the fact that the latest GRV as well as the conventional parametrizations can accommodate new HERA data [9, 10] at low x and in a broad Q^2 range equally well. On the other hand the characteristic Q^2 dependence of the exponent $\lambda(Q)$ remains essentially unchanged as it is still given by the basic radiation pattern (35). Similarly even in the refined versions of DGPD [6, 7] $F_2^{ep}(x,Q)$ approaches the initial structure function $F_2^{ep}(x,\mu)$ in same nonuniform manner as discussed in Section 5 and displayed in Figs 2 and 5. The available experimental data either reach sufficiently low values of $x \approx 10^{-4}$ but stop at $Q^2 \geq 2$ GeV^2 (H1 and ZEUS at HERA), or reach low Q^2 but only touch the crucial region of $x \leq 10^{-3}$ (E665 at Fermilab). Despite its limited x range the recent E665 data indicate deviation from the latest GRV parametrizations for x below 10^{-2} and $Q^2 \leq 1$ GeV^2 [29]. New quantitative tests are expected from upgrades of H1 and ZEUS detectors,

which will extend the region of accessible Q^2 down to a fraction of GeV^2 and could soon throw some light on the questions discussed in this paper.

8. Summary and conclusions

In this paper I have discussed the basic idea and consequences of the DGPD from physical as well as mathematical points of view. I have argued that it runs into problems if we attempt to interpret the properties of initial distributions in physical terms. The only way to avoid these problems, and the one adopted by GRV, is to assume that the initial scale μ lies outside the range of validity of leading twist perturbative QCD. All the peculiar properties discussed in Sections 4, 5 are then of no concern, but at the same time the approach looses its physical justification and becomes primarily an exercise in mathematics. Investigating the consequences of different initial parton distributions on the solutions of LO/NLO DGLAP evolution equations is an interesting mathematical problem of its own and I have therefore devoted the second part of this paper to it. In particular I have looked for signatures in the behavior of $F_2^{\mathrm{ep}}(x,Q^2)$ that would provide clear signals that the GRV dynamics is at work. The only, but on the other hand rather unique signature of this kind is the characteristic Q^2 dependence of the exponent $\lambda(Q)$ in (51). To best way of pinning down this dependence at HERA is to extend the Q^2 range below 1 GeV^2, where the variation of $\lambda(Q)$ is strongest, and study in detail the Q^2 variation of $\lambda(Q)$ in the whole available Q^2 range. But even then a significant increase in the precision of measuring λ is necessary before a definite conclusion on the Q^2-dependence of λ can be drawn.

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