NUCLEON SPIN STRUCTURE FROM THE INSTANTON VACUUM*

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We discuss the evaluation of the nucleon isoscalar axial coupling, $g_A^{(0)}$, in the instanton vacuum using the $1/N_c$ expansion. This approach allows a fully consistent treatment of the $\mathrm{U}(1)_A$ anomaly. We compute the nucleon matrix element of the topological charge, $\langle N|F\widetilde{F}|N\rangle$, and show that it reduces to the matrix element of the isoscalar axial quark current. Our arguments show that the usual evaluation of $g_A^{(0)}$ in the chiral quark soliton model is consistent with the $\mathrm{U}(1)_A$ anomaly in leading order of $1/N_c$. Such calculations give $g_A^{(0)} = 0.36$, which is in agreement with the recent estimate by Ellis and Karliner.

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The so-called "proton spin crisis" has attracted a lot of interest in recent years [1]. The moments of the measured polarized proton structure function, g_1, combined with information from neutron and hyperon beta decay, allows to extract the value of the isosinglet axial coupling constant of the nucleon, $g_A^{(0)}$. This quantity is defined as the nucleon matrix element of the isosinglet axial current (we assume N_f quark flavors),

$$J_{5\mu}(x) = \sum_f^{N_f} \bar{\psi}_f(x)\gamma_\mu\gamma_5\psi_f(x), \tag{1}$$

$$\langle N|J_{5\mu}(0)|N\rangle = g_A^{(0)}\bar{u}\gamma_\mu\gamma_5 u. \tag{2}$$

In the naive parton model, $g_A^{(0)}$ can be interpreted as the fraction of the nucleon spin carried by the quarks, commonly denoted by $\Delta\Sigma$. The value

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originally obtained by EMC was consistent with zero [2], while an analysis of newly available data by Ellis and Karliner comes to a value of $g_A^{(0)} = 0.27 \pm 0.04 \pm \ldots$ [1]. These results have prompted many theoretical investigations, which provided new insights about the relation of the parton model language to QCD.

The question about a parton interpretation aside, it remains a challenge to understand the small value of $g_A^{(0)}$ in QCD. Following the work of Brodsky *et al.* [3], who showed that $g_A^{(0)} = 0$ in the Skyrme model, this quantity has been estimated in effective chiral models of the nucleon with explicit quark degrees of freedom, such as the chiral bag model [4] and the chiral quark soliton model [5, 6], which give non-zero results in the subleading order of $1/N_c$. In these approaches one faces the question how the U(1) axial current operator of QCD, Eq. (1), is to be represented consistently in terms of the degrees of freedom of the effective model. This problem does not arise for conserved currents related to "good" symmetries of the strong interactions, like the electromagnetic or isovector axial current. The U(1) axial current of QCD, however, has an anomalous divergence,

$$\partial_\mu J_{5\mu}(x) = \frac{N_f}{16\pi^2} F\widetilde{F}(x) + 2i \sum_f^{N_f} m_f \bar{\psi}_f \gamma_5 \psi_f(x) , \qquad (3)$$

which is non-zero in the chiral limit, $m_f \to 0$. Obviously, a consistent definition of the U(1) axial current in an effective description of QCD requires an understanding of the role of gluonic degrees of freedom — in particular, of topological fluctuations.

In this note we want to show how a fully consistent and quantitative description of the nucleon isoscalar axial coupling can be achieved in the framework of the instanton vacuum. The relevance of instantons has been amply demonstrated by phenomenology as well as by lattice calculations [7, 9]. In particular, instantons describe the spontaneous breaking of chiral symmetry, which is the most important non-perturbative phenomenon determining the structure of light hadrons, including the nucleon [8, 9]. Using a $1/N_c$-expansion, one derives from the instanton vacuum an effective quark action in the form of a Nambu–Jona-Lasinio model, which provides a convenient tool for computing correlation functions of meson and baryon currents. This leads to a description of baryons as chiral solitons — N_c "valence" quarks moving in a self-consistent meson field [10]. Such an approach gives a very good description of the static properties of non-strange and strange baryons as well as their various form factors [11].

On the other hand, the instanton vacuum provides a microscopic picture of the non-perturbative fluctuations of the gluon field. It therefore offers the possibility to include the effects of the $U(1)_A$ anomaly in an effective description of hadrons. In the meson sector, the solution of the so-called $U(1)_A$ problem — the large mass difference between the η and η' meson — has been one of the first successes of the instanton vacuum [12]. Recently, a method has been developed to evaluate nucleon matrix elements of gluon operators in the instanton vacuum. This method is based on the variational description of the instanton ensemble by Diakonov and Petrov [13] and the $1/N_c$-expansion in the quark sector. Within this approach, we can evaluate $g_A^{(0)}$ in a way which is fully consistent with the $U(1)_A$ anomaly. The consistency may be demonstrated as follows. In QCD, the anomaly equation, Eq. (3) is an operator equation, and $g_A^{(0)}$ may be equivalently expressed as the nucleon matrix element of the topological charge density, $F\tilde{F}(x)$,

$$\langle N|F\tilde{F}(0)|N\rangle \;=\; \frac{32\pi^2}{N_f} g_A^{(0)} m_N \, \bar{u}\, i\gamma_5 u \, . \tag{4}$$

Using the method of [14], we can now compute the nucleon matrix element of $F\tilde{F}(x)$ in the instanton vacuum and show explicitly that the value of $g_A^{(0)}$ obtained from Eq. (4) agrees with the nucleon matrix element of the usual isosinglet axial quark current. In other words, the instanton vacuum allows to evaluate the nucleon matrix element of both the left- and the right-hand side of the anomaly equation, Eq. (3), and we verify that both agree within our scheme of approximations.

In the following, we give a brief outline of the approach developed in [14]. We first discuss how the $U(1)_A$-anomaly determines the dispersion of topological fluctuations of the numbers of instantons and antiinsantons in the ensemble. We then formulate the prescription to calculate the nucleon matrix element of $F\tilde{F}$, and verify that the $U(1)_A$ anomaly is realized at the level of nucleon matrix elements. Finally, we consider the implications of this result for calculations of $g_A^{(0)}$ within the chiral soliton model [5, 6].

$U(1)_A$-anomaly and fluctuations of the numbers of instantons. In the instanton approach the partition function of Euclidean Yang–Mills theory is reduced to a statistical ensemble of instantons (I) and antiinstantons ($\bar{I}$). An essential property of this description is that the number of "pseudoparticles" is not fixed (grand canonical ensemble). This is a necessary consequence of the trace and $U(1)_A$ anomalies of QCD. In fact, the anomalies unambiguously determine the dispersion of the fluctuations of the number of I's (N_+) and $\bar{I}$'s (N_-) in the ensemble. The dispersion of fluctuations of the total nuber of instantons, $N_+ + N_-$, is governed by the trace anomaly (see [14] for a discussion). Fluctuations of the difference of the number of

I's and $\bar{I}$'s,

$$\Delta \equiv N_+ - N_- \neq 0, \tag{5}$$

are determined by the topological susceptibility of the vacuum,

$$\langle \Delta^2 \rangle = \langle Q_t^2 \rangle, \qquad Q_t \equiv \frac{1}{32\pi^2} \int d^4x \, F\tilde{F}(x). \tag{6}$$

This follows from the fact that a single $I(\bar{I})$ has topological charge ± 1,

$$\frac{1}{32\pi^2} \left(\int d^4x \, F\tilde{F}(x) \right)_{\text{single } I(\bar{I})} = \pm 1, \tag{7}$$

if we assume that the VEV of Q_t^2 is entirely due to instanton fluctuations.

In QCD (with fermions), the topological susceptibility is in the chiral limit determined completely by the $U(1)_A$ anomaly. It is possible to derive the limiting behaviour by saturating the Ward identity for the correlation function of two topological charge densities with pseudoscalar meson states [15]. One finds (V is the four-dimensional volume)

$$\frac{\langle Q_t^2 \rangle}{V} = -\langle \bar{\psi}\psi \rangle \left(\sum_f^{N_f} m_f^{-1} \right)^{-1}, \qquad \langle \bar{\psi}\psi \rangle \equiv \frac{1}{N_f} \sum_f^{N_f} \langle \bar{\psi}_f \psi_f \rangle. \tag{8}$$

The topological susceptibility is proportional to the harmonic average of the quark masses and thus vanishes if one or more of the fermion flavors becomes massless.

The instanton ensemble with fermions exhibits precisely this behavior. The vanishing of the topological susceptibility in the chiral limit is a consequence of the presence of "unbalanced" zero modes in the fermion determinant for $N_+ \neq N_-$. In fact, a calculation of the fermion determinant in zero-mode approximation (keeping only the zero mode in the interaction of the fermions with instantons [8]) results in a probability distribution of Δ with a dispersion identical to Eq. (8) [14]. Thus, the instanton vacuum correctly describes the topological susceptibility of the QCD vacuum in the chiral limit.

Nucleon matrix element of $F\tilde{F}$. The instanton vacuum gives a quantitative prescription for evaluating the nucleon matrix elements not only of quark, but also of gluon operators. Specifically, we are interested in the zero-momentum transfer nucleon matrix element of the topological charge density, $F\tilde{F}(x)$. It is extracted from the correlation function

$$\left\langle J_N \, J_N^\dagger \, Q_t \right\rangle, \tag{9}$$

where $J_N \equiv J_N(x_1), J_N^\dagger \equiv J_N^\dagger(x_2)$ are nucleon currents consisting of N_c quark fields coupled to a color singlet. The correlation function is to be computed as an average over the grand canonical ensemble of instantons with fermions. The averaging consists of two steps:

I) average over the instanton coordinates and the fermion field of an ensemble with fixed N_+, N_- (canonical ensemble), allowing for $N_+ \neq N_-$.

II) average over fluctuations of $\Delta = N_+ - N_-$, with the dispersion $\langle\Delta^2\rangle = \langle Q_t^2\rangle$ given by Eq. (8).

Step I: The fixed-$N_\pm$ average can conveniently be carried out with the help of the effective quark action, which is obtained by integrating over the instanton coordinates first [14, 16]. It has the form of a Nambu–Jona-Lasinio model,

$$S_{\text{eff}}[\bar\psi,\psi] = \int d^4x \sum_f^{N_f} \bar\psi_f \partial\!\!\!/ \psi_f - Y_+ - Y_- - S_\Delta\,, \tag{10}$$

where $Y_\pm$ are $2N_f$-fermion vertices of the form of the 't Hooft interaction (determinant in flavor),

$$\begin{aligned} Y_\pm[\bar\psi,\psi] &= \left(\frac{2V}{N}\right)^{N_f-1} (iM)^{N_f} \int d^4x \, \det_{fg} J_\pm(x)_{fg}\,, \\ J_\pm(x)_{fg} &= -i\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4l}{(2\pi)^4}\exp(-i(k-l)\cdot x) \\ &\times\; F(k)F(l)\,\bar\psi_f(k)\tfrac{1}{2}(1\pm\gamma_5)\psi_g(l)\,. \end{aligned} \tag{11}$$

Eq. (10) is derived by evaluating the fermion determinant of the ensemble of $N_\pm$ instantons in saddle-point approximation ($1/N_c$-expansion). Here, M is the dynamical quark mass, and $F(k)$ a form factor proportional to the wave function of the instanton zero mode. For unequal numbers of I's and $\bar I$'s, the effective quark action contains a CP-violating fermion vertex proportional to Δ,

$$\begin{aligned} S_\Delta[\bar\psi,\psi] &= -\frac{\Delta}{V\langle\bar\psi\psi\rangle}\left(\sum_f^{N_f} m_f^{-1}\right)(Y_+ - Y_-) \\ &= \frac{\Delta}{\langle\Delta^2\rangle}(Y_+ - Y_-)\,. \end{aligned} \tag{12}$$

This coupling of the quarks to topological fluctuations of the number of instantons, which emerges naturally in a saddle-point evaluation of the fermion determinant for $N_+ \neq N_-$, plays a crucial role in the realization of the $U(1)_A$ anomaly, as seen below. An effective action similar to Eqs (10, 12) has also been suggested by Nowak *et al.* [17]. These authors introduce a local (space-time dependent) density of instantons through a coarse-graining procedure. Such a concept is not required in our approach.

With the help of the effective quark action, Eq. (10), the fixed-$N_\pm$ average of two nucleon currents can be represented as

$$\left\langle J_N\, J_N^\dagger \right\rangle_{\text{fixed}-N_\pm} \;=\; \frac{\displaystyle\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, J_N\, J_N^\dagger\, \exp\left(-S_{\text{eff}}[\bar{\psi},\psi]\right)}{\displaystyle\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, \exp\left(-S_{\text{eff}}[\bar{\psi},\psi]\right)} \;\equiv\; \left\langle J_N\, J_N^\dagger \right\rangle_{\text{eff}} . \tag{13}$$

In the limit of large Euclidean times this correlation function is dominated by the saddle point corresponding to the static chiral soliton [10]. Also correlation functions with gluon operators can be systematically expressed as averages in the effective quark theory [14]. In the case of the topological charge, Eq. (9) the result is particularly simple,

$$\left\langle J_N\, J_N^\dagger\, Q_t \right\rangle_{\text{fixed}-N_\pm} \;=\; \Delta \left\langle J_N\, J_N^\dagger \right\rangle_{\text{eff}} . \tag{14}$$

The free nucleon correlation function is simply multiplied by the topological charge of the fixed-$N_\pm$ instanton ensemble, Δ, which seems intuitively plausible.

Step II: Having computed the average over the fixed-$N_\pm$ ensemble, we now must average the result over fluctuations of Δ,

$$\left\langle J_N\, J_N^\dagger\, Q_t \right\rangle \;\equiv\; \sum_\Delta P(\Delta) \left\langle J_N\, J_N^\dagger\, Q_t \right\rangle_{\text{fixed}-N_\pm} , \tag{15}$$

where $P(\Delta)$ is the probability distribution of Δ in the grand ensemble, describing the dispersion Eq. (8). Since the dispersion is $O(m)$ and thus small in the chiral limit, it is sufficient to evaluate the fixed-$N_\pm$ average on the r.h.s. of Eq. (15) to the first non-vanishing order in Δ. For our correlation function Eq. (14) this means

$$\left\langle J_N\, J_N^\dagger\, Q_t \right\rangle \;=\; \langle \Delta^2 \rangle \frac{d}{d\Delta} \left\langle J_N\, J_N^\dagger \right\rangle_{\text{eff}} \Big|_{\Delta=0} \tag{16}$$

$$=\; \left\langle J_N\, J_N^\dagger\, (Y_+ - Y_-) \right\rangle_{\text{eff}} \Big|_{\Delta=0} \tag{17}$$

$$=\; \left\langle J_N\, J_N^\dagger\, \frac{1}{2N_f} \int d^4x\, \partial_\mu J_{5\mu}(x) \right\rangle_{\text{eff}} \Big|_{\Delta=0} . \tag{18}$$

It should be noted, that the factor $\langle\Delta^2\rangle$ has cancelled with a corresponding factor in Eq. (12), so that the result is finite in the chiral limit. In Eq. (18) we have used the fact the $Y_+ - Y_-$ is nothing but the divergence of the isosinglet axial current of the effective quark theory, Eq. (10),

$$\int d^4x \, \partial_\mu J_{5\mu}(x) \quad = \quad 2N_f(Y_+ - Y_-)\,, \tag{19}$$

as follows from the equations of motion in leading order of $1/N_c$.

Eq. (18) now shows how the $\mathrm{U}(1)_A$-anomaly is realized: the grand canonical correlation function with the topological charge reduces to the Nambu–Jona-Lasinio correlation function with an insertion of the divergence of the usual isosinglet axial current of the massive "constituent" quark field. Taking the limit of large Euclidean times, one obtains from Eq. (18) the corresponding equation for the nucleon matrix elements.

Conclusions. We have demonstrated how the instanton vacuum naturally includes the $\mathrm{U}(1)_A$-anomaly in the effective description of the nucleon. The mechanism are the fluctuations of the difference of the number of I's and $\bar{I}$'s in the ensemble, the dispersion of which is governed by the $\mathrm{U}(1)_A$-anomaly. Our argument shows that a calculation of $g_A^{(0)}$ in the chiral quark soliton model may be regarded as a consistent estimate of the nucleon matrix element of the topological charge, Eq. (4), in the instanton vacuum. We have established this equality in the leading order of the $1/N_c$ expansion. A systematic treatment of higher-order corrections in $1/N_c$ is in principle possible, but difficult. For instance, the 't Hooft form of the effective quark interaction, Eq. (11), applies only to the leading order in $1/N_c$.

Calculations in the chiral quark soliton model with SU(3) flavor give $g_A^{(0)} \sim 0.36$ [6]. This result is in agreement with the value obtained in the recent analysis by Ellis and Karliner [1].

The correct description of the $\mathrm{U}(1)_A$ anomaly is a crucial test for the general method for evaluating matrix elements of gluon operators developed in [14]. This method allows also to evaluate the nucleon matrix elements of QCD operators of leading and non-leading twist, as are encountered in the OPE-description of deep-inelastic scattering. Such an approach opens the prospect of describing the deep-inelastic structure of the nucleon consistently with its hadronic structure, in one well-defined scheme of approximations. Calculations of the nucleon structure functions using this method are in progress.

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