SUMMARY AND OUTLOOK*

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The main problems discussed during the Conference were connected with the following topics:

1. Deep inelastic scattering at low x.
2. Deep inelastic diffraction.
3. Nuclear effects in deep inelastic scattering.
4. High energy limit of QCD and formal theoretical issues.

The experimental data have been reviewed in talks given by Barbara Badełek, Albert De Roeck, Witek Krasny and Andrzej Eskreys [1–4].Barbara Badełek reviewed the fixed target data which, thanks to their precision, are the basic source of experimental input for the QCD improved parton model analysis. Several new data are available on $F_2^p, F_2^d, {F_2}^A$ as well as on the spin dependent structure functions g_1^p and g_1^d. Several experiments cover the very interesting transition region of low Q^2 and low x. The HERA data have been reviewed by Albert De Roeck, Witek Krasny and Andrzej Eskreys. The highlights of HERA results are: increase of the structure functions with decreasing x and the large rapidity (or diffractive) events. The new preliminary results on the charm component F_2^c of the proton structure function F_2 from HERA have also been for the first time reported at this Conference [2]. The experimental data from HERA on the structure of the final states in deep inelastic scattering and in photoproduction have also been reviewed [2, 3]. They play important role for revealing the details of the underlying dynamics.

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Perturbative QCD predicts that several new phenomena will occur when the parameter x specifying the longitudinal momentum fraction of a hadron carried by a parton (*i.e.* by a quark or by a gluon) becomes very small [5, 6]. The main expectation is that the gluon densities should strongly grow in this limit, eventually leading to the parton saturation effects [5–8]. The small x behaviour of the structure functions is driven by the gluon through the $g \to q\bar{q}$ transition and the increase of gluon distributions with decreasing x implies a similar increase of the deep inelastic lepton-proton scattering structure function F_2 as the Bjorken parameter x decreases [11]. The Bjorken parameter x is, as usual, defined as $x = Q^2/(2pq)$ where p is the proton four momentum, q the four momentum transfer between the leptons and $Q^2 = -q^2$. The recent experimental data are consistent with this perturbative QCD prediction that the structure function $F_2(x, Q^2)$ should strongly grow with the decreasing Bjorken parameter x [12, 13].

Small x behaviour of structure functions for fixed Q^2 reflects the high energy behaviour of the virtual Compton scattering total cross-section with increasing total CM energy squared W^2 since $W^2 = Q^2(1/x - 1)$. The appropriate framework for the theoretical description of this behaviour is the Regge pole exchange picture [15].

The high energy behaviour of the total hadronic and (real) photoproduction cross-sections can be economically described by two contributions: an (effective) pomeron with its intercept slightly above unity (~ 1.08) and the leading meson Regge trajectories with intercept $\alpha_R(0) \approx 0.5$ [14]. The reggeons can be identified as corresponding to ρ, ω, f or A_2 exchange(s) depending upon the quantum numbers involved. All these reggeons have approximately the same intercept. One refers to the pomeron obtained from the phenomenological analysis of hadronic total cross sections as the "soft" pomeron since the bulk of the processes building-up the cross sections are low p_t (soft) processes.

The Regge pole model gives the following parametrization of the deep inelastic scattering structure function $F_2(x, Q^2)$ at small x

$$F_2(x, Q^2) = \sum_i \tilde{\beta}_i(Q^2) x^{1-\alpha_i(0)}. \tag{1}$$

The relevant reggeons are those which can couple to two (virtual) photons. The (singlet) part of the structure function F_2 is controlled at small x by pomeron exchange, while the non-singlet part $F_2^{\rm NS} = F_2^p - F_2^n$ by the A_2 reggeon. Neither pomeron nor A_2 reggeons couple to the spin structure function $g_1(x, Q^2)$ which is described at small x by the exchange of reggeons corresponding to axial vector mesons [16, 17] *i.e.* to A_1 exchange for the non-singlet part $g_1^{\rm NS} = g_1^p - g_1^n$ *etc.*

$$g_1^{\rm NS}(x, Q^2) = \gamma(Q^2) x^{-\alpha_{A_1}(0)}. \tag{2}$$

The reggeons which correspond to axial vector mesons are expected to have very low intercept (*i.e.* $\alpha_{A_1} \leq 0$ *etc.*).

The growth of structure functions with decreasing parameter x is much stronger than that which would follow from the expectations based on the "soft" pomeron exchange mechanism with the soft pomeron intercept $\alpha_{\rm soft} \approx 1.08$. The high energy behaviour which follows from perturbative QCD is often referred to as being related to the "hard" pomeron in contrast to the soft pomeron describing the high energy behaviour of hadronic and photoproduction cross-sections.

At small x the dominant role is played by the gluons and the basic dynamical quantity is the unintegrated gluon distribution $f(x, Q_t^2)$ where x denotes the momentum fraction of a parent hadron carried by a gluon and Q_t its transverse momentum. The unintegrated distribution $f(x, Q_t^2)$ is related in the following way to the more familiar scale dependent gluon distribution $g(x, Q^2)$:

$$xg(x, Q^2) = \int^{Q^2} \frac{dQ_t^2}{Q_t^2} f(x, Q_t^2). \tag{3}$$

In the leading $\ln(1/x)$ approximation the unintegrated distribution $f(x, Q_t^2)$ satisfies the BFKL equation [9, 10, 19] which has the following form:

$$f(x, Q_t^2) = f^0(x, Q_t^2)$$
$$+\bar{\alpha}_s \int_x^1 \frac{dx'}{x'} \int \frac{d^2q}{\pi q^2} \left[\frac{Q_t^2}{(\boldsymbol{q}+\boldsymbol{Q_t})^2} f(x', (\boldsymbol{q}+\boldsymbol{Q_t})^2) - f(x', Q_t^2)\Theta(Q_t^2 - q^2) \right], \tag{4}$$

where

$$\bar{\alpha}_s = \frac{3\alpha_s}{\pi} \tag{5}$$

The first and the second terms on the right hand side of Eq. (4) correspond to real gluon emission with q being the transverse momentum of the emitted gluon, and to the virtual corrections respectively. $f^0(x, Q_t^2)$ is a suitably defined inhomogeneous term.

After resuming the virtual corrections and "unresolvable" gluon emissions ($q^2 < \mu^2$) where μ is the resolution defining the "resolvable" radiation, equation (4) can be rearranged into the following "folded" form:

$$f(x, Q_t^2) = \hat{f}^0(x, Q_t^2)\bar{\alpha}_s \int_x^1 \frac{dx'}{x'} \int \frac{d^2q}{\pi q^2} \Theta(q^2 - \mu^2)$$
$$\times \Delta_R\left(\frac{x}{x'}, Q_t^2\right) \frac{Q_t^2}{(\boldsymbol{q}+\boldsymbol{Q_t})^2} f(x', (\boldsymbol{q}+\boldsymbol{Q_t})^2) + O\left(\frac{\mu^2}{Q_t^2}\right), \tag{6}$$

where Δ_R which screens the $1/z$ singularity is given by:

$$\Delta_R(z,Q_t^2) = z^{\bar{\alpha}_s \ln(Q_t^2/\mu^2)} = \exp\left(-\bar{\alpha}_s \int\limits_z^1 \frac{dz'}{z'} \int\limits_{\mu^2}^{Q_t^2} \frac{dq^2}{q^2}\right) \tag{7}$$

and

$$\hat{f}^0(x,Q_t^2) = \int\limits_x^1 \frac{dx'}{x'} \Delta_R\left(\frac{x}{x'},Q_t^2\right) \frac{df^0(x',Q_t^2)}{d\ln(1/x')}\,. \tag{8}$$

Equation (6) sums the ladder diagrams with the reggeized gluon exchange along the chain with the gluon trajectory $\alpha_G(Q_t^2) = 1 - (\bar{\alpha}_s/2)\ln(Q_t^2/\mu^2)$.

For the fixed coupling case Eq. (4) can be solved analytically and the leading behaviour of its solution at small x is given by the following expression:

$$f(x,Q_t^2) \sim (Q_t^2)^{\frac{1}{2}} \frac{x^{-\lambda_{\rm BFKL}}}{\sqrt{\ln(\frac{1}{x})}} \exp\left(-\frac{\ln^2(Q_t^2/\bar{Q}^2)}{2\lambda'' \ln(1/x)}\right) \tag{9}$$

with

$$\lambda_{\rm BFKL} = 4\ln(2)\bar{\alpha}_s\,, \tag{10}$$

$$\lambda'' = \bar{\alpha}_s 28\zeta(3)\,, \tag{11}$$

where the Riemann zeta function $\zeta(3) \approx 1.202$. The parameter $\bar{Q}$ is of nonperturbative origin.

The quantity $1+\lambda_{\rm BFKL}$ is equal to the intercept of the so - called BFKL pomeron. Its potentially large magnitude (~ 1.5) should be contrasted with the intercept $\alpha_{\rm soft} \approx 1.08$ of the (effective) "soft" pomeron which has been determined from the phenomenological analysis of the high energy behaviour of hadronic and photoproduction total cross-sections [14].

The solution of the BFKL equation reflects its diffusion pattern which is the direct consequence of the absence of transverse momentum ordering along the gluon chain. The interrelation between the diffusion of transverse momenta towards both the infrared and ultraviolet regions **and** the increase of gluon distributions with decreasing x is a characteristic property of QCD at low x. It has important consequences for the structure of the hadronic final state in deep inelastic scattering at small x.

In practice one introduces the running coupling $\bar{\alpha}_s(Q_t^2)$ in the BFKL equation (4). This requires the introduction of an infrared cut-off to prevent entering the infrared region where the coupling becomes large. The effective intercept $\lambda_{\rm BFKL}$ found by numerically solving the equation depends weakly on the magnitude of this cut-off [28]. The impact of the momentum cut-offs on the solution of the BFKL equation has also been discussed in Refs [29, 30].

It should finally be emphasized that in impact parameter representation the BFKL equation offers an interesting interpretation in terms of colour dipoles [31, 32].

The structure functions $F_{2,L}(x, Q^2)$ are driven at small x by the gluons and are related in the following way to the unintegrated distribution f:

$$F_{2,L}(x, Q^2) = \int_x^1 \frac{dx'}{x'} \int \frac{dQ_t^2}{Q_t^2} F_{2,L}^{\text{box}}(x', Q_t^2, Q^2) f(\frac{x}{x'}, Q_t^2). \tag{12}$$

The functions $F_{2,L}^{\text{box}}(x', Q_t^2, Q^2)$ may be regarded as the structure functions of the off-shell gluons with virtuality Q_t^2. They are described by the quark box (and crossed box) diagram contributions to the photon-gluon interaction. The small x behaviour of the structure functions reflects the small z ($z = x/x'$) behaviour of the gluon distribution $f(z, Q_t^2)$.

Equation (12) is an example of the "k_t factorization theorem" which relates measurable quantities (like DIS structure functions) to the convolution in both longitudinal as well as in transverse momenta of the universal gluon distribution $f(z, Q_t^2)$ with the cross-section (or structure function) describing the interaction of the "off-shell" gluon with the hard probe [25, 24]. The k_t factorization theorem is the basic tool for calculating the observable quantities in the small x region in terms of the (unintegrated) gluon distribution f which is the solution of the BFKL equation.

The leading — twist part of the k_t factorization formula can be rewritten in a collinear factorization form. The leading small x effects are then automatically resumed in the splitting functions and in the coefficient functions. The k_t factorization theorem can in fact be used as the tool for calculating these quantities. Thus, for instance, the moment function $\bar{P}_{qg}(\omega, \alpha_s)$ of the splitting $P_{qg}(z, \alpha_s)$ function is represented in the following form (in the DIS scheme):

$$\bar{P}_{qg}(\omega, \alpha_s) = \frac{\gamma_{gg}^2(\frac{\bar{\alpha}_s}{\omega}) \tilde{F}_2^{\text{box}}(\omega = 0, \gamma = \gamma_{gg}(\frac{\bar{\alpha}_s}{\omega}))}{2\sum_i e_i^2}, \tag{13}$$

where $\tilde{F}_2^{\text{box}}(\omega, \gamma)$ is the Mellin transform of the moment function $\bar{F}_2^{\text{box}}(\omega, Q_t^2, Q^2)$ *i.e.*

$$\bar{F}_2^{\text{box}}(\omega, Q_t^2, Q^2) = \frac{1}{2\pi i} \int_{1/2-i\infty}^{1/2+i\infty} d\gamma \tilde{F}_2^{\text{box}}(\omega, \gamma) \left(\frac{Q^2}{Q_t^2}\right)^{\gamma} \tag{14}$$

and the anomalous dimension $\gamma_{gg}(\bar{\alpha}_s/\omega)$ has the following expansion [27];

$$\gamma_{gg}(\frac{\bar{\alpha}_s}{\omega}) = \sum_{n=1}^{\infty} c_n \left(\frac{\bar{\alpha}_s}{\omega}\right)^n \tag{15}$$

This expansion gives the following expansion of the splitting function P_{gg}

$$zP_{gg}(z,\alpha_s)=\sum_1^{\infty}c_n\frac{\left[\alpha_s\ln\left(\frac{1}{z}\right)\right]^{n-1}}{(n-1)!}. \tag{16}$$

Representation (13) generates the following expansion of the splitting function $P_{qg}(z,\alpha_s)$ at small z:

$$zP_{qg}(z,\alpha_s)=\frac{\alpha_s}{2\pi}zP^{(0)}(z)+(\bar{\alpha}_s)^2\sum_{n=1}^{\infty}b_n\frac{[\bar{\alpha}_s ln(1/z)]^{n-1}}{(n-1)!}. \tag{17}$$

The first term on the right hand side of Eq. (17) vanishes at $z=0$. It should be noted that the splitting function P_{qg} is formally non-leading at small z when compared with the splitting function P_{gg} . For moderately small values of z however, when the first few terms in the expansions (15) and (17) dominate, the BFKL effects can be much more important in P_{qg} than in P_{gg}. This comes from the fact that in the expansion (17) all coefficients b_n are different from zero while in Eq. (15) we have $c_2=c_3=0$ [27]. The small x resummation effects within the conventional QCD evolution formalism have recently been discussed in Refs [33–36]. One finds in general that at the moderately small values of x which are relevant for the HERA measurements, the small x resummation effects in the splitting function P_{qg} have a much stronger impact on F_2 than the small x resummation in the splitting function P_{gg}. This reflects the fact, which has already been mentioned above, that in the expansion (17) all coefficients b_n are different from zero while in Eq. (15) we have $c_2=c_3=0$. The k_t factorization and small x resummation effects have been summarized in this Conference by Francesco Hautmann [37]. The application of the k_t factorization for the calculation of the structure function F_L at low x including its extrapolation to the region of low Q^2 was discussed in the talk given by Anna Staśto [38].

A more general treatment of the gluon ladder than that which follows from the BFKL formalism is provided by the CCFM equation based on angular ordering along the gluon chain [20, 21]. This equation embodies both the BFKL equation at small x and the conventional Altarelli-Parisi evolution at large x. The unintegrated gluon distribution f now acquires dependence upon an additional scale Q which specifies the maximal angle of gluon emission. The CCFM equation has a form analogous to that of the "folded" BFKL equation (6):

$$f(x,Q_t^2,Q^2)=\hat{f}^0(x,Q_t^2,Q^2)$$
$$+\bar{\alpha}_s\int_x^1\frac{dx'}{x'}\int\frac{d^2q}{\pi q^2}\Theta\left(Q-\frac{qx}{x'}\right)\Delta_R\left(\frac{x}{x'},Q_t^2,q^2\right)\frac{Q_t^2}{(\boldsymbol{q}+\boldsymbol{Q_t})^2}f(x',(\boldsymbol{q}+\boldsymbol{Q_t})^2,q^2)),$$

(18)

where the theta function $\Theta(Q - qx/x')$ reflects the angular ordering constraint on the emitted gluon. The "non-Sudakov" form-factor $\Delta_R(z, Q_t^2, q^2)$ is now given by the following formula:

$$\Delta_R(z, Q_t^2, q^2) = \exp\left[-\bar{\alpha}_s \int_z^1 \frac{dz'}{z'} \int \frac{dq'^2}{q'^2} \Theta(q'^2 - (qz')^2)\Theta(Q_t^2 - q'^2)\right]. \quad (19)$$

Eq. (18) still contains only the singular term of the $g \to gg$ splitting function at small z. Its generalization which would include remaining parts of this vertex (as well as quarks) is possible. The numerical analysis of this equation was presented in Ref. [21] and the detailed discussion of the QCD coherence effects which lead to angular ordering has been summarized in a talk given by Peter Sutton [39].

The CCFM equation which is the generalization of the BFKL equation generates the steep $x^{-\lambda}$ type of behaviour for the deep inelastic structure functions as the effect of the resummation of the leading $\ln(1/x)$ resummation [39, 40]. One can however obtain satisfactory description of the HERA data staying within the scheme based on the Altarelli-Parisi equations alone without the small x resummation effects being included in the formalism [41–44]. In the latter case the singular small x behaviour of the gluon and sea quark distributions has to be introduced in the parametrization of the starting distributions at the moderately large reference scale $Q^2 = Q_0^2$ (*i.e.* $Q_0^2 \approx 4$ GeV^2 or so) [41, 43]. One can also generate steep behaviour dynamically starting from non-singular "valence-like" parton distributions at some very low scale $Q_0^2 = 0.35$ GeV^2 [42, 44, 45]. In the latter case the gluon and sea quark distributions exhibit "double logarithmic behaviour" [46]

$$F_2(x, Q^2) \sim \exp\left(2\sqrt{\xi(Q^2, Q_0^2)\ln(1/x)}\right), \quad (20)$$

where

$$\xi(Q^2, Q_0^2) = \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2}\frac{3\alpha_s(q^2)}{\pi}. \quad (21)$$

For very small values of the scale Q_0^2 the evolution length $\xi(Q^2, Q_0^2)$ can become large for moderate and large values of Q^2 and the "double logarithmic" behaviour (20) is, within the limited region of x, similar to that corresponding to the power like increase of the type $x^{-\lambda}$, $\lambda \approx 0.3$. The present status of the conventional QCD improved parton model analysis of the deep inelastic scattering and of related processes has been summarized in talks given by Alan Martin [43] and Ewald Reya [44].

The discussion presented above concerned the small x behaviour of the singlet structure function which was driven by the gluon through the $g \to q\bar{q}$ transition. The gluons of course decouple from the non-singlet channel and the mechanism of generating the small x behaviour in this case is different. The simple Regge pole exchange model predicts in this case that

$$F_2^{\rm NS}(x,Q^2) = F_2^p(x,Q^2) - F_2^n(x,Q^2) \sim x^{1-\alpha_{A_2}(0)} \tag{22}$$

where $\alpha_{A_2}(0)$ is the intercept of the A_2 Regge trajectory. For $\alpha_{A_2}(0) \approx 1/2$ this behaviour is stable against leading order QCD evolution. This follows from the fact that the leading singularity of the moment $\gamma_{qq}(\omega)$ of the splitting function $P_{qq}(z)$:

$$\gamma(\omega) = \int_0^1 \frac{dz}{z} z^\omega P_{qq}(z) \tag{23}$$

is located at $\omega = 0$ and so the (nonperturbative) A_2 Regge pole at $\omega = \alpha_{A_2}(0) \approx 1/2$ remains the leading singularity controlling the small x behaviour of the non-singlet structure function.

The novel feature of the non-singlet channel is the appearance of the **double** logarithmic terms *i.e.* powers of $\alpha_s \ln^2(1/x)$ at each order of the perturbative expansion [47–51].These double logarithmic terms are generated by the ladder diagrams with quark (antiquark) exchange along the chain. The ladder diagrams can acquire corrections from the "bremsstrahlung" contributions [49, 51] which do not vanish for the polarized structure function $g_1^{\rm NS}(x,Q^2)$ [51].

In the approximation where the leading double logarithmic terms are generated by ladder diagrams with quark (antiquark) exchange along the chain the unintegrated non-singlet quark distribution $f_q^{\rm NS}(x,k_t^2)$ ($q^{\rm NS} = u + \bar{u} - d - \bar{d}$) satisfies the following integral equation :

$$f_q^{\rm NS}(x,Q_t^2) = f_{q0}^{\rm NS}(x,Q_t^2) + \tilde{\alpha}_s \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{Q_t^2/z} \frac{dQ_t'^2}{Q_t'^2} f_q^{\rm NS}\left(\frac{x}{z},Q_t'^2\right), \tag{24}$$

where

$$\tilde{\alpha}_s = \frac{2}{3\pi}\alpha_s \tag{25}$$

and Q_0^2 is the infrared cut-off parameter. The unintegrated distribution $f_q^{\rm NS}(x,Q_t^2)$ is, as usual, related in the following way to the scale dependent (nonsinglet) quark distribution $q^{\mathrm{NS}}(x,Q^2)$:

$$q^{\mathrm{NS}}(x,Q^2) = \int^{Q^2} \frac{dQ_t^2}{Q_t^2} f_q^{\mathrm{NS}}(x,Q_t^2). \tag{26}$$

The upper limit Q_t^2/z in the integral equation (24) follows from the requirement that the virtuality of the quark at the end of the chain is dominated by Q_t^2. A possible non-perturbative A_2 reggeon contribution has to be introduced in the driving term *i.e.*

$$f_{q0}^{\mathrm{NS}}(x,Q_t^2) \sim x^{-\alpha_{A_2}(0)} \tag{27}$$

at small x.

Equation (24) implies the following equation for the moment function $\bar{f}_q^{\mathrm{NS}}(\omega,Q_t^2)$

$$\begin{aligned} \bar{f}_q^{\mathrm{NS}}(\omega,Q_t^2) &= \bar{f}_{q0}^{\mathrm{NS}}(\omega,Q_t^2) \\ &+\frac{\tilde{\alpha}_s}{\omega}\left[\int_{Q_0^2}^{Q_t^2} \frac{dQ_t'^2}{Q_t'^2}\bar{f}_q^{\mathrm{NS}}(\omega,Q_t'^2) + \int_{Q_t^2}^{\infty} \frac{dQ_t'^2}{Q_t'^2}\left(\frac{Q_t^2}{Q_t'^2}\right)^{\omega} \bar{f}_q^{\mathrm{NS}}(\omega,Q_t'^2)\right]. \end{aligned} \tag{28}$$

Equation (28) follows from (24) after taking into account the following relation:

$$\int_0^1 \frac{dz}{z} z^{\omega}\Theta\left(\frac{Q_t^2}{Q_t'^2} - z\right) = \frac{1}{\omega}\left[\Theta(Q_t^2 - Q_t'^2) + \left(\frac{Q_t^2}{Q_t'^2}\right)^{\omega}\Theta(Q_t'^2 - Q_t^2)\right]. \tag{29}$$

For fixed coupling $\tilde{\alpha}_s$ equation (28) can be solved analytically. Assuming for simplicity that the inhomogeneous term is independent of Q_t^2 (*i.e.* that $\bar{f}_{q0}^{\mathrm{NS}}(\omega,Q_t^2) = C(\omega)$) we get the following solution of Eq. (28):

$$\bar{f}_q^{\mathrm{NS}}(\omega,Q_t^2) = C(\omega)R(\tilde{\alpha}_s,\omega)\left(\frac{Q_t^2}{Q_0^2}\right)^{\gamma^-(\tilde{\alpha}_s,\omega)}, \tag{30}$$

where

$$\gamma^-(\tilde{\alpha}_s,\omega) = \frac{\omega - \sqrt{\omega^2 - 4\tilde{\alpha}_s}}{2} \tag{31}$$

and

$$R(\tilde{\alpha}_s,\omega) = \frac{\omega\gamma^-(\tilde{\alpha}_s,\omega)}{\tilde{\alpha}_s}. \tag{32}$$

Equation (31) defines the anomalous dimension of the moment of the non-singlet quark distribution in which the double logarithmic $\ln(1/x)$ terms *i.e.* the powers of α_s/ω^2 have been resummed to all orders. It can be seen from (31) that this anomalous dimension has a (square root) branch point singularity at $\omega = \bar{\omega}$ where

$$\bar{\omega} = 2\sqrt{\bar{\alpha}_s}. \tag{33}$$

This singularity will of course be also present in the moment function $\bar{f}_q^{NS}(\omega, Q_t^2)$ itself. It should be noted that in contrast to the BFKL singularity whose position above unity was proportional to α_s, $\bar{\omega}$ is proportional to $\sqrt{\alpha_s}$ — this being the straightforward consequence of the fact that equation (28) sums double logarithmic terms $(\alpha_s/\omega^2)^n$. This singularity gives the following contribution to the non-singlet quark distribution $f_q^{NS}(x, Q_t^2)$ at small x:

$$f_q^{NS}(x, Q_t^2) \sim \frac{x^{-\bar{\omega}}}{\ln^{3/2}\left(\frac{1}{x}\right)}. \tag{34}$$

For small values of the QCD coupling this contribution remains non - leading in comparison to the contribution of the A_2 Regge pole.

As has been mentioned above the corresponding integral equation which resumes the double logarithmic terms in the spin dependent quark distributions is more complicated than the simple ladder equation (24) due to non-vanishing contributions coming from bremsstrahlung diagrams. It may however be shown that , at least as far as the non-singlet structure function is concerned, these contributions give only a relatively small correction to $\bar{\omega}$. The main interest in applying the QCD evolution equations to study the spin structure function is that the naive Regge pole expectations based on the exchange of low-lying Regge trajectories become unstable against the QCD perturbative "corrections". The relevant reggeon which contributes to $g_1^{NS}(x, Q^2)$ is the A_1 exchange which is expected to have a very low intercept $\alpha_{A_1}(0) \leq 0$. The perturbative singularity generated by the double logarithmic $\ln(1/x)$ resummation can therefore become much more important than in the case of the unpolarized case where it is hidden behind leading A_2 exchange contribution. Even if we restrict ourselves to leading order QCD evolution [52, 53] then the non-singular $x^{-\alpha_{A_1}(0)}$ behaviour (with $\alpha_{A_1}(0) \leq 0$) becomes unstable as well and the polarized quark densities acquire singular behaviour:

$$\Delta q^{NS}(x, Q^2) \sim \exp(2\sqrt{\xi^{NS}(Q^2)\ln(1/x)}), \tag{35}$$

where

$$\xi^{NS}(Q^2) = \int^{Q^2} \frac{dq^2}{q^2}\bar{\alpha}_s(q^2). \tag{36}$$

This follows from the fact that $\Delta P_{qq}(z) = P_{qq}(z)$ where $P_{qq}(z)$ and $\Delta P_{qq}(z)$ are the splitting functions describing the evolution of the spin independent and spin dependent quark distributions respectively and from the fact that $P_{qq}(z) \to$ const. as $z \to 0$.

The introduction of the running coupling effects in equation (28) turns the branch point singularity into the series of poles which accumulate at $\omega = 0$ [48]. The numerical analysis of the corresponding integral equation, with the running coupling effects taken into account, gives an effective slope,

$$\lambda(x, Q_t^2) = \frac{d \ln \Delta f_q^{NS}(x, Q_t^2)}{d \ln(1/x)} \tag{37}$$

with magnitude $\lambda(x, Q_t^2) \approx 0.2 - 0.3$ at small x [54]. The result of this estimate suggest that a reasonable extrapolation of the (non-singlet) polarized quark densities would be to assume an $x^{-\lambda}$ behaviour with $\lambda \approx 0.2 - 0.3$. Similar extrapolations of the spin-dependent quark distributions towards the small x region have been assumed in several recent parametrizations of parton densities [55–58].The perturbative QCD effects become significantly amplified for the singlet spin structure function due to mixing with the gluons. The simple ladder equation may not however be applicable for an accurate description of the double logarithmic terms in the polarized gluon distribution ΔG [59]. The spin dependent structure functions have been reviewed in this Conference by Ewald Reya [44] and the effects of the small x resummations in the nonsinglet structure functions F_1 and g_1 were discussed by Johannes Blümlein [60]. The small x behaviour of the spin dependent structure function g_1 has also been discussed in Refs [61, 62]. Important recent theoretical development in the case of the spin dependent structure functions was the formulation of the complete NLO formalism which allowed the QCD analysis of polarized deep inelastic scattering to reach theoretical accuracy compatible with that in the unpolarized case.

It is expected that absence of transverse momentum ordering along the gluon chain, which leads to the correlation between the increase of the structure function with decreasing x and the diffusion of transverse momentum should reflect itself in the behaviour of less inclusive quantities than the structure function $F_2(x, Q^2)$. The dedicated measurements of low x physics which are particularly sensitive to this correlation are the deep inelastic plus jet events, transverse energy flow in deep inelastic scattering, production of jets separated by the large rapidity gap and dijet production in deep inelastic scattering.

In principle deep inelastic lepton scattering containing a measured jet can provide a very clear test of the BFKL dynamics at low x [43, 64, 65, 67]. The idea is to study deep inelastic (x, Q^2) events which contain an identified jet (x_j, k_{Tj}^2) where $x \ll x_j$ and $Q^2 \approx k_{Tj}^2$. Since we choose events with

$Q^2 \approx k_{Tj}^2$ the leading order QCD evolution (from k_{Tj}^2 to Q^2) is neutralized and attention is focussed on the small x, or rather small x/x_j behaviour. Choosing the configuration $Q^2 \approx k_{Tj}^2$ we eliminate by definition gluon emission which corresponds to strongly ordered transverse momenta *i.e.* that emission which is responsible for the LO QCD evolution. The measurement of jet production in this configuration may therefore test more directly the $(x/x_j)^{-\lambda}$ behaviour which is generated by the BFKL equation where the transverse momenta are not ordered. The recent H1 results concerning deep inelastic plus jest events are consistent with the increase of the cross-section with decreasing x as predicted by the BFKL dynamics [2, 68].

A conceptually similar process is that of the two-jet production, with the jets separated by a large rapidity gap Δy, in hadronic collisions or in photoproduction [69, 70]. Besides the characteristic $\exp(\lambda\Delta y)$ dependence of the two-jet cross-section one expects significant weakening of the azimuthal back-to-back correlations of the two jets. Another measurement which should be sensitive to the QCD dynamics at small x is that of the transverse energy flow in deep inelastic lepton scattering in the central region away from the current jet and from the proton remnant [71]. The BFKL dynamics predicts in this case a substantial amount of transverse energy which should increase with decreasing x. The experimental data are consistent with this theoretical expectation [68]. Absence of transverse momentum ordering also implies weakening of the back-to-back azimuthal correlation of dijets produced close to the photon fragmentation region [72, 73]. The experimental data on the hadronic final state and in particular the comparison of the deep inelastic scattering and photoproduction data were summarized by Witek Krasny [3].

Another important process which is sensitive to the small x dynamics is the deep inelastic diffraction [74, 75]. Deep inelastic diffraction in *ep* inelastic scattering is a process:

$$e(p_e) + p(p) \to e'(p_e') + X + p'(p') \tag{38}$$

where there is a large rapidity gap between the recoil proton (or excited proton) and the hadronic system X. To be precise process (38) reflects the diffractive dissociation of the virtual photon. Diffractive dissociation is described by the following kinematical variables:

$$\beta = \frac{Q^2}{2(p-p')q}, \tag{39}$$

$$x_P = \frac{x}{\beta}, \tag{40}$$

$$t = (p-p')^2. \tag{41}$$

Assuming that diffraction dissociation is dominated by the pomeron exchange and that the pomeron is described by a Regge pole one gets the following factorizable expression for the diffractive structure function [77–80, 82]:

$$\frac{\partial F_2^{\rm diff}}{\partial x_P \partial t} = f(x_P, t) F_2^P(\beta, Q^2, t)\,, \tag{42}$$

where the "flux factor" $f(x_P, t)$ is given by the following formula:

$$f(x_P, t) = N \frac{B^2(t)}{16\pi} x_P^{1-2\alpha_P(t)} \tag{43}$$

with $B(t)$ describing the pomeron coupling to a proton and N being the normalization factor. The function $F_2^P(\beta, Q^2, t)$ is the pomeron structure function which in the (QCD improved) parton model is related in a standard way to the quark and antiquark distribution functions in a pomeron.

$$F_2^P(\beta, Q^2, t) = \beta \sum e_i^2 [q_i^P(\beta, Q^2, t) + \bar{q}_i^P(\beta, Q^2, t)] \tag{44}$$

with $q_i^P(\beta, Q^2, t) = \bar{q}_i^P(\beta, Q^2, t)$. The variable β which is the Bjorken scaling variable appropriate for deep inelastic lepton-pomeron "scattering", has the meaning of the momentum fraction of the pomeron carried by the probed quark (antiquark). The quark distributions in a pomeron are assumed to obey the standard Altarelli–Parisi evolution equations:

$$Q^2 \frac{\partial q^P}{\partial Q^2} = P_{qq} \otimes q^P + P_{qg} \otimes g^P \tag{45}$$

with a similar equation for the evolution of the gluon distribution in a pomeron. The first term on the right hand side of the Eq. (45) becomes negative at large β, while the second term remains positive and is usually very small at large β unless the gluon distributions are large and have a hard spectrum.

The data suggest that the slope of F_2^P as the function of Q^2 does not change sign even at relatively large values of β. This favours the hard gluon spectrum in a pomeron [83–85], and should be contrasted with the behaviour of the structure function of the proton which, at large x, decreases with increasing Q^2. The data on inclusive diffractive production favour the soft pomeron with relatively low intercept. The diffractive production of vector mesons seems to require a "hard" pomeron contribution [86–88]. It has also been pointed out that the factorization property (42) may not hold in models based entirely on perturbative QCD when the pomeron is represented by the BFKL ladder [89, 90]. There exist also models of deep inelastic diffraction which do not rely on the pomeron exchange picture [91, 92].

The experimental data on deep inelastic diffraction have been summarized by Andrzej Eskreys [4] and their phenomenolgical description within the soft pomeron exchange mechanism and QCD by Krzysztof Golec-Biernat [93].

Interesting and complementary information on the underlying dynamics can be obtained from studying deep inelastic scattering on nuclear targets. The unified picture of this process which embodied both the nuclear shadowing at low x together with other effects at moderate and large values of x was presented in the talks given by G. Piller and W. Melnitchouk [94, 95] and the experimental data on nuclear shadowing were summarized by Barbara Badełek [1]. The nuclear shadowing is closely related to the deep inelastic diffraction. At low Q^2 it is mainly controlled by the rescattering of vector mesons which couple to virtual photons while at large Q^2 the rescattering of the high-mass $q\bar{q}$ component of the virtual photon dominates. In the infinite momentum frame *i.e.* in a frame where the momentum $p = p_A/A$ is very large, the latter mechanism can be interpreted in terms of the screening effects of partons from different nucleons in the nucleus [96, 97]. Presence of the partonic mechanism in nuclear shadowing implies that it should contain the leading twist component which is (approximately) independent of Q^2 for large Q^2 and this property of nuclear shadowing is confirmed by the experimental data [1, 97]. At very small values of x which can possibly be reached at HERA, analysis of nuclear shadowing can be very useful for studying the parton recombination effects [5], which can become significantly amplified in the nuclear medium. The nuclear effects which affect the nuclear structure functions for moderate and large values of x are the pion contribution, binding and "off-shell" effects and finally the Fermi motion [94, 95].

Interesting contribution concerning the nucleon deep inelastic structure from the instanton vacuum was presented by Christian Weiss [98].

Several new interesting results have been obtained in the formal studies of the high energy (or small x) limit in QCD which go beyond the leading logarithmic approximation [10, 99, 100, 105, 106]. The important theoretical tool in this case is the effective field theory where the basic objects are the reggeized gluons and the effective action of this effective field theory obeys conformal invariance [99, 100]. Theoretical analysis simplifies in the large N_c limit. One can discuss both the pomeron which appears as the bound state of two (reggeized) gluons, and the odderon (*i.e.* the bound state of three reggeized gluons), as well as bound states of many reggeized gluons [101]. Conformal invariance is also very useful for analysing the BFKL pomeron away from the forward direction [102, 103] as well as the triple pomeron and more complicated vertices [104].

The genuine next-to-leading $\ln(1/x)$ corrections to the BFKL equation can be present in all relevant quantities *i.e.* in the particle-particle-reggeon vertex, the reggeon-reggeon-particle vertex and in the gluon Regge trajectory [105, 106]. (The reggeon here corresponds to the reggeized gluon). Besides that one has also to include additional region of phase-space which goes beyond strong ordering of longitudinal momenta. The recent developments in the study of the high energy limits of QCD were reviewed in this Conference by Lev Lipatov [107] and the mathematical problems related to the odderon singularities were presented by Roman Janik [108]. The complete calculation of the next-to-leading $\ln(1/x)$ corrections to the BFKL equation has recently been presented in Ref. [109].

To sum up we have had an opportunity to hear excellent reviews of almost all experimental and theoretical results concerning deep inelastic scattering. There, of course, still remain several interesting problems which require better understanding and clarification like, for instance, the role of the small x resummation effects, possible role of the shadowing corrections, interplay between "soft" and "hard" pomerons *etc.* The latter problem is closely linked with the change of the behaviour of the nucleon structure function with the scale Q^2 which is observed in the data. This change may be related to the transition from the perturbative to non-perturbative domain and certainly needs detailed dynamical explanation.

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