PRESENT STATUS OF PRECISE CALCULATION FOR SMALL-ANGLE BHABHA SCATTERING*,**

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The problem of obtaining a theoretical description of Bhabha scattering at small angle, to monitor the luminosity at the precision 0.1% requested by LEP experiments, is discussed and recent results are presented.

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Bhabha scattering is the process

$$e^+e^- \to e^+e^-(\gamma, \gamma, \ldots) \ , \tag{1}$$

where any number of photons (observed or not) is in the final state.

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The smallness of the mass of the electron is the reason of its much larger probability of photon emission than for other leptons or quarks. Therefore in this process, not only initial state emission is relevant, but also final state emission and, when the scattering angles are small, also the interference between the two states.

The process has two Born (lowest order) Feynman diagrams contributing to the amplitude, usually referred to as s-channel (or annihilation) diagram and t-channel diagram, furthermore each of them can occur with a photon or a Z^0 boson in the intermediate state. As a result the cross-section has already from the Born order 10 different channel combinations:

$$\begin{aligned} &\sigma_0^{(1)}[\gamma(s),\gamma(s)]\,, \quad \sigma_0^{(2)}[\gamma(s),\gamma(t)]\,, \quad \sigma_0^{(3)}[\gamma(t),\gamma(t)]\,, \\ &\sigma_0^{(4)}[\gamma(s),Z(t)]\,, \quad \sigma_0^{(5)}[\gamma(t),Z(t)]\,, \quad \sigma_0^{(6)}[Z(t),Z(t)]\,, \\ &\sigma_0^{(7)}[Z(s),\gamma(s)]\,, \quad \sigma_0^{(8)}[Z(s),\gamma(t)]\,, \\ &\sigma_0^{(9)}[Z(s),Z(t)]\,, \quad \sigma_0^{(10)}[Z(s),Z(s)]\,. \end{aligned} \tag{2}$$

At LEP1 and for large scattering angles (namely between 40° and 140°) the s-channel dominates, because at this energy the annihilation into the Z^0 boson has the resonance peak. Therefore in this configuration the Bhabha scattering is used by LEP-experiments to measure the Z^0 properties (this aspect is discussed in [1]). At LEP2 still for large-angles the photon t-channel dominates.

At small scattering angles (namely less than 10°) the photon t-channel dominates at all energies, because of the coulomb peak, and its main contribution to the cross-section $d\sigma_0^{(3)}[\gamma(t),\gamma(t)]/d\theta \propto 1/\theta^3$ amounts to almost the whole cross-section, the Z^0 boson exchange plays only a small correction role. For this reason the Bhabha scattering at small angle has been chosen by the LEP-experiments as the luminosity monitoring process. This talk is dedicated to the theoretical aspect for having such a measurement at the requested accuracy level.

In a e^+e^- colliding beams experiment the luminosity is determined by measuring the number of the events for a monitoring process, which is given by

$$N = \int \mathcal{L}\sigma dt \simeq \mathcal{L}\sigma\Delta t\,, \tag{3}$$

where $\mathcal{L}$ is the luminosity and σ the cross-section for the selected process (Bhabha scattering usually and in this case). N is measured experimentally for the time Δt where the conditions for $\mathcal{L}$ and σ do not change, so $\mathcal{L}$ is immediately determined if σ is provided by a theoretical calculation.

An important behavior of the cross-section is that the dependence on various experimental cuts is flatter when the conditions chosen are more inclusive. To measure the luminosity with smaller error is therefore necessary to identify the cuts configurations where this behaviour is maximized and in general to include as many events with emission of photons as possible (loose cuts), excluding however regions of the phase space with a sizeable background.

The accuracy obtained for the luminosity at PEP and PETRA was about 3%, and also at LEP the project accuracy was 2% [2]. But since the very beginning in '89 the experimental contribution to the luminosity error went under 1%, decreasing continuously for all the experimental groups in the following years. In '92-'93 started the installation of second generation of luminometers, which took the experimental contribution to luminosity error down to 0.1% or better.

At LEP starting in '89 theory had calculations with radiative corrections at $\mathcal{O}(\alpha)$(see [3] for references to early calculations) and also some event generators were available [4, 5].

Soon by use of different resummation techniques for multi-photon emission the theoretical error went under 1% for a number of numerical integrating programs and event generators : ALIBABA [6], OLDBIS+LUMLOG [7, 8], BHLUMI [9], 40THIEVES [10], BHAGEN [11].

Since then a lot of work has been done, struggling for a bit of improvement to reach the present perspective of 0.1% theoretical precision (see references in [12, 13]).

In this theory versus experiment confrontation a necessary remark is that at this precision level no theoretical improvement is truly completed if not implemented in an event generator, which only allows for an enough accurate description of real experimental conditions.

In the fitting of Z^0 properties, the experimental wisdom suggests to choose ratios of observables, such as the ratio of the hadronic to the leptonic widths, $R_h = \Gamma_{\mathbf{had}}/\Gamma_{\mathbf{lep}}$, or asymmetries to eliminate luminosity error dependence and other systematic errors. The mass of the Z^0, M_Z, is not fitted inside a ratio, but fortunately its dependence from the luminosity error is very small.

Also the hadronic cross-section at the Z^0 peak, $\sigma_0^{\mathbf{had}}$, is not fitted inside a ratio, but in this case the dependence from the luminosity error is relevant and today it is the main source of its error. Through that important observable the luminosity error propagates to many other interesting observables, such as the number of light neutrino types, N_ν, which has many speculative interpretations.

Having bird-eye watched to the motivations for a very precise theoretical investigation of the Bhabha cross-section for small scattering angles, let us now go to examine the precision level obtained in the theoretical investigations. We introduce the differential cross-section for the Bhabha scattering without visible photon emission ($\equiv$ BS)

$$\frac{d\sigma}{d\Omega} = \sum_{i=1}^{10} \frac{d\bar{\sigma}_0^{(i)}}{d\Omega} \left(1 + \delta^{(i)}\right) , \tag{4}$$

where the various terms (channels) correspond to the different combinations of possibilities shown in Eq. (2). As already remarked, at small scattering-angle the cross-section is mostly given by the photon t-channel contribution $\sigma_0^{(3)}[\gamma(t), \gamma(t)]$, so the following considerations, although rather general, are essentially referred to that channel.

With $d\bar{\sigma}_0^{(i)}/d\Omega$ is indicated the differential Born cross-section, improved with the vacuum polarization radiative corrections, Dyson resummed to all orders, so that the coupling α becomes a running coupling depending on the channel invariant s or t.

The factor $\left(1 + \delta^{(i)}\right)$, which accounts for radiative corrections, is different for every channel, but has the same structure shown in the first row of Table I. There are sketched, for the dominant channel, all the contributions of the radiative corrections up to $\mathcal{O}(\alpha^3)$, which are necessary to obtain the requested accuracy of 0.1%. As usual let us call E_b the beam energy, $s = 4E_b^2$, $L(s) = \ln(s/m_e^2)$, $\beta_e(s) = 2\,(\alpha/\pi)\,(L(s) - 1)$, $\beta_{\rm int}(\theta) = 4\,(\alpha/\pi) \ln \tan \theta/2$, $\beta(s, \theta) = \beta_e(s) + \beta_{\rm int}(\theta)$. For large-angles $\beta \simeq \beta_e(s)$, while for small-angles $\beta \simeq \beta_e(|t|)$. Δ and ε are the maximum photon energy-fractions for the real emission of hard and soft photons respectively. In the first row of Table I is the Born term 1, plus the radiative corrections contributions coming from calculations of Feynman graphs with virtual and real soft emission of photons at $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha^3)$. At $\mathcal{O}(\alpha)$the correction has to be included completely, and there is separated into the infrared term $2\beta \ln \varepsilon$, the term proportional to the big logarithm $L(s)$ and all the rest just proportional to α.

For typical values at LEP1 (for $E_b \simeq 50$GeV, $L \simeq 23, \beta \simeq 0.1$ and only a little smaller values come in the case of interest, where $|t|$ has to be taken in place of s), $\varepsilon \leq 10^{-4}$ to safely omit terms proportional to ε in soft photon approximated terms, and $0.1 \leq \Delta \leq 0.5$ to have experimental relevance, it can be easily verified that the correction $2\beta \ln \varepsilon \simeq -1.4$ overwhelms the Born term (as the remaining $\mathcal{O}(\alpha)$terms are not large enough) and can take the cross-section to unphysical negative values.

TABLE I

Sketch of the main contributions to the dominant channel of the Bhabha scattering process at small-angle. In the first row are the contributions to the factorized radiative correction $(1+\delta^{(3)})$ to the lowest order differential cross-section $(d\bar{\sigma}_0^{(3)}/d\Omega)$, in the second row is the same for the one photon radiated Bhabha scattering process, and so on. In the last row are the sums of the infrared terms in each column (order by order in α), in the last column each term is the infinite resummation of the leading infrared terms in each row. The last term in the last row and last column is the infinite resummation of all the other terms in the last row or last column.

Born	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha^2)$	
1	$+2\beta\ln\varepsilon+\alpha La_{11}+\alpha a_{10}$	$+\frac{1}{2!}(2\beta\ln\varepsilon)^2+\alpha^2L^2a_{22}+\alpha^2La_{21}+\ldots$	↓
	$+[2\beta\ln\frac{\Delta}{\varepsilon}+\alpha Lb'_{11}+\alpha b'_{10}]\{1$	$+2\beta\ln\varepsilon+\alpha Lb_{11}+\alpha b_{10}$	↓
		$+[\frac{1}{2!}(2\beta\ln\frac{\Delta}{\varepsilon})^2+\alpha^2L^2c'_{22}+\alpha^2Lc'_{21}+\ldots]\{1$	↓
1	$+2\beta\ln\Delta+\ldots$	$+\frac{1}{2!}(2\beta\ln\Delta)^2+\ldots$	↓

$\mathcal{O}(\alpha^3)$	Resummed
$+\frac{1}{3!}(2\beta\ln\varepsilon)^3+\alpha^3L^3a_{33}+\ldots$	$\varepsilon^{2\beta}\{1+\ldots\}$
$+\frac{1}{2!}(2\beta\ln\varepsilon)^2+\alpha^2L^2b_{22}+\ldots\}$	$\varepsilon^{2\beta}\{2\beta\ln\frac{\Delta}{\varepsilon}+\ldots\}$
$+2\beta\ln\varepsilon+\alpha Lc_{11}+\ldots\}$	$\varepsilon^{2\beta}\{\frac{1}{2!}(2\beta\ln\frac{\Delta}{\varepsilon})^2+\ldots\}$
$+[\frac{1}{3!}(2\beta\ln\frac{\Delta}{\varepsilon})^3+\alpha^3L^3d'_{33}+\ldots]\{1+\ldots\}$	$\varepsilon^{2\beta}\{\frac{1}{3!}(2\beta\ln\frac{\Delta}{\varepsilon})^3+\ldots\}$
$+\frac{1}{3!}(2\beta\ln\Delta)^3+\ldots$	$\Delta^{\beta}\{1+\ldots\}$

This is the well known infrared problem which was solved long time ago by arguing that is necessary to add all the infinite series of infrared terms, which are shown in the first row up to $\mathcal{O}(\alpha^3)$. The result of the infinite sum

$$1+2\beta\ln\varepsilon+\frac{1}{2!}(2\beta\ln\varepsilon)^2+\frac{1}{3!}(2\beta\ln\varepsilon)^3+\ldots=\varepsilon^{2\beta}\,, \tag{5}$$

has the exponentiated form $\varepsilon^{2\beta}$, which is reported in the last term of the first row and has the nice property of being positive, no-matter how small is ε and, factorized out, restores the classical limit of vanishing the cross-section without radiation emission ($\varepsilon=0$).

The other terms indicated in the first row are necessary to achieve the mentioned accuracy of 0.1%. The coefficients like a_{kl} are in general complicated expressions, they can be obtained only through the appropriate Feynman graph calculations, where big cancellations occur (for example intermediate calculations have αL^2, $\alpha^2 L^4$ and $\alpha^2 L^3$ terms which cancel in the sum) and they are expected to have values of order of a few units.

In the second row of Table I is the contribution to the Bhabha cross-section coming from the process which has one hard proton (with an energy-fraction ω such that $\varepsilon \leq \omega \leq \Delta$). This process (one photon radiated Bhabha scattering $\equiv$ 1RBS) has a Born term which starts at $\mathcal{O}(\alpha)$, respect to BS, and in a rough approximation has in comparison to that Born expression the factor

$$2\beta \int_{\varepsilon}^{\Delta} \frac{d\omega}{\omega} = 2\beta \ln \frac{\Delta}{\varepsilon}, \tag{6}$$

which has then to be corrected for next orders by virtual and real soft emission of photons. Again in the second row the requested terms to achieve the 0.1% precision are indicated. The infinite resummation of the infrared terms in curled brackets bring again the $\varepsilon^{2\beta}$ factor in the last term. By summing up the terms in the second column it can be verified the cancellation of the infrared terms, the $\mathcal{O}(\alpha)$correction depends on Δ and can amount up to a few of 10%. If on the contrary the process is taken inclusive ($\Delta \to 1$) the $\mathcal{O}(\alpha)$correction becomes rather small (a few %).

In the third row is the contribution from the process with two hard photons emitted ($\equiv$ 2RBS) and the rough factor to BS Born term is this time

$$\frac{1}{2!}(2\beta)^2 \int_{\varepsilon}^{\Delta} \frac{d\omega_1}{\omega_1} \int_{\varepsilon}^{\Delta+\varepsilon-\omega_1} \frac{d\omega_2}{\omega_2} = \frac{1}{2!}(2\beta)^2 \left[\ln^2 \frac{\Delta}{\varepsilon} - \frac{\pi^2}{6}\right], \tag{7}$$

which therefore starts from $\mathcal{O}(\alpha^2)$ column. The corrections to this Born term, necessary to 0.1% precision, are indicated and infinite resummation over infrared terms again bring to $\varepsilon^{2\beta}$ factor in the last term. Again the sum of terms in $\mathcal{O}(\alpha^2)$ column shows the vanishing of the infrared terms and the remaining contribution is usually up to few percent, depending on Δ.

Finally in the fourth row is the contribution from the process with three hard photons emitted ($\equiv$ 3RBS) and this time only the rough factor to BS Born term is reported, even if we can imagine corrections and infrared terms resummation as reported in the last term of the row. In the $\mathcal{O}(\alpha^3)$ column sum is the usual disappearing of infrared terms, while the estimation of the contribution can be this time of the order of 0.1% or less.

The last term in the last row and last column is the infinite resummation of all the other terms in the last row or last column. Just to obtain a theoretical prediction of the cross-section the best is in principle to use the terms in the last row, where the infrared cutoff has disappeared and the contributions, order by order in the perturbative expansion, are lowering rather quickly.

The problem is that to accurately describe the experimental conditions it is necessary to simulate the process, *i.e.* to produce the requested amount of events, matching the apparatus condition, for each of the processes BS, 1RBS, 2RBS, 3RBS, ... This requires the construction of an event generator for each of the processes, which have an increasing number of particles in the final state (this aspect is not evident in Table I, where each process is represented by a row).

Furthermore, as already remarked, for small values of the cut-off parameter ε the $\mathcal{O}(\alpha)$correction can cause negative values to the BS cross-section. Adding the next $\mathcal{O}(\alpha^2)$ term, which is positive, can slightly improve the situation, but the corresponding term in the next row has also to be added to the corresponding 1RBS process, whose cross-section becomes negative. Proceeding in this way one simply push the negative cross-section effect to a process with more photons in the final state, therefore less important, with the purpose to disregard it.

Another possibility is to use the resummed terms in the last column of Table 1, which are always positive. But in this case, to eliminate the cut-off dependent factor $\varepsilon^{2\beta}$, one has to sum all the infrared terms in the hard photons factor of the kind

$$+\frac{1}{n!}\left(2\beta\ln\frac{\Delta}{\varepsilon}\right)^n, \tag{8}$$

or at least to include these terms for as many photons as is requested by the accuracy. For the possible choice of parameters for LEP1 already used ($\Delta = 0.5, \varepsilon = 10^{-4}, \beta = 0.1$) the term in Eq. (8) has the following values for $n = 1-8$: 1.2429, 0.7724, 0.3200, 0.0994, 0.0247, 0.0051, 0.0009, 0.0001, suggesting that up to the process with 7 hard photon emission has to be accounted in order to have the 0.1% accuracy.

Finally the partially resummed expression (somehow represented in the sketch of Table 1 by the last term in the last row or last column) is available in the form of structure functions integrals, but also in this case some approximations are introduced and have to be recovered to obtain the desired 0.1% precision.

All the exposed problems are examined and solved in part by different groups in various ways, producing also different codes.

BHLUMI 4 [14] is an event generator based on YFS-exponentiation method, which in the sketch of Table 1 is somehow illustrated by the sequence of terms in the last column.

OLDBIS+LUMLOG [7, 8], SABSPV [15] and BHAGEN95 [16] are Monte Carlo integration programs and use structure function approach with corrections to recover the approximations, introduced differently in the various programs, to obtain the resummation.

While the leading-logarithm coefficients $a_{11}, a_{22}, a_{33}, \ldots$ were known since a long time (see references in [17]), only recently the coefficients of subleading-logarithm (like a_{21}) were calculated analytically up to $\mathcal{O}(\alpha^2)$ and coded in a numerical program NLLBHA [18] in the way represented in the sketch of Table I by the terms of the last row.

Some comparisons between the different programs were done at $\mathcal{O}(\alpha)$ to verify the general behaviour of different codes (technical precision) [19, 13]. In the course of the LEP200 Working Group many tests were done to investigate the differences in the results of the codes, while the event selections, from some very simple choices, were more and more approaching the real experimental ones [13]. In a test for a simplified event selection (so called BARE1) the difference between NLLBHA and BHLUMI remains inside 0.1% for the interesting values of the parameters.

Comparisons are usually done for just a part of the cross section (although it is the most relevant one), but to obtain the programmed 0.1% precision also other corrections have to be included (like $\sigma_0^{(8)}[Z(s), \gamma(t)]$ with its $\mathcal{O}(\alpha)$radiative corrections, $\sigma_0^{(2)}[\gamma(s), \gamma(t)], \ldots$).

Vacuum polarization corrections are included accurately and the hadronic contribution, obtained through dispersion integration of the fit to experimental measurements of e^+e^- into hadrons [20], brings an error which is 0.07% for first generation luminometers and 0.04% for second generation luminometers (smaller angles). This error has to be considered as the ultimate limit in theoretical precision.

In Fig. 1 (taken from [13]) is shown the comparison between BHLUMI, SABSPV and BHAGEN95, with all corrections included and for a rather realistic event selection of calorimetric type, called CALO2, for three different angular acceptances for final positron-electron called Wide-Wide, Narrow-Narrow and Wide-Narrow.

One can see that the results stay inside the cut-line box of 0.1% height for values of the minimal admitted final electron or positron energy fraction $z_{\min}$ of experimental relevance. The asymmetric angular acceptance Wide-Narrow has results which are closer one another, confirming that this event selection (which is similar to the one adopted by the experiments) squeezes the effect of the higher order corrections introduced without completeness (therefore differently) in the various approaches.

In conclusion the recent work on Bhabha scattering at small-angle gives very good control of the technical agreement at $\mathcal{O}(\alpha)$, 0.1% agreement at least in the configurations chosen by the experiments, which squeeze the higher order effects, the control on $\mathcal{O}(\alpha^2)$ subleading contributions being necessary in the general case and for an even better (but rather difficult to obtain) theoretical accuracy.

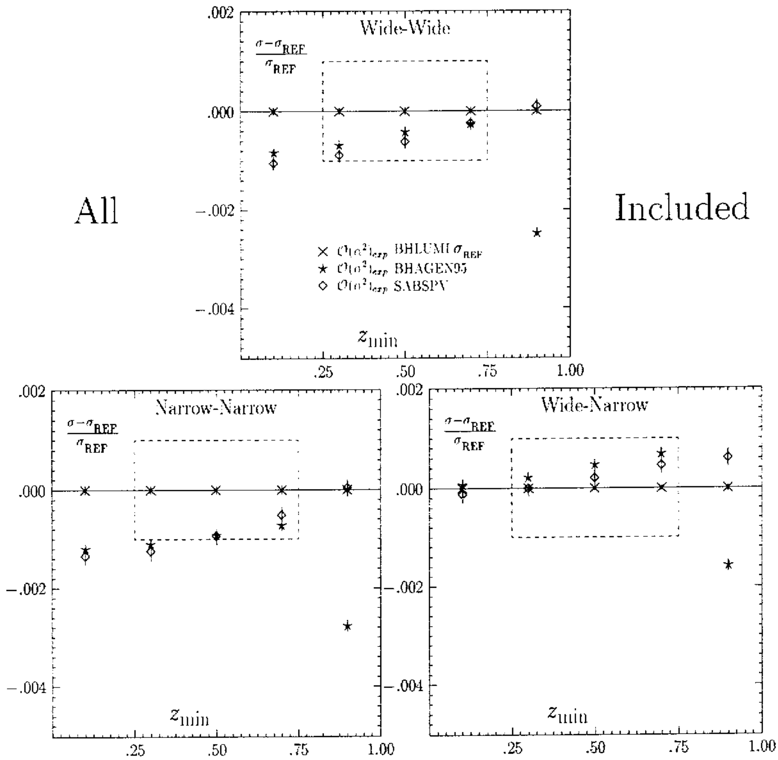

Fig. 1. Monte Carlo results for various symmetric/asymmetric versions of the CALO2 ES, for matrix elements beyond first order. Z exchange, up-down interference and vacuum polarization are switched ON. The center of mass energy is $\sqrt{s} = 92.3$ GeV. Not available x-sections are set zero. In the plot, the $\mathcal{O}(\alpha^2)^{\rm YFS}_{\rm exp}$ cross section $\sigma_{\rm BHL}$ from BHLUMI 4.x is used as a reference section.

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REFERENCES

[1] Lectures by G. Passarino and D. Schaile, this Conference.

[2] G. Altarelli *et al.*, *Physics at LEP*, eds J. Ellis and R. Peccei, CERN 86-02 1986, p.3.

[3] M. Caffo, E. Remiddi, F. Semeria, Bhabha Scattering, Z Physics at LEP 1, G. Altarelli, R. Kleiss and C. Verzegnassi Eds, CERN 89-08 Vol. 1 (1989) 171.

[4] F.A. Berends, R. Kleiss, *Nucl. Phys.* **B228**, 537 (1983).

[5] F.A. Berends,R. Kleiss, W. Hollik, *Nucl. Phys.* **B304**, 712 (1988).

[6] W. Beenakker, F.A. Berends, S.C. van der Marck, *Nucl. Phys.* **B355**, 281 (1991).
[7] S. Jadach, E. Richter-Wąs, B.F.L. Ward, Z. Wąs, *Phys. Lett.* **B253**, 469 (1991).
[8] S. Jadach, E. Richter-Wąs, B.F.L. Ward, Z. Wąs, *Phys. Lett.* **B260**, 438 (1991).
[9] S. Jadach, E. Richter-Wąs, B.F.L. Ward, Z. Wąs, *Phys. Lett.* **B268**, 253 (1991); *Comput. Phys. Commun.* **70**, 305 (1992).
[10] M. Cacciari, A. Deandrea, G. Montagna, O. Nicrosini, L. Trentadue, *Phys. Lett.* **B271**, 431 (1991).
[11] M. Caffo, H. Czyż, E. Remiddi, *Nuovo Cim. A* **105**, 277 (1992), preprint DFUB 91-01 (April 1991); *Int. J. Mod. Phys.* **C4**, 591 (1993).
[12] *Part III: Small Angle Bhabha Scattering, Reports of the working Group on Precision Calculations for the Z Resonance*, eds D. Bardin, W. Hollik and G. Passarino, CERN 95-03 1995, p.341.
[13] H. Anlauf *et al.*, Proceedings of the CERN Workshop on Physics at LEP2, Bhabha scattering, G. Altarelli, T. Sjöstrand and F. Zwirner Eds, CERN 96-01 1996.
[14] S. Jadach, E. Richter-Wąs, B.F.L. Ward, Z. Wąs, *Phys. Lett.* **B353**, 362 (1995); preprint CERN-TH/95-38; long write-up in [13].
[15] M. Cacciari, G. Montagna, O. Nicrosini, F. Piccinini, in [12] p.389; *Comput. Phys. Commun.* **90**, 301 (1995); preprint CERN-TH/95-169; long write-up in [13].
[16] M. Caffo, H. Czyż, E. Remiddi, in [12] p.361; long write-up in [13].
[17] M. Caffo, H. Czyż, E. Remiddi, *Phys. Lett.* **B327**, 369 (1994).
[18] A. Arbuzov *et al.*, *JETP* **108**, 1 (1995); in [12] p.369; long write-up in [13].
[19] M. Caffo, H. Czyż, E. Remiddi, in [12] p.361.
[20] S. Eidelman, F. Jegerlehner, *Z. Phys.* **C67**, 585 (1995); H. Burkhardt, B. Pietrzyk, *Phys. Lett.* **B356**, 398 (1995); see also references in [13].