

## CHIRAL PERTURBATION THEORY\*

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An introduction to the methods and ideas of Chiral Perturbation Theory is presented in this talk. The discussion is illustrated with some phenomenological predictions that can be compared with available experimental results.

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### 1. Introduction

Chiral Perturbation Theory (ChPT) is an effective theory that describes in a consistent and systematic way the strong, electromagnetic and weak interactions involving the lower mass pseudoscalar particles. It is based on a non-proved theorem that states that apart from causality and unitarity, the contents of a quantum field theory is dictated by the symmetries it possesses [1]. The idea is, thus, to replace the quarks and gluons of QCD by the pseudoscalar mesons and write down the most general lagrangian involving these particles that has the same symmetries as the QCD lagrangian. The ChPT generating functional admits an expansion in powers of external momenta and quark masses. Although ChPT is not a renormalizable theory the results can be rendered finite order by order in the expansion. The prize to pay is that new terms (with unknown constants) have to be included in the lagrangian at each order in the expansion.

I cannot cover in a single talk all the exciting results related with ChPT obtained during the last years. I should rather discuss here only a few of them and refer the interested reader to some of the more recent, excellent reviews available in the literature [2–8].

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This talk is organized in the following way: In the next section I present the lowest order lagrangian. In Section 3, I present the way to calculate the next order corrections,  $O(p^4)$ , and discuss some results at  $O(p^6)$ . Section 4 deals with the Wess-Zumino term and its  $O(p^6)$  corrections. Finally, the last section contains a brief summary of the talk and a list (not an exhaustive one!) of the present active research lines related with Chiral Perturbation Theory.

## 2. Lowest order lagrangian

The QCD lagrangian can be written in terms of  $q = \text{column}(u \ d \ s)$  in the form:

$$\begin{aligned} \mathcal{L}_{\text{QCD}} = & -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + i\bar{q}_L\gamma^\mu D_\mu q_L + i\bar{q}_R\gamma^\mu D_\mu q_R \\ & + \bar{q}_L m_q q_R + \bar{q}_R m_q q_L + \mathcal{L}_{\text{QCD}}^{\text{HF}}, \end{aligned} \quad (1)$$

where the term  $\mathcal{L}_{\text{QCD}}^{\text{HF}}$  includes the contribution from the heavy quarks ( $c$ ,  $b$  and  $t$ ) and we have explicitly separated the contributions for the left-handed,  $q_L$ , and right-handed,  $q_R$ , light quark fields. These components appear always separated except in the two terms proportional to the quark mass matrix,  $m_q = \text{diag}(m_u \ m_d \ m_s)$ . It is, thus, clear that in the limit where  $m_q = 0$  the lagrangian is invariant under independent transformations of the left and right-handed quark fields, *i.e.* under the group  $\text{SU}(3)_L \times \text{SU}(3)_R$ :

$$q_L \rightarrow g_L q_L \quad q_R \rightarrow g_R q_R \quad \text{with} \quad g_L, g_R \in \text{SU}(3)_{L,R}. \quad (2)$$

In view of this symmetry, one would expect all the hadrons to appear in multiplets of opposite parity, where all the particle should have approximately the same mass. However, there is no evidence for a particle with the same quantum numbers as the proton, but opposite parity and similar mass (the lightest  $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$  state has a mass  $m \sim 1535 \text{ MeV}$ ). Similar comparisons can be made for all the other hadronic states. We cannot blame the small quark masses  $m_u$  and  $m_d$  for this big effect. Instead, before the appearance of QCD it was already recognized that  $\text{SU}(3)$  was a rather good symmetry [9]. The chiral symmetry is, thus, spontaneously broken:  $\text{SU}(3)_L \times \text{SU}(3)_R \rightarrow \text{SU}(3)_V$ , *i.e.* the vacuum is symmetric only under  $\text{SU}(3)_V$  transformations. This breaking is produced through the non-zero value of the quark condensate

$$\langle 0|\bar{u}u|0 \rangle = \langle 0|\bar{d}d|0 \rangle = \langle 0|\bar{s}s|0 \rangle \sim -(250 \text{ MeV})^{3/2}, \quad (3)$$

which becomes the order parameter of the spontaneous chiral symmetry breaking. The Goldstone theorem assures that in this process eight goldstone bosons appear (one for each broken generator) [10]. These bosons are massless in the limit of massless quarks, but the small explicit chiral symmetry breaking through the quark masses gives a small mass to the goldstone bosons.

The idea of ChPT is to write down an effective lagrangian where the quarks and gluons have been replaced by the goldstone bosons appearing in the spontaneous chiral symmetry breaking. A convenient parameterization is in terms of a  $3 \times 3$  unitary matrix:

$$\Sigma = e^{2iM/f} \quad \text{with} \quad M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}, \quad (4)$$

and  $f$  is a free constant. This matrix transforms under  $SU(3)_L \times SU(3)_R$  as:

$$\Sigma \rightarrow g_L \Sigma g_R^\dagger. \quad (5)$$

The effective lagrangian contains an infinite number of terms, but it can be expanded according to the number of derivatives. This is something more than a convenient classification. Physically, it means an expansion in terms of powers of momenta that have to be small compared with the chiral symmetry breaking scale, which is  $\sim 1$  GeV. Lorentz invariance requires the number of derivatives to be even. Thus, the first term is:

$$\mathcal{L}_2 = \frac{f^2}{8} \text{tr} \partial_\mu \Sigma \partial^\mu \Sigma^\dagger. \quad (6)$$

This is the only relevant term with two derivatives, because other possible terms one can think off, such as  $\Sigma \partial_\mu \partial^\mu \Sigma^\dagger$ , differ from (6) only in a total derivative. Expanding  $\Sigma$  it is obvious that the lagrangian in Eq. (6) contains the kinetic terms for all the pseudoscalar mesons and interaction terms involving 4, 6 and a larger number of pseudoscalars. Moreover, taking the axial current one has:

$$\langle 0 | J_\mu^{L1+i2} | \pi^+ \rangle = -\frac{i}{\sqrt{2}} f_\pi P_{\pi\mu} \quad \text{with} \quad J_\mu^{L_a} = -\frac{if^2}{4} \text{tr} (T^a \partial_\mu \Sigma \Sigma^\dagger), \quad (7)$$

leading to the identification at this order of the free constant  $f$  with the well-known pion decay constant  $f_\pi = f = 132$  MeV. Note that at this point there is a complete  $SU(3)$  symmetry among the three decay constants:  $f_\pi = f_\eta = f_K$ .

The effects of the explicit chiral symmetry breaking through the non-vanishing values of the quark masses can be included in the lagrangian (6) adding some new terms:

$$\mathcal{L}_2 = \frac{f^2}{8} \text{tr} \left( \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + (\Sigma \chi^\dagger + \chi \Sigma^\dagger) \right), \quad (8)$$

where  $\chi$  contains the external scalar and pseudoscalar fields in the following way:

$$\chi = B(s - ip), \quad \text{where} \quad s = m_q + \dots \quad (9)$$

$B$  is again a free constant that can be calculated in terms of the pseudoscalar and quark masses:

$$B = \frac{2m_\pi^2}{m_u + m_d} = \frac{2m_K^2}{m_u + m_s} = \frac{6m_\eta^2}{m_u + m_d + m_s}. \quad (10)$$

From this relations, eliminating the quark masses, one can obtain the Gell-Mann-Okubo mass relation [11]

$$4m_K^2 - m_\pi^2 = 3m_\eta^2. \quad (11)$$

The new term in the lagrangian also contains more interaction terms, which are proportional to the pseudoscalar masses. The expansion, thus, is not only in powers of the momenta, but also in powers of the pseudoscalar masses.

External vector fields can be introduced in the theory converting the derivatives appearing in the lagrangian in covariant derivatives:

$$\begin{aligned} \mathcal{L}_2 &= \frac{f^2}{8} \text{tr} (D_\mu \Sigma D^\mu \Sigma^\dagger + (\Sigma \chi^\dagger + \chi \Sigma^\dagger)); \\ D_\mu \Sigma &= \partial_\mu \Sigma + iL_\mu \Sigma - i\Sigma R_\mu, \end{aligned} \quad (12)$$

and adding the appropriate kinetic terms for the vector fields  $L_\mu$  and  $R_\mu$ . These fields transform under  $\text{SU}(3)_L \times \text{SU}(3)_R$  as:

$$\begin{aligned} L_\mu &\rightarrow g_L L_\mu g_L^\dagger - i g_L \partial_\mu g_L^\dagger, \\ R_\mu &\rightarrow g_R R_\mu g_R^\dagger - i g_R \partial_\mu g_R^\dagger. \end{aligned} \quad (13)$$

In particular, we can introduce electromagnetic interactions involving photons and pseudoscalars with the identification  $L_\mu = R_\mu = e A_\mu Q$ , where  $Q$  is the quark charge matrix:

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}. \quad (14)$$

In this way we complete the description of the lowest order ChPT lagrangian. With this lagrangian we can reproduce all the Current Algebra results obtained in the 60's. For instance, it is a very simple exercise to obtain from Eq. (12) the Weinberg amplitude [12]:

$$A(s, t, u) = \frac{s - m_\pi^2}{f_\pi^2}, \quad (15)$$

that fixes the scattering amplitude for the process  $\pi^a(p_a) \pi^b(p_b) \rightarrow \pi^c(p_c) \pi^d(p_d)$  through the isospin decomposition

$$T_{ab,cd} = \delta_{ab}\delta_{cd}A(s, t, u) + \delta_{ac}\delta_{bd}A(t, s, u) + \delta_{ad}\delta_{bc}A(u, t, s), \quad (16)$$

with  $s = (p_a + p_b)^2$ ,  $t = (p_a - p_c)^2$  and  $u = (p_a - p_d)^2$ . The amplitudes of definite isospin can be expanded in partial wave amplitudes according to:

$$A^I(s, \cos \theta) = i \frac{32\pi\sqrt{s}}{\sqrt{s - 4m_\pi^2}} \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) (1 - e^{2i\delta_l^i(s)}), \quad (17)$$

where  $\delta_l^i$  are the phase shifts. The corresponding scattering lengths,  $a_l^i$ , are defined as the slope of the phase shifts at threshold. The lowest order predictions from Eq. (15) are:

$$a_0^0 = 0.156 \qquad a_0^0 - a_2^0 = 0.201 \quad (18)$$

to be compared with the experimental results:

$$a_0^0 = 0.26 \pm 0.05 \qquad a_0^0 - a_2^0 = 0.29 \pm 0.04. \quad (19)$$

It is clearly important to evaluate what are the corrections to these lowest order results.

### 3. Higher order corrections

The advantage of ChPT is that it provides a consistent way to calculate the quantum corrections to the tree level results of Current Algebra. The key point is that loop diagrams always contribute to a higher order in the momentum expansion. For instance, one loop diagrams with vertices derived from the lagrangian  $\mathcal{L}_2$  are  $O(p^4)$ . In any one-loop diagram the number of vertices is the same as the number of internal lines. Since each internal line contributes at  $O(p^{-2})$ , the total dimension of the diagram is given by the

momentum integral, *i.e.*  $O(p^4)$ . This result can be easily generalized to any  $L$ -loop diagram containing  $N_d$  vertices of dimension  $d$

$$D = 2L + 2 + \sum_d (d - 2)N_d. \quad (20)$$

The result of a loop calculation is, in general, divergent as one expects from dimensional counting. However, consistency in the momentum expansion requires the introduction of the terms of the effective lagrangian that are of the same order as the loop result. These terms are multiplied by free constants. So, we can use these constants to remove all the divergences, just splitting all of them into a finite, renormalized piece and an infinite one that is tuned to absorb all the divergences appearing at a given order in the expansion. Since we have built the effective lagrangian in such a way that contains *all* the possible terms at each order, we are assured that we will be able to remove all the divergences. The theory, however, is non-renormalizable because we are forced to introduce new counterterms at each order in the chiral expansion, in contrast to a renormalizable theory where only a finite number of counterterms are needed. We should remark here that both, the finite part of the constants and the loop contributions depend on the renormalization scale  $\mu$ . The physical amplitudes, however, are independent of this scale.

TABLE I

The values of the  $L_i$  coefficients and the input used to determine them, they are quoted at a scale  $\mu = m_\rho$ .

$L_i$	Value $\cdot 10^3$	Input
1	$0.4 \pm 0.3$	$K_{e4}$ and $\pi\pi \rightarrow \pi\pi$
2	$1.35 \pm 0.3$	$K_{e4}$ and $\pi\pi \rightarrow \pi\pi$
3	$-3.5 \pm 1.1$	$K_{e4}$ and $\pi\pi \rightarrow \pi\pi$
4	$-0.3 \pm 0.5$	$1/N_c$ arguments
5	$1.4 \pm 0.5$	$F_K/F_\pi$
6	$-0.2 \pm 0.3$	$1/N_c$ arguments
7	$-0.4 \pm 0.2$	Gell-Mann-Okubo, $L_5$ , $L_8$
8	$0.9 \pm 0.3$	$m_{K^0} - m_{K^+}$ , $L_5$ , baryon mass ratios
9	$6.9 \pm 0.7$	pion electromagnetic charge radius
10	$-5.5 \pm 0.7$	$\pi \rightarrow e\nu\gamma$

The lagrangian at  $O(p^4)$  contains 10 terms contributing to the same processes as the lagrangian  $\mathcal{L}_2$  [13]. In addition there are two more terms that do not contain any pseudoscalar field and, thus, they cannot be measured.

The values of the 10 free constants can be determined from experimental data [13, 4]. Their values at the scale  $\mu = m_\rho$ , together with an indication of the process where they have been determined is shown in Table I. Actually, they turn out to be of the expected order of magnitude. Indeed, assuming that the chiral symmetry breaking scale,  $\Lambda_\chi$ , is  $O(1 \text{ GeV})$  and taking into account that the constant in the lagrangian  $\mathcal{L}_2$  is  $f_\pi/8$ , we would expect

$$L_i \sim \frac{f_\pi}{8\Lambda_\chi} = 2 \times 10^{-3}. \quad (21)$$

A first result at  $O(p^4)$  is the  $SU(3)$  breaking in the decay constants:

$$\begin{aligned} f_\pi &= f \left[ 1 - 2\mu_\pi - \mu_K + \frac{4m_\pi^2}{f^2} L_5(\mu) + \frac{8m_K^2 + 4m_\pi^2}{f^2} L_4(\mu) \right] \\ f_K &= f \left[ 1 - \frac{3}{4}\mu_\pi - \frac{3}{2}\mu_K - \frac{3}{4}\mu_{\eta_8} + \frac{4m_K^2}{f^2} L_5(\mu) + \frac{8m_K^2 + 4m_\pi^2}{f^2} L_4(\mu) \right] \\ f_{\eta_8} &= f \left[ 1 - 3\mu_K + \frac{4m_{\eta_8}^2}{f^2} L_5(\mu) + \frac{8m_K^2 + 4m_\pi^2}{f^2} L_4(\mu) \right], \end{aligned} \quad (22)$$

where  $\mu_P$  arises from the loop contributions and is given by:

$$\mu_P = \frac{m_P^2}{16\pi^2 f^2} \log \left( \frac{m_P^2}{\mu^2} \right). \quad (23)$$

The ratios among the decay constants are almost independent of the value of  $L_4$ , which is expected to be zero because it is of higher order in the  $1/N_c$  expansion. Thus, from the experimental value

$$\frac{f_K}{f_\pi} = 1.22 \pm 0.01 \quad (24)$$

we can fix the constant  $L_5$  to the value quoted in Table I and predict the ratio

$$\frac{f_{\eta_8}}{f_\pi} = 1.30 \pm 0.05. \quad (25)$$

The constants in the lagrangian can, a priori, be calculated from QCD but certainly a non-perturbative method is required. There are some interesting attempts to investigate the chiral lagrangian with lattice QCD but, up to now the quenched approximation has always been used and, thus, a direct comparison with the numbers in Table I is meaningless and a reformulation of the chiral lagrangian in the quenched approximation is needed [15]. A different approach to evaluate the  $L_i$  was adopted in [16, 17]. The

assumption was that the constants  $L_i$  are saturated by the contribution of the lowest mass resonances after they have been integrated out. The predicted values of the constants under this assumption is shown in Table II. Since the couplings of the resonances to the pseudoscalar mesons are also unknown constants we have to use three of the  $L_i$  constants to fix these couplings. In any case, we can see from the table that the agreement with the experimental values is excellent. Although it is not shown in the table it can be seen that, as one would expect, the dominant contribution arises from the vector meson nonet. This assumption is particularly useful when calculating  $O(p^6)$  corrections, where the number of free constants is very large to be fixed by experimental data and strict (not implemented with this assumption) Chiral Perturbation Theory loses predictive power.

TABLE II

The values of the  $L_i$  coefficients compared with the predictions obtained assuming their saturation by the resonances. The asterisks mark the input parameters.

$L_i$	Value $\cdot 10^3$	Resonance saturation prediction
1	$0.4 \pm 0.3$	0.6
2	$1.35 \pm 0.3$	1.2
3	$-3.5 \pm 1.1$	-3.0
4	$-0.3 \pm 0.5$	0.0
5	$1.4 \pm 0.5$	1.4*
6	$-0.2 \pm 0.3$	0.0
7	$-0.4 \pm 0.2$	-0.3
8	$0.9 \pm 0.3$	0.9*
9	$6.9 \pm 0.7$	6.9*
10	$-5.5 \pm 0.7$	-6.0

Before closing this section let me comment on two recent two loop calculations. We showed in the previous section that the lowest order prediction for the scattering length  $a_0^0$  was slightly out of the experimental value. The one-loop,  $O(p^4)$  correction was calculated long ago [18–20] and recently the two-loop calculation has been performed [21]. We show in Fig. 1 the phase shift difference  $\delta_0^0 - \delta_1^1$  as a function of the center of mass energy of the two incoming pions. The contribution from the constants in  $\mathcal{L}_6$  has been estimated to be negligible and, thus, the figure has been drawn assuming that they cancel. The scattering lengths also show the same improvement:

$$a_0^0 = 0.156 + 0.044 + 0.017 = 0.217$$
$$a_0^0 - a_0^2 = 0.201 + 0.042 + 0.016 = 0.258, \tag{26}$$



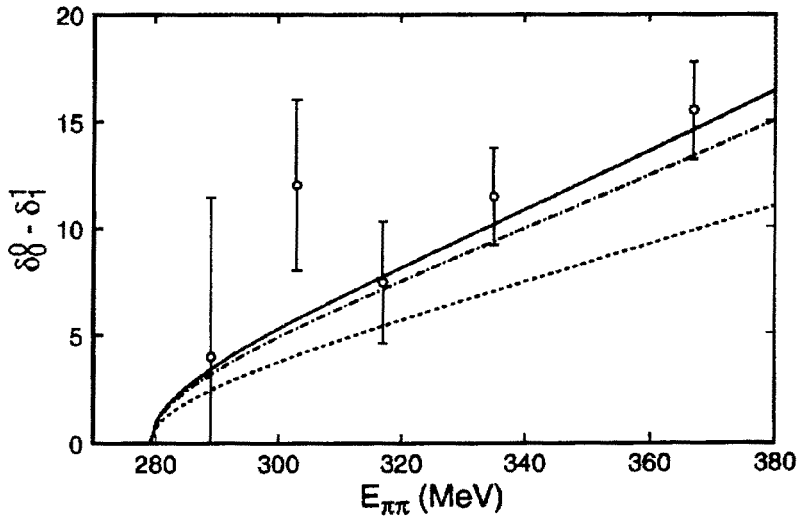


Fig. 1. Phase shifts difference as a function of the center of mass energy. The dashed line stands for the lowest order result, the dash-dotted line for the one-loop result and the full line for the two-loops result, assuming that the constants of the  $O(p^6)$  lagrangian vanish.

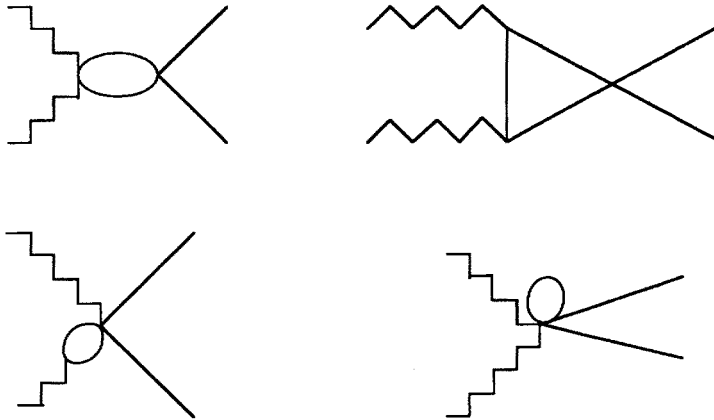


Fig. 2. Feynman diagrams contributing at lowest order to the process  $\gamma\gamma \rightarrow \pi^0\pi^0$ .

where the first term in the addition correspond to the lowest order result, the second to the one-loop correction and the third to the two-loop correction.

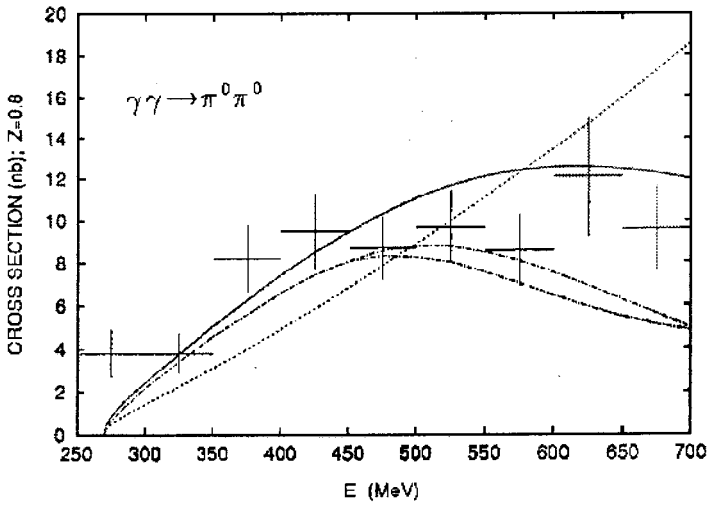


Fig. 3. Total cross section for the lowest order (dashed line) and  $O(p^6)$  (solid line). The dash-dot band is a dispersive calculation from Ref. [25].

The process  $\gamma\gamma \rightarrow \pi^0\pi^0$  presents a very interesting situation. Inspecting the lagrangians  $\mathcal{L}_2$  and  $\mathcal{L}_4$  one can see that there are no  $\gamma\pi^0\pi^0$  nor  $\gamma\gamma\pi^0\pi^0$  interaction terms. Thus, the lowest order contribution to  $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$  is given by the loop diagrams shown in Fig. 2, where the particles circulating in the loops are charged pions and kaons. This is an  $O(p^4)$  contribution. Thus, the result from the loop calculation must be finite (because there are no terms in  $\mathcal{L}_4$  contributing to this process that can be used to remove the divergences). Indeed, although each one of the diagrams shown in Fig. 2 is divergent, when adding all of them the divergences cancel and we obtain a parameter free prediction for the cross-section. The result is shown in Fig. 3 (dashed line) [22] compared with the experimental data from the Crystall Ball Collaboration [23]. Although the order of magnitude of the theoretical prediction is correct, it is a factor  $\sim 2$  too small compared with the experimental data near threshold. The inclusion of the  $O(p^6)$  terms improves the agreement between the theoretical prediction and the experimental data [24] (solid line). These corrections receive contributions from two-loop diagrams constructed with vertices from  $\mathcal{L}_2$ , one loop diagrams with one vertex from  $\mathcal{L}_4$  and tree level contributions from  $\mathcal{L}_6$ . Again, these last contributions contain free parameters that have fixed assuming their saturation by resonances.

#### 4. The Wess–Zumino term and higher order corrections

The lagrangian at  $O(p^4)$  contains an additional term originated by the chiral anomaly: the Wess–Zumino term [26]. This term contributes to processes with an odd number of pseudoscalar fields, in contrast with the lagrangians we dealt with in the previous sections that contribute to processes with an even number of pseudoscalars. The most characteristic process is the decay  $\pi^0 \rightarrow \gamma\gamma$ . Indeed, it is this process the one that is used to fix the constant in the Wess–Zumino term to be the number of colors <sup>1</sup>.

The Wess–Zumino term provides a good description of the decay width  $\Gamma(\pi^0 \rightarrow \gamma\gamma)$  assuming  $f = f_\pi$ . The situation, however, is rather different for the decay  $\eta \rightarrow \gamma\gamma$ . The physical  $\eta$  is a mixture of the octet and singlet pieces:

$$\begin{aligned}\eta &= \cos\theta\eta_8 + \sin\theta\eta_1 \\ \eta' &= -\sin\theta\eta_8 + \cos\theta\eta_1\end{aligned}\tag{27}$$

with  $\theta = -19.5^\circ$  [28]. Fixing  $f_1 = 1.1f_\pi$  from the experimental value  $\Gamma(\eta' \rightarrow \gamma\gamma) = (4.47 \pm 0.39)\text{keV}$  and using  $f_8 = 1.3f_\pi$ , as predicted by ChPT at  $O(p^4)$ , we obtain  $\Gamma(\eta \rightarrow \gamma\gamma) = 0.44\text{keV}$ . This result is in very good agreement with the experimental value  $\Gamma(\eta \rightarrow \gamma\gamma)_{\text{exp}} = (0.41 \pm 0.07)\text{keV}$ . However, the use of the next to leading order prediction for  $f_8$  is inconsistent with a lowest order prediction!. In order to take  $f_8 \neq f_\pi$  in a consistent way one has to include the whole, next order,  $O(p^6)$  corrections. This was done in Refs. [29] and [30], where the explicit cancellation of all the divergences and the absence of contributions from the  $O(p^6)$  lagrangian was shown. The only effect of the next order corrections is the modification of the value of  $f_8$ , thus justifying the procedure followed to obtain the ChPT prediction for the  $\eta \rightarrow \gamma\gamma$  decay width.

The cancellation of the corrections to the Wess–Zumino term is not a general feature. In [29] it was explicitly shown that the cancellation of the divergences appearing in one-loop diagrams contributing to the process  $P \rightarrow \gamma\gamma^*$  (where  $P$  stands for a neutral pseudoscalar meson and  $\gamma^*$  is an off mass shell photon) requires the introduction of counterterms. The  $O(p^6)$  lagrangian contributing to anomalous processes and the coefficients needed to cancel all the divergences are known [31–33]. The number of terms in the lagrangian is again very large to determine them experimentally. However, with the assumption of their saturation by the contribution of the lowest-mass resonances a very good description of the  $q^2$ -dependence in the decay  $P \rightarrow \gamma\gamma^*$  has been achieved [34].

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<sup>1</sup> Due to the origin of the Wess–Zumino term, the free constant it contains must be an integer [27].

The  $O(p^6)$  corrections clearly improve the situation for the decay  $\eta \rightarrow \pi^+\pi^-\gamma$ . The lowest order prediction for the decay width turns out smaller than the experimental value:

$$\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)_{\text{LO}} = 35 \text{ eV} \quad \Gamma(\eta \rightarrow \pi^+\pi^-\gamma)_{\text{EXP}} = (53 \pm 10) \text{ eV}. \quad (28)$$

Moreover, the predicted photon energy spectrum does not fit the experimental one [35]. The  $O(p^6)$  corrections have two effects. First of all, they increase the value of the decay width to

$$\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)_{O(p^6)} = 47 \text{ eV} \quad (29)$$

in such a way that it is now compatible with the experimental value (28). Second, they soften the photon spectrum as it is required by the experimental data.

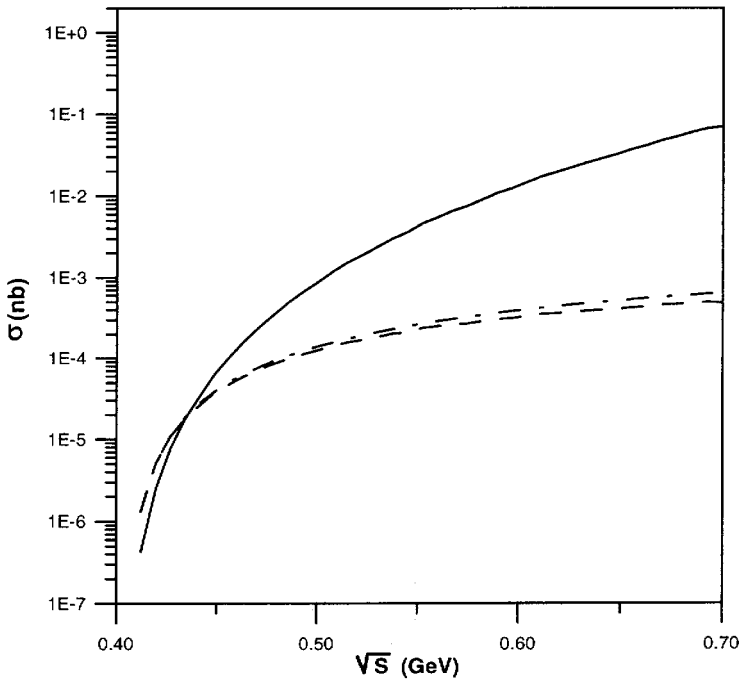


Fig. 4.  $\gamma\gamma \rightarrow \pi^0\pi^0\pi^0$  cross section at lowest order (dashed line) and  $O(p^6)$  (full line) as a function of the center of mass energy. The dot-dash line corresponds to the non-relativistic tree level approximation.

Finally, a short comment on a case where the effects of the corrections to the Wess–Zumino term are extremely important: the cross section for  $\gamma\gamma \rightarrow \pi^0\pi^0\pi^0$ . The lowest order amplitude turns out to be proportional to  $m_\pi^2$ , thus predicting a very small cross section. At  $O(p^6)$  terms proportional to the center of mass energy squared appear giving rise to huge corrections that increase the lowest order predictions in two orders of magnitude for a center of mass energy around 600 GeV as can be seen in Fig. 4 [36]. Certainly, questions about the convergence of the expansion arise at this point. However, there should be no problem because the huge corrections are due to the smallness of the lowest order prediction. Indeed, in the chiral limit, the lowest order amplitude vanishes, while the  $O(p^6)$  contribution is different from 0.

## 5. Conclusions

Chiral Perturbation Theory is an effective, low energy theory of QCD. It allows to calculate cross-sections and decay widths for processes involving pseudoscalar mesons. The expansion parameter is the momenta and masses involved in the process compared to the chiral symmetry breaking scale (around 1 GeV). It is a non-renormalizable theory, but results can be rendered finite order by order in the perturbative expansion. The price to pay is the introduction of new free constants in each order. This fact certainly limits the predictivity of the theory. However, this is not important until a high order is reached,  $O(p^6)$ . But even in this case, interesting phenomenological results can be obtained assuming the saturation of the free constants by the low mass resonances. The validity of this assumption has been verified for those constants in the Chiral Lagrangian that can be fixed by experimental data.

Let me finish with a small list of research subjects in Chiral Perturbation Theory that are being followed nowadays:

1. Two-loop calculations in the meson sector.
2. Introduction of Vector Mesons and other resonances in the Chiral Lagrangian.
3. Kaon leptonic and non-leptonic decays.
4. Pion-nucleon interactions.
5. Chiral symmetry and underlying quark models.
6. Heavy Quark applications of chiral symmetry.

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