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FROM LEP 1 PHYSICS TO LEP 2 PHYSICS THE GAUGE CONNECTION*

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The most relevant theoretical aspects associated with the experimental results obtained at LEP 1 and foreseen at LEP 2 are discussed. In particular the quest for a fully gauge invariant formulation of radiative corrections, both for two fermion and four fermion processes at LEP 2 energies is addressed. The outcome of the analysis clearly shows that such a formulation is indeed possible and some of the subtleties are discussed.

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1. Theoretical basics for LEP 1 Physics

About 1.5×10^7 Z decays have been recorded and analysed during the years of operation of the four LEP experiments — from autumn of 1989 to the end of 1994. A completely revised analysis of radiative corrections at the Z resonance is therefore needed in order to match the reached experimental precision. The theoretical goal must be to estimate the intrinsic theoretical uncertainties of the results emerging from different approaches, which are mainly caused by the neglect of higher order contributions.

The results, which have been presented in Ref. [1], are based on several different approaches and on a comparison of their numerical predictions. The findings of the Report are based on the following computer codes:

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BHM [2]

Burgers, Hollik, Martinez, Teubert;

LEPTOP — ITEP Moscow group [3]

Novikov, Okun, Rozanov, Vysotsky;

TOPAZ0 — Torino–Pavia group [4]

Montagna, Nicrosini, Passarino, Piccinini, Pittau;

WOH [5]

Beenakker, Burgers, Hollik;

ZFITTER — Dubna-Zeuthen group [6]

Bardin, Bilenky, Chizhov, Olchevsky, S.Riemann, T.Riemann, Sachwitz, Sazonov, Sedykh, Sheer.

Conclusions from this study are that the differences between results of different codes are small compared to existing experimental uncertainties. Thus improvement of experimental accuracy at LEP 1 and SLC is welcome even at the present level of theoretical accuracy. At present the most promising are measurements of g_V/g_A in various P- and C- violating asymmetries and polarizations. The real bottleneck for improved theoretical accuracy in g_V/g_A is presented by the uncertainty of the input parameter $\alpha(M_Z)$. The improved accuracy of this important parameter calls for new accurate measurements of the cross-section $e^+e^- \to$ hadrons at low energies (Novosibirsk and Bejing accelerators, *etc.*)

The estimates of theoretical uncertainties are highly subjective and their values partly reflect the internal philosophy of the actual implementation of radiative corrections in a given code. In many cases the one-loop approximation in the electroweak gauge coupling is adequate enough at the present level of experimental accuracy. At the same time, however, it should be stressed that a complete evaluation of the sub-leading corrections, $\mathcal{O}(G_\mu^2 M_Z^2 m_t^2)$ would greatly reduce the uncertainty that we observe, one way or the other, for all observables.

In case the next generation of experiments at LEP 1 and SLC improves accuracy considerably (a problem not only of statistics but mainly of systematics) the full program of two-loop electroweak calculations should be carried out.

1.1. Calculational schemes

It is important to underline how an estimate of the theoretical uncertainty emerges from the many sets of numbers obtained with the five codes. First of all, one may distinguish between intrinsic and parametric uncertainties. The latter are normally associated with a variation of the input parameters according to the precision with which they are known. Typically, we have $|\Delta\alpha^{-1}(M_Z^2)| = 0.12$, $|\Delta m_b| = 0.3\,\text{GeV}$, $|\Delta m_c| = 0.35\,\text{GeV}$ etc. These uncertainties will eventually shrink when more accurate measurements become available. Intrinsic uncertainties associated with missing non-leading higher-order corrections. An essential ingredient of all calculations for radiative corrections to physical observables is the choice of the renormalization scheme.
There are many renormalization schemes in the literature:

- the on-shell schemes in various realizations [7–13, 3]
- the G_μ scheme [14–16]
- the * scheme [17]
- the $\overline{MS}$ scheme [18–19].

One cannot simulate the shift of a given quantity due to a change in the renormalization scheme with one code alone. Thus the corresponding theoretical band in that quantity will be obtained from the differences in the prediction of the codes, which use different renormalization schemes. On top of that we should also take into account the possibility of having different implementations of the full radiative corrections within one code, — within one well specified renormalization scheme.
There are common features of and main differences between the electroweak libraries of the five codes.

Common features:

- All five codes use as input parameters the most accurately known electroweak parameters G_μ, M_Z and $\alpha(M_Z)$, in order to calculate the less precisely measured pseudo-observables.
- They use the same expressions for final state QED and QCD corrections (radiation factors).
- All codes include essentially the same internal gluon corrections of the order of $\alpha\alpha_s$ in the W and Z self-energy quark loops.

- All codes include leading two-loop corrections of the order of $G_\mu^2 m_t^4$; all gluonic corrections of the order of $\alpha_s G_\mu m_t^2$; leading gluonic corrections $\alpha\alpha_s^2$ in the vector boson self-energies.

Main differences:

- Each code uses a different renormalization framework.

- Some codes define an electroweak Born approximation, others give no physical emphasis to a Born approximation and employ only the notion of an *Improved Born Approximation (IBA)* which includes the leading electroweak loop corrections.

- They differ by the choice of the definition of the weak mixing angle.

1.2. Options, theoretical uncertainties

There is a need to quantify the effect of our partial lack of knowledge of the missing higher-order terms in radiative corrections. Thus we have introduced the notion of *option*. There are two main components in each observable:

$$O = O_B + \Delta O\,. \tag{1}$$

The term O_B, giving the bulk of the answer, is often called the Born approximation, or improved Born approximation, or the leading contribution to O. The term ΔO represents a small perturbative correction, often called remainder or non-leading contribution. Different realizations usually have different ways of performing this splitting so that, while they agree at the $\mathcal{O}(\alpha)$, there are differences which start at $\mathcal{O}(\alpha^2)$.

- Leading–Remainder splitting
  Generally speaking, the effective couplings contain a leading and usually re-summed part and a non-leading (remainder) one, quite independent of the specific realization. The way in which the non-leading terms can be treated and the exact form of the *leading–remainder* splitting give rise to several possible options in the actual implementation of radiative corrections that in turn become another source of theoretical uncertainty.

- Scale in vertex corrections
  Another possible option has to do with the scale of α in the non-leading corrections, in particular the vertex corrections. The difference between possible identifications of coupling constants in the $\mathcal{O}(\alpha)$ corrections

represents, of course, effects of $\mathcal{O}(\alpha^2)$. The fact that the neutral current amplitude is automatically expressed in terms of $G_\mu M_Z^2$ is a possible heuristic argument to adopt the same strategy in the evaluation of the presently known $\mathcal{O}(\alpha)$ corrections, but in order to be on the safe side, the differences should be considered as a theoretical uncertainty.

- Linearization
  In almost any realization we have the possibility of using an *expanded* versus a *non-expanded* option — to linearize our expressions.

- More generally, the difference between the two options in the evaluation of O^2, where O is given by Eq. (1), is equal to $(\Delta O)^2$, a two-loop reducible but non-leading contribution and by comparing the two options we obtain a rough estimate of the importance of the missing non-leading two-loop effects.

- Resummation
  There are different ways of implementing the resummation of the vector boson self-energies. These choices in turn are deeply related to the proper definition of remainder. Resummation is very often the main recipe for separating a small remainder from the bulk of the effect.

One choice consists in a resummation which includes the square of the $Z-\gamma$ mixing-term with the option of strictly keeping in the resummation only the one-loop irreducible terms. Even more generally we can distinguish among

1. complete expansion of the one-loop self-energies,
2. partial resummation of fermionic self-energies or
3. partial inclusion of bosonic self-energies in the resummation.

There are two considerations to be made at this point. Sometimes accidental cancellations occur among the fermionic and the bosonic sectors, which would suggest a similar treatment for both. However, the bosonic sector is not gauge invariant by itself. Thus any resummation of bosonic parts must properly identify some numerically relevant but gauge invariant sub-set.

1.3. $e^+e^- \to \bar{f}f$ at LEP 2 energies

There is an important question to be answered when we consider the e^+e^- annihilation into 2-fermions at LEP 2 energies: what is the fate of the familiar parametrization in terms of running effective couplings? In order to give an answer we must remember that only a proper arrangement of radiative corrections, including all contributions up to a given order, is gauge invariant. Thus every procedure designed for subtracting some part from the whole (de-convolution, de-boxization, ...) must respect gauge invariance.

Formally the amplitude can be written as

$$A(s) = \frac{V_i V_f}{s - M_0^2 + S_{ZZ}} + B\,, \tag{2}$$

where S_{ZZ} is the full 1 PI self-energy, $V_{i,f}$ is the full i, f state vertex and B represents the multiparticle exchange diagrams. From the bare Lagrangian we derive the complex pole

$$s_p - M_0^2 + S_{ZZ}(s_p) = 0\,, \tag{3}$$

and a Laurant expansion of the amplitude will follow

$$A(s) = \frac{M_{if}(s_p)}{s - s_p} + \frac{M_{if}(s) - M_{if}(s_p)}{s - s_p} + B\,, \tag{4}$$

$$M_{if} = V_i \frac{s - s_p}{s - M_0^2 + S_{ZZ}} V_f\,, \tag{5}$$

which gives the pole, the residue and the non-resonant part as separately gauge-invariant objects. The key point is that at LEP 2 energies the non-resonant background is large and non negligible. Thus we must examine the box diagrams, in particular the WW box with a W propagator which in the R_ξ-gauge is given by

$$\frac{1}{p^2 + M_W^2}\left[\delta_{\mu\nu} + (\xi^2 - 1)\frac{p_\mu p_\nu}{p^2 + \xi^2 M_W^2}\right]. \tag{6}$$

It follows

$$B_{WW}(\xi) = B_{WW}(\xi = 1) + (\xi^2 - 1)\frac{\delta_\xi}{M_W^2}\gamma^\mu(1 + \gamma^5) \otimes \gamma^\mu(1 + \gamma^5)\,, \tag{7}$$

and schematically we can write

$$B_\xi = B_1 + (\xi^2 - 1)\Delta(\xi)\,, \tag{8}$$

$$B_\xi = B_{\xi_0} + \bar{\Delta}(\xi, \xi_0)\,, \tag{9}$$

$$\bar{\Delta}(\xi, \xi_0) = (\xi^2 - 1)\Delta(\xi) - (\xi_0 - 1)\Delta(\xi_0)\,. \tag{10}$$

This splitting is however not unique since we can always rearrange it as

$$B_\xi = B_{\xi_0} + \delta B + \bar{\Delta}(\xi, \xi_0) - \delta B \,. \tag{11}$$

In any case a pragmatic proposal is available. We could use the R_ξ gauge and incorporate $\bar{\Delta}(\xi, \xi_0)$ in the rest of the ξ-dependent amplitude so that the WW box is computed explicitly in the ξ_0 gauge. This procedure gives a ξ-independent answer which is also unambiguous, unless weak boxes are not convoluted or unless a re-summation is performed. A proposal for de-boxization is therefore that

1. We all agree on subtracting $B(\xi = 1)$.
2. Those working in the $\xi = 1$ gauge stop here.
3. Those working in any ξ_0 gauge compensate the amplitude (without weak boxes) with $\bar{\Delta}(\xi_0, 1)$.

Alternatively one could use the ZFITTER option of inserting the Weak Boxes into the Form Factors.

2. Theoretical basics for LEP 2 physics

For LEP 1 physics or, more generally, for $2f$-physics we need relatively few diagrams with one loop corrections and leading higher loop contributions. To the contrary for $4f$-physics we need many diagrams at the tree level. Typically we have

1. 10 diagrams for $e^+e^- \to \mu^-\bar{\nu}_\mu u\bar{d}$
2. 20 diagrams for $e^+e^- \to e^-\bar{\nu}_e u\bar{d}$
3. 56 diagrams for $e^+e^- \to e^+e^-\nu_e\bar{\nu}_e$

Different groups are already producing results for the LEP 2 WW/EG Working Group [20], however we should ask how far one can go without loop corrections [21].

2.1. The question of gauge invariance and the process $e^+e^- \to W^+W^-$

Although the proper treatment of the problem should be discussed in the contest of unstable (off-shell) W's we can already show all the elements of the method in a simpler process where we unrealistically assume that the W are on-shell. With the simple process $e^+e^- \to W^+W^-$ we intend to mimic a more realistic situation ($e^+e^- \to 4\,\text{fermions}$) where however the full one loop calculation is not available. Even in this unrealistic case the Z boson will appear as an unstable particle in the s-channel and this is already enough to spoil the unitarity cancellation if the proper care is not applied. First of all we write a Ward identity (WI) for the process $e^+(p_+)e^-(p_-) \to W^+(q_+)W^-(q_-)$ at the three level. All particles are on mass-shell and in general Higgs ghosts (ϕ) must be included. It is easy to show that the following identity holds, where W is the sum of all contracted diagrams

$$W = \frac{1}{4}\,g^2\,\frac{\Gamma_Z(s)}{s - M_Z^2 + i\Gamma_Z(s)}\,\bar{u}(p_+)\gamma^\mu\left(4\,s_\theta^2 - 1 - \gamma^5\right)v(p_-)e_\mu(q_-)\,,$$
$$\Gamma_Z(s) = \gamma_Z\,s, \quad \gamma_Z = \frac{g^2}{48\,\pi c_\theta^2}\sum_f N_f\left[\left(I_f^{(3)} - 2\,Q_f s_\theta^2\right)^2 + \frac{1}{4}\right], \tag{12}$$

where the sum is over all the fermions which satisfy the condition $s > 4m_f^2$. Moreover $I_f^{(3)}, Q_f, N_f$ denote the third component of isospin, the electric charge (in units of the proton charge) and the color number for the fermion. It is perhaps useful to comment at this point on the width to be inserted in the Z propagator when s is well above the WW threshold. Let S_{ZZ} be the Z self-energy, then the inverse Z propagator will become $\chi(s) = s - M_0^2 + S_{ZZ}(s)$. The complex pole s_p is a solution of

$$\chi(s_p) = 0\,. \tag{13}$$

If we write $s_p = m^2 - i\Gamma\,m$ then, by neglecting fermion masses, it follows

$$m^2 = M_Z^2 - \Gamma^2, \qquad M_Z^2 = M_0^2 - ReS_{ZZ}\left(M_Z^2\right)\,,$$
$$\mathrm{Im}\,S_{ZZ}(s) = \frac{s}{m}\,\Gamma, \qquad s < 4\,M_W^2\,,$$
$$\Gamma_Z = \frac{M_Z}{m}\,\Gamma, \qquad M_Z\Gamma_Z = \mathrm{Im}\,S_{ZZ}\left(M_Z^2\right)\,. \tag{14}$$

We can therefore write

$$\frac{1}{s - s_p} = \frac{1 + i\,\Gamma_Z/M_Z}{s - M_Z^2 + i\,s\Gamma_Z/M_Z^2}\,. \tag{15}$$

and take into account that the width Γ_Z is always computed for $s = m^2 \approx M_Z^2$. In the renormalization of the theory one will encounter terms like

$$\frac{S_{ZZ}(s) - ReS_{ZZ}(s_p)}{s - s_p}\,, \tag{16}$$

where the imaginary parts of $S_{ZZ}(s)$ due to W loops must be expanded up to first order to be combined with the corresponding imaginary part coming from the ZWW vertex diagram.

$$\frac{1}{s-M_0^2+S_{ZZ}(s)}=\frac{1}{s-M_Z^2+i\,\frac{s}{M_Z}\,\Gamma_Z}\left[1+\frac{W(s)}{s-s_p}\right],$$
$$W(s)=S_{ZZ}(s_p)-S_{ZZ}(s)+i\,\frac{\Gamma_Z}{M_Z}\,(s-s_p)\,. \tag{17}$$

Since $\operatorname{Im}S_{ZZ}(s)=(\Gamma/m)s=(\Gamma_Z/M_Z)s$ then the imaginary part of $W(s)$ will receive contributions from thresholds such that $s>M_Z^2$. We must stress that one should write the amplitude in terms of physical input parameters, *i.e.* $\alpha(0),G_\mu,M_W,M_Z,\ldots$. Then the question of what to use for s_θ^2 will arise and from the WI relative to $\gamma_Z=0$ we learn that gauge invariance is violated unless $c_\theta^2=M_W^2/M_Z^2$. In order to understand the content of the WI, we write it again in the R_ξ gauge where the relevant gauge fixing terms and the W and ϕ propagators are:

$$C^a=-\frac{1}{\xi}\,\partial_\mu B^a_\mu+\xi M_W\phi^a\,,$$
$$\Delta_W^{\mu\nu}=\frac{1}{p^2+M_W^2}\left[\delta^{\mu\nu}+(\xi^2-1)\,\frac{p^\mu p^\nu}{p^2+\xi^2M_W^2}\right],$$
$$\Delta_\phi=\frac{1}{p^2+\xi^2M_W^2}\,. \tag{18}$$

If A_W^μ and A_ϕ are the relative amplitude, what we compute is

$$X_\xi=\frac{1}{\xi}\,p_\mu A_W^\mu+i\xi M_W\,A_\phi\,. \tag{19}$$

Another way of putting it is to say that C^a is a free field. It is convenient to extract the W and ϕ propagators, such that

$$A_W^\mu=\Delta_W^{\mu\nu}\,a_W^\nu\,,$$
$$A_\phi=\Delta_\phi\,a_\phi\,, \tag{20}$$

to obtain

$$X_\xi=\frac{p^\mu}{\xi\left(p^2+M_W^2\right)}\left[\delta_{\mu\nu}+(\xi^2-1)\,\frac{p^\mu p^\nu}{p^2+\xi^2M_W^2}\right]a_W^\nu$$
$$+\,i\xi M_W\,\frac{a_\phi}{p^2+\xi^2M_W^2}$$
$$=\frac{\xi}{p^2+\xi^2M_W^2}\,\left(p\cdot a_W+iM_W a_\phi\right)\,. \tag{21}$$

This shows explicitly how the WI is satisfied in all R_ξ gauges when it is satisfied in the renormalizable ($\xi = 1$) gauge, the only other modification being the Z propagator where however the $p_\mu p_\nu$ part gives a term proportional to the electron mass. This is usually neglected or otherwise we insert correspondingly a ϕ^0-line. In turns this means that in all gauges we must respect $c_\theta^2 = M_W^2/M_Z^2$. The violation of the WI, consequence of a non zero γ_Z has an immediate counterparts in terms of the asymptotic behavior of the total cross section when $s \to \infty$.

Indeed from the three basic diagrams contributing to the process in lowest order we find

$$\sigma(s) \approx \frac{g^4}{256\,\pi^2 s}\,\beta f(s)\,,$$

$$f(s) = \Delta\gamma \left[f_1 + f_2\,\frac{s}{M_W^2} + f_3\,\frac{s^2}{M_W^4}\right] + \dots . \tag{22}$$

The violating term $\Delta\gamma$ is given by

$$\Delta\gamma = -\frac{\gamma_Z}{1+\gamma_Z^2}\,. \tag{23}$$

Clearly this is a rather academical problem since it would suffice to neglect the Z width from the beginning. Nevertheless we insist in our discussion since it is relatively simple and moreover the solution has everything in common with the more realistic cases. There are three additional one loop diagrams to be considered or rather their imaginary parts. The first is given by the $Z-\gamma$ transition induced by fermionic loops. We easily find

$$-2\ \mathrm{Im}\ \Sigma_{Z\gamma}(p^2) = -\frac{4}{3}\,\frac{g^2 s_\theta}{c_\theta}\pi^3\,p^2 \sum_f N_f Q_f \left(I_f^{(3)} - 2\,Q_f s_\theta^2\right)$$
$$= -\frac{g^2 s_\theta}{c_\theta}\left(8 - \frac{64}{3}s_\theta^2\right)\pi^3 p^2\,. \tag{24}$$

In this equation we have assumed that $-p^2 = s \gg 4m_f^2$, so that all fermions are assumed to be massless. Also the ZW^+W^- vertex, corrected with a fermionic loop should be considered. The first result is that the imaginary parts do not factorize into the lowest order, but two additional form factors are needed. We find

$$
\begin{aligned}
-2\ \mathrm{Im}\ V_{ZWW} &= 12\, g^3 c_\theta \pi^3 \sum_i v_i W^i_{\mu\alpha\beta}\,, \\
V^1_{\mu\alpha\beta} &= 2\ \left[\delta_{\mu\beta} p_\alpha - \delta_{\mu\alpha} p_\beta - \delta_{\alpha\beta} q_{-\mu}\right]\,, \\
V^2_{\mu\alpha\beta} &= 2\ \left[\delta_{\mu\beta} p_\alpha - \delta_{\mu\alpha} p_\beta\right]\,, \\
V^3_{\mu\alpha\beta} &= 2\ \left[\delta_{\mu\beta} p_\alpha + \delta_{\mu\alpha} p_\beta\right]\,, \\
V^4_{\mu\alpha\beta} &= -\frac{2}{s}\, q_{-\mu} p_\alpha p_\beta\,,
\end{aligned} \tag{25}
$$

where the coefficients are given by

$$
\begin{aligned}
v_1 &= \frac{2}{3} + \frac{2}{3}\frac{\mu^2}{\beta^2} - 4\,\frac{\mu^4}{\beta^4} + 8\,\frac{\mu^6}{\beta^5}\, l_0\,, \\
v_2 &= \frac{\mu^2}{\beta^2}\left[1 + 6\,\frac{\mu^2}{\beta^2} - 4\,\frac{\mu^2}{\beta}\left(1 + 3\,\frac{\mu^2}{\beta^2}\right) l_0\right]\,, \\
v_3 &= 0\,, \\
v_4 &= \frac{\mu^2}{\beta^4}\left[-\frac{8}{3} - 24\,\frac{\mu^2}{\beta^2} + 16\,\frac{\mu^4}{\beta^2} + 16\,\frac{\mu^2}{\beta}\left(1 + 2\,\frac{\mu^2}{\beta^2} + 2\,\frac{\mu^4}{\beta^2}\right) l_0\right]\,,
\end{aligned} \tag{26}
$$

and where we have used

$$
\mu^2 = M_W^2/s, \qquad \beta^2 = 1 - 4\,\mu^2, \qquad l_0 = \ln\frac{1-\beta}{1+\beta}\,. \tag{27}
$$

Notice that the CP-odd coefficient v_3 is zero. Next we consider the sum $W + W_{\mathbf{add}}$, where $W_{\mathbf{add}}$ contains the additional diagrams obtained by inserting the loop corrections into the Z-exchange diagram. As a result we find that for each isospin doublet $d = (f, f')$ the following identity holds

$$
\begin{aligned}
W + W_{\mathbf{add}} =& \frac{1}{4} g^2 \frac{1}{s - M_Z^2 + i\Gamma_Z(s)} \bar{u}\gamma^\mu \left(4 s_\theta^2 - 1 - \gamma^5\right) v \frac{g^2}{96\,\pi c_\theta^2} \sum_{ff'} W_{ff'}\,, \\
W_{ff'} =& 2\left(v_f^2 + v_{f'}^2\right) + 1 + \left(v_{f'} - v_f\right) + \left(I_{f'}^{(3)} - I_f^{(3)}\right) \\
&+ 4\left(Q_{f'} v_{f'} + Q_f v_f\right) s_\theta^2\,,
\end{aligned} \tag{28}
$$

where $v_f = I_f^{(3)} - 2Q_f s_\theta^2$. From the fact that $I_f^{(3)} - I_{f'}^{(3)} = 1$ and $I_f^{(3)} - v_f = 2Q_f s_\theta^2$ it follows

$$W + W_{\mathrm{add}} = 0\,. \tag{29}$$

Thus we have found the minimal set of one loop diagrams which must be included in order to respect the corresponding WI. This set does not include all fermionic loop insertion into the basic diagrams of the three level amplitude, indeed we should also consider the corrections to the $\gamma-\gamma$, $\gamma-Z$ transitions and to the γWW vertex. However we find that the collection of all these diagrams is given by

$$\begin{aligned} W' &= g^2 \bar{u}\gamma^\mu v e_\mu \sum_f N_f \left[W'_{\gamma Z}(f) + W'_{\gamma WW}(f) + W'_{\gamma\gamma}(f)\right], \\ W'_{\gamma Z}(f) &= \frac{1}{s - M_Z^2 + i\Gamma_Z(s)} \frac{g^2}{24\pi} Q_f v_f \left[M_Z^2 s_\theta^4 - \left(s - M_W^2\right) s_\theta^2\right], \\ W'_{\gamma WW}(f) &= \frac{1}{s}\frac{g^2}{24\pi} Q_f I_f^{(3)} s_\theta^2\,, \\ W'_{\gamma\gamma} &= -\frac{g^2}{12\pi} Q_f^2 s_\theta^4\,. \end{aligned} \tag{30}$$

By collecting the various terms we obtain that for each fermion

$$W' \propto g^4 \sum_f N_f Q_f v_f s_\theta^2 \left(M_W^2 - c_\theta^2 M_Z^2 + \mathcal{O}(g^2)\right). \tag{31}$$

Once more the WI is $W' = \mathcal{O}(g^6)$ if we have defined from the beginning $c_\theta^2 = M_W^2/M_Z^2$. A more complete treatment is needed in order to achieve $W' = 0$ (*i.e.* to all orders) and it requires a comprehensive study of full one-loop propagators in the neutral sector of the theory, including real parts. It is important however to observe that we have defined a minimal set of additional diagrams, needed to restore the Ward identities. Obviously the imaginary part of the triangle VWW ($V = Z, \gamma$) is the sum of all cuts over intermediate states. Two additional contributions have been left out in our considerations, which will be denoted by $E_\pm^{\mu\alpha\beta}$,

$$\begin{aligned} E_+^{\mu\alpha\beta} &= -2i\,(a+b) \int d^4q\, \mathrm{tr}\left[\gamma^\mu\gamma^\rho\gamma^\alpha\gamma^\sigma\gamma^\beta\gamma^\tau\right] \\ &\quad \times q_\rho\,(q+q_+)_\sigma\,(q+p)_\tau\,\frac{1}{(q+p)^2}\,, \\ E_-^{\mu\alpha\beta} &= 2i\,(a+b) \int d^4q\, \mathrm{tr}\left[\gamma^\mu\gamma^\rho\gamma^\alpha\gamma^\sigma\gamma^\beta\gamma^\tau\right] \\ &\quad \times (q+p)_\rho\,(q+q_-)_\sigma\,q_\tau\,\frac{1}{(q+p)^2}\,, \end{aligned} \tag{32}$$

where $a(b)$ denotes the vector(axial-vector) coupling of V to down-fermions and where all coupling constant factors have been left out. It is easy to observe that these additional imaginary parts have nothing to do with the Z width but rather with the W width. As such they must certainly be included for any realistic process $e^+e^- \to 4f$ in order to compensate for the off-shell W's but not for real W's. Let us indeed consider their contribution to our WI, namely $q_+^\alpha E_{\pm\mu\alpha\beta}$. It is immediate to find

$$q_+^\alpha E_{+\mu\alpha\beta} = 0, \qquad q_+^\alpha E_{-\mu\alpha\beta} = -\frac{8}{3}\, i(a+b) M_W^2 I_0 \delta_{\mu\beta}\,. \tag{33}$$

Suppose that this extra term is inserted into the original WI, actually both for the Z and for the γ, and that we sum over all fermions. It is clear that the parts proportional to Γ_Z continue to cancel, while new contributions will appear. These new contributions are exactly cancelled by the procedure of keeping M_W^2 in all $Z(\gamma)W\phi$ vertices and of replacing $q_\pm^2 = -M_W^2$ with $q_\pm^2 = -M_W^2 + i\, M_W \Gamma_W$. The presence of the two additional cuts requires off-shell W's to satisfy the WI. In other words we include the additional contributions only when the W's are off-shell and their width is inserted into the Feynman diagrams.

Our last comment will be devoted to the correct treatment of the imaginary part of the triangle diagram. One of our results is that this imaginary part does not factorize into the lowest order. In computing the relative contribution to the WI, particular care must be used in keeping terms proportional to q_+^α. The resulting expression is

$$-2\ \mathrm{Im}\ V_{ZWW} = 12\, g^3 c_\theta \pi^3 \sum_{i=1,6} v_i W^i_{\mu\alpha\beta}\,, \tag{34}$$

where we have introduced additional form factors

$$\begin{aligned} W^5_{\mu\alpha\beta} &= -2\, \delta_{\mu\beta} q_{+\alpha}\,, \\ W^6_{\mu\alpha\beta} &= 2\, q_{-\mu} q_{+\alpha} p_\beta\,, \end{aligned} \tag{35}$$

which would give zero when contracted with a physical source. The corresponding contribution to the WI (contraction with q_+^α) gives

$$\frac{g^3}{96\,\pi}\, c_\theta\, \delta_{\mu\beta}\,, \tag{36}$$

which is what the WI requires. If we neglect the $W^{2,4}$ form factors then additional terms will remain, proportional to $q_{-\mu} p_\beta$ and once more the WI is violated. If in addition one also neglects the $W^{5,6}$ form factors then the right structure $\delta_{\mu\beta}$ is obtained but with a wrong coefficient.

2.2. The processes $\gamma W^- \to e^- \bar{\nu}_e$, $e^+ W^- \to e^+ \mu^- \bar{\nu}_\mu$.

In order to analyze other sets of Ward identities associated with situations where their violation corresponds to large numerical errors, we use again processes which are simple enough to discuss everything from an analytical point of view. We start with the e.m. WI for $\gamma W^- \to e^- \bar{\nu}_e$ where all particles, including the photon, are on their mass-shell. Four diagrams suffice in this case since the $\gamma - Z$ transition induced by fermions is zero for an on-shell photon. We get that the sum of the diagrams is

$$W = \frac{g^2 s_\theta}{2\sqrt{2}} \bar{u}\gamma^\mu \left(1 + \gamma^5\right) v e_\mu \frac{-\gamma_W + \sum_d \frac{g^2}{48\,\pi}}{s - M_W^2 + i\gamma_W s} s, \tag{37}$$

where the sum is again over all doublets $d = (f, f')$ and where N_d is the total numbers of isospin doublets active in the process. Since

$$\gamma_W = \sum_d \frac{g^2}{48\,\pi}, \tag{38}$$

we observe that the introduction of the imaginary part of the γWW vertex is enough to guarantee the validity of the WI, *i.e.* $W = 0$. Now we consider another WI related to the W boson where we let the photon to be off-shell and also allow for a photon propagator in the diagrams. If the vertex correction is not included we find

$$\frac{g^2 s_\theta}{2\sqrt{2}} \bar{u}\gamma^\mu \left(1 + \gamma^5\right) v e_\mu \frac{1}{q^2} \frac{q^2 - i\gamma_W s}{s - M_W^2 + i\gamma_W s}, \tag{39}$$

where the momenta are q for the photon and p for the W boson. The fact that this WI is violated even for zero width is well known. As soon as the photon is off-shell, other non q^2-resonating diagram will be present, thus we expect a constant violating term. Alternatively one should remember that for off-shell particles a Ward identity must be compensated by the coupling of the Faddeev–Popov ghosts to the sources. Indeed we consider the corresponding WI with off-shell photon and W boson. The three contributing diagrams give

$$\frac{i\,g^2 s_\theta}{2\sqrt{2}} \bar{u}\gamma^\mu \left(1 + \gamma^5\right) v e_\mu \frac{q^2}{q^2 \left(p^2 + M_W^2\right) \left(s - M_W^2\right)}, \tag{40}$$

where we have included the W and γ propagators and $(p+q)^2 = K^2 = -s$. There is an additional diagram where the photon source emits both a W-line and a Faddeev–Popov X ghost-line. The rule for this extra vertex can be found following a gauge transformation of the fields.

$$A_\mu \to A_\mu + i\, g s_\theta \left(W_\mu^+ X^- - W_\mu^- X^+\right) - \partial_\mu X^0 . \tag{41}$$

We obtain

$$\frac{-i\, g^2 s_\theta}{2\sqrt{2}}\, \bar{u}\gamma^\mu \left(1+\gamma^5\right) v e_\mu \frac{1}{\left(p^2 + M_W^2\right)\left(s - M_W^2\right)}, \tag{42}$$

which exactly compensates for the on-shell photon. As for the e.m. off-shell WI one would obtain

$$\frac{i\, g^2 s_\theta}{2\sqrt{2}}\, \bar{u}\gamma^\mu \left(1+\gamma^5\right) v \frac{p^2 + M_W^2}{q^2\left(p^2 + M_W^2\right)\left(-s + M_W^2\right)}, \tag{43}$$

with an additional diagram where the W source emits both a W-line and a ghost Y-line giving

$$\frac{-i\, g^2 s_\theta}{2\sqrt{2}}\, \bar{u}\gamma^\mu \left(1+\gamma^5\right) v \frac{1}{q^2\left(-s + M_W^2\right)} . \tag{44}$$

However the introduction of a W-width will violate the identity with a term proportional to $\Gamma_W s/q^2$. This in turns means that in a realistic process as $e^+e^- \to e^+\nu_e\bar{u}d$ the presence of a resonating W will require the introduction of a width but at the same time an almost on-shell photon will give rise to terms proportional to Γ_W/m_e.

Next we show explicitly a WI for the process $e^+W^- \to e^+\mu^-\bar{\nu}_\mu$, where there are six diagrams in lowest order. Two of this diagrams are nothing else that $\gamma W^- \to \mu\bar{\nu}_\mu$ embedded with the positron line. We assume to be far from the W-resonance so that the W width can be safely neglected. In this case we obtain that the diagrams add up to

$$W = \frac{g^3}{\sqrt{2}} \sum_{i=1,5} W_i ,$$

$$W_1 = \frac{1}{2} s_\theta^2 \left[\frac{q^2 - K^2 - M_W^2}{K^2 + M_W^2} + 1\right] \frac{1}{q^2}\, \gamma^\mu \left(1+\gamma^5\right) \otimes \gamma_\mu ,$$

$$W_2 = -\frac{1}{8}\frac{1}{K^2 + M_W^2}\, \gamma^\mu \left(1+\gamma^5\right) \otimes \gamma^\mu \left(1+\gamma^5\right) ,$$

$$W_3 = -\frac{1}{8}\frac{q^2 - K^2 + s_\theta^2 M_Z^2}{\left(K^2 + M_W^2\right)\left(q^2 + M_Z^2\right)}\gamma^\mu\left(1+\gamma^5\right)\otimes\gamma_\mu\left(4s_\theta^2 - 1 - \gamma^5\right),$$

$$W_4 = \frac{1}{16}\frac{2s_\theta^2 - 1}{c_\theta^2}\frac{1}{q^2 + M_Z^2}\gamma^\mu\left(1+\gamma^5\right)\otimes\gamma_\mu\left(4s_\theta^2 - 1 - \gamma^5\right),$$

$$W_5 = -\frac{1}{16}\frac{1}{c_\theta^2}\frac{1}{q^2 + M_Z^2}\gamma^\mu\left(1+\gamma^5\right)\otimes\gamma_\mu\left(4s_\theta^2 - 1 - \gamma^5\right). \tag{45}$$

From $M_W^2 = c_\theta^2 M_Z^2$ it follows $W = 0$. Thus we have verified that the single W diagrams are not separately gauge invariant but all the background must be included, otherwise the Ward identity will give rise to a q^2 independent term. As soon as the W-width is considered we must include other four diagrams in the WI with a fermionic triangle in the two charge assignment and the positron line coupled to a γ or a Z. The Ward identity becomes

$$W = \frac{g^3}{\sqrt{2}}\sum_{i=1,7} W_i, \tag{46}$$

where the $W_i, i = 1,5$ are the same as before with now $K^2 + M_W^2 \to K^2 + M_W^2 - i\,\gamma_W K^2$. The two new contributions are

$$W_6 = -\,i\frac{1}{2}s_\theta^2\frac{K^2}{q^2\left(K^2 + M_W^2 - i\gamma_W K^2\right)}\gamma_W^{(\gamma)}\gamma^\mu\left(1+\gamma^5\right)\otimes\gamma_\mu,$$

$$W_7 = i\frac{1}{2}s_\theta^2\frac{K^2}{\left(q^2 + M_Z^2\right)\left(K^2 + M_W^2 - i\gamma_W K^2\right)} \times \gamma_W^{(Z)}\gamma^\mu\left(1+\gamma^5\right)\otimes\gamma_\mu\left(4s_\theta^2 - 1 - \gamma^5\right). \tag{47}$$

The imaginary parts for each isospin doublet are

$$\gamma_W = \frac{g^2}{48\,\pi},$$

$$\gamma_W^{(\gamma)} = \frac{g^2}{48\,\pi}\left(Q_{f'} - Q_f\right) = -\gamma_W,$$

$$\gamma_W^{(Z)} = \frac{g^2}{384\,\pi}\left(v_{f'} - v_f - 1\right) = -\frac{1}{4}\gamma_W c_\theta^2. \tag{48}$$

The full WI, *i.e.* $W = 0$, is now satisfied since both the q^2-dependent terms and the constants cancel.

2.3. Bremsstrahlung

One of the first examples of the interplay between Ward identities and bremsstrahlung processes is given by the calculation of $e^+e^- \to W^+W^-\gamma$. Suppose than one is interested in the soft photon approximation and that moreover the renormalizable ($\xi = 1$) gauge it is used.

In the soft photon limit, the expressions for the diagrams which have the photon emitted from an external fermion line, factorize into some coefficient and the corresponding lowest order amplitude. The factorization for diagrams, which have the photon emitted from an external vector boson line, is not immediately obvious in the $\xi = 1$ gauge. As a matter of fact the non-factorizing part of the bremsstrahlung amplitude vanishes due to a Ward identity for the lowest order amplitude. In a more general process, as $e^+e^- \to 4f\gamma$, where double-resonating diagrams will occur, the introduction of a W-width will eventually spoil the WI and interfere with usual aspects of the bremsstrahlung process, like factorization in the soft photon limit. In order to analyze this phenomenon we consider again a much simpler process. Let us start with the standard model in the limit $c_\theta \to 0$ and consider the decay $Z \to W^+W^-\gamma$. There are five diagrams in the $\xi = 1$ gauge, for which the e.m. WI can be easily proved.

For on-shell external W's there is actually no need to insert a width in the internal W propagators, but assume for a moment that the modification is done. As a consequence the e.m. WI is violated and, even worst, the non-zero term are different if computed in the renormalizable gauge or in the unitary gauge. This fact follows simply from the presence of internal ϕ-lines in the renormalizable gauge. As a next step we could consider $Z \to 4f\gamma$ and look for anomalous effects due to the W width. Alternatively we can keep $Z \to W^+W^-\gamma$ but with off mass shell W's and compute the e.m. WI. This example is really instructive since now the two W are considered off mass shell and the coupling of the Faddeev–Popov ghosts to the sources must be taken into account. We simply use the fact that the Feynman rules for these vertices are provided by the gauge transformation of the fields, like for instance

$$W_\mu^- \to W_\mu^- - ig\left(c_\theta Z_\mu + s_\theta A_\mu\right) X^- + igW_\mu^-\left(c_\theta X^0 + s_\theta Y\right) - \partial_\mu X^- , \quad (49)$$

where $X^\pm, X^0$ and Y are the Faddeev–Popov ghosts of the standard model. Let us start again from the WI for $Z \to W^+W^-\gamma$ with on-shell W's. The momenta are defined by

$$Z(k) \to W^+(q_+) + W^-(q_-) + \gamma(q) , \quad (50)$$

and we also introduce

$$p_\pm = q + q_\pm , \quad (51)$$

and obtain that the five diagrams contributing to the WI can be written as

$$d_{\mu\lambda\alpha\beta} = g^2 s_\theta c_\theta \sum_{i=1,5} d^i_{\mu\lambda\alpha\beta}\,, \tag{52}$$

where

$$\begin{aligned}
d^1_{\mu\lambda\alpha\beta} &= v_{\lambda\beta\sigma}\left(k,-q_-,-p_+\right) v_{\mu\sigma\lambda}\left(-q,p_+,-q_+\right) \Delta_W\left(p_+\right)\,,\\
d^2_{\mu\lambda\alpha\beta} &= v_{\lambda\sigma\alpha}\left(k,-p_-,-q_+\right) v_{\mu\beta\sigma}\left(-q,-q_-,p_-\right) \Delta_W\left(p_-\right)\,,\\
d^3_{\mu\lambda\alpha\beta} &= -M_Z^2 s_\theta^2\, \delta_{\lambda\beta}\delta_{\mu\alpha}\, \Delta_\phi\left(p_+\right)\,,\\
d^4_{\mu\lambda\alpha\beta} &= -M_Z^2 s_\theta^2\, \delta_{\lambda\alpha}\delta_{\mu\beta}\, \Delta_\phi\left(p_-\right)\,,\\
d^5_{\mu\lambda\alpha\beta} &= -2\,\delta_{\mu\lambda}\delta_{\alpha\beta} + \delta_{\mu\beta}\delta_{\lambda\alpha} + \delta_{\mu\alpha}\delta_{\lambda\beta}\,,
\end{aligned} \tag{53}$$

where we have introduced

$$\begin{aligned}
\Delta_W(p) = \Delta_\phi(p) &= \frac{1}{p^2+M_W^2}\,,\\
v_{\mu\nu\lambda}\left(p^1,p^2,p^3\right) &= -\,\delta_{\mu\lambda}\left(p^1_\nu - p^3_\nu\right)\\
&\quad -\delta_{\lambda\nu}\left(p^3_\mu - p^2_\mu\right)\\
&\quad -\delta_{\mu\nu}\left(p^2_\lambda - p^1_\lambda\right)\,.
\end{aligned} \tag{54}$$

It is simple to prove that $q^\mu d_{\mu\lambda\alpha\beta} = 0$. If the W's are of mass shell then the kinematics is completely specified by

$$\begin{aligned}
q_\pm^2 &= -s_\pm\,,\\
q_\pm\cdot q &= \frac{1}{2}\,x_\pm\,,\\
k\cdot q_\pm &= \frac{1}{2}\left(-x_\mp \mp s_+ \pm s_- - M_Z^2\right)\,,\\
q_+\cdot q_- &= \frac{1}{2}\left(-x_+ - x_- + s_+ s_- - M_Z^2\right)\,,\\
\delta_\pm &= s_\pm - M_W^2\,.
\end{aligned} \tag{55}$$

Since the W-lines are off mass shell (but not the Z-line), we allow for propagators in those external lines. Being off mass shell means that $q_\pm \neq -M_W^2$, but we still retain gauge invariant W sources, *i.e.* $\partial_\mu J^\mu = 0$. In this case the e.m. WI becomes

$$q^\mu d_{\mu\lambda\alpha\beta} = g^2 s_\theta c_\theta\, \Delta_W\left(p_+\right) \Delta_W\left(p_-\right)\left(\delta_+ f_+ + \delta_- f_-\right)\,, \tag{56}$$

where $f_\pm$ are certain functions of the kinematical invariants. We must now add the contribution due to the ghosts, always keeping in mind that the Z is on-shell and that the off-shell W sources are gauge invariants. There are four additional contributions

$$D^1_{\lambda\alpha\beta} = v_{\lambda\beta\alpha}(k, -q_-, -p_+)\, \Delta_W(p_-)\, \Delta_W(q_+)\ ,$$

$$D^2_{\lambda\alpha\beta} = -v_{\lambda\beta\alpha}(k, -p_-, -q_+)\, \Delta_W(p_+)\, \Delta_W(q_-)\ ,$$

$$D^3_{\lambda\alpha\beta} = -\delta_{\lambda\alpha} p_{-\beta} \Delta_X(p_-)\, \Delta_W(q_-)\ ,$$

$$D^4_{\lambda\alpha\beta} = -\delta_{\lambda\beta} p_{+\alpha} \Delta_X(p_+)\, \Delta_W(q_+)\ , \tag{57}$$

where $\Delta_X = \Delta_W$ is the charged-ghost propagator. It follows

$$g^2 s_\theta c_\theta \left[\Delta_W(q_+)\, \Delta_W(q_-)\, q^\mu \sum_{i=1,5} d^i_{\mu\lambda\alpha\beta} + \sum_{i=1,4} D^i_{\lambda\beta\alpha}\right] = 0\,. \tag{58}$$

Thus the e.m. WI for off-shell W external lines has been proved. If we now introduce a width for the internal W-lines, this will differentiate Δ_W from Δ_ϕ and Δ_X with an obvious violation of the WI. The latter can only be restored if we include the imaginary parts from:

- $Z - \gamma$ transition,
- ZWW and γWW vertices,
- $Z\gamma WW$ boxes.

The same imaginary parts must therefore be included in the calculation of the process $e^+e^- \to 4f + \gamma$.

REFERENCES

[1] Reports of the Working Group on Precision Calculations for the Z-resonance, D. Bardin, W. Hollik and G. Passarino eds., CERN preprint 95-03, March 1995.

[2] G. Burgers, W. Hollik, M. Martinez, program BHM; W. Hollik, *Fortschr. Phys.* **38**, 3, 165 (1990); M. Consoli, W. Hollik, F. Jegerlehner: Proceedings of the Workshop on Z physics at LEP I, CERN Report 89-08 Vol. I, 7; G. Burgers, F. Jegerlehner, B. Kniehl, J.H.K ühn: CERN Report 89-08 Vol. I, 55.

[3] V.A. Novikov, L.B. Okun, M.I. Vysotsky, *Nucl. Phys.* **B397**, 35 (1993); V.A. Novikov, L.B. Okun, A.N. Rozanov M.I. Vysotsky, V. Yurov *Phys. Lett.* **B331**, 433 (1994); V.A. Novikov, L.B. Okun, A.N. Rozanov, M.I. Vysotsky, CERN preprint CERN-TH.7217/94, *Mod. Phys. Lett.* **A9**, 2641 (1994).
[4] G. Montagna, O. Nicrosini, G. Passarino, F. Piccinini, R. Pittau, *Nucl. Phys.* **B401**, 3 (1993); G. Montagna, O. Nicrosini, G. Passarino, F. Piccinini, R. Pittau, program TOPAZO, *Comput. Phys. Commun.* **76**, 328 (1993).
[5] W. Hollik, program WOH.
[6] D. Bardin *et al.*, program ZFITTER 4.9, *Nucl. Phys.* **B351**, 1 (1991); *Z. Phys.* **C44**, 493 (1989); *Phys. Lett.* **B255**, 290 (1991); CERN-TH.6443/1992, May 1992; hep-ph/9412201.
[7] G. Passarino, M. Veltman, *Nucl. Phys.* **B160**, 151 (1979).
[8] A. Sirlin, *Phys. Rev.* **D22**, 971 (1980); W.J. Marciano, A. Sirlin, *Phys. Rev.* **D22**, 2695 (1980).
[9] D.Yu. Bardin, P.Ch. Christova, O.M. Fedorenko, *Nucl. Phys.* **B175**, 235 (1980); D.Yu. Bardin, P.Ch. Christova, O.M. Fedorenko, *Nucl. Phys.* **B197**, 1 (1982); D.Yu. Bardin, M.S. Bilenky, G.V. Mitselmakher, T. Riemann, M. Sachwitz, *Z. Phys.* **C44**, 493 (1989).
[10] J. Fleischer, F. Jegerlehner, *Phys. Rev.* **D23**, 2001 (1981).
[11] K.I. Aoki, Z. Hioki, R. Kawabe, M. Konuma, T. Muta, *Suppl. Prog. Theor. Phys.* **73**, 1 (1982).
[12] M. Consoli, S. LoPresti, L. Maiani, *Nucl. Phys.* **B223**, 474 (1983).
[13] M. Böhm, W. Hollik, H. Spiesberger, *Fortschr. Phys.* **34**, 687 (1986); W. Hollik, *Fortschr. Phys.* **38**, 165 (1990).
[14] F. Jegerlehner, *Z.Phys.* **C32**, 425 (1986).
[15] W. Hollik, H.J. Timme, *Z. Phys.* **C33**, 125 1986).
[16] G. Passarino, R. Pittau, *Phys. Lett.* **B228**, 89 (1989); G. Passarino, *Nucl Phys.* **B361**, 351 (1991).
[17] D.C. Kennedy, B.W. Lynn, *Nucl. Phys.* **B322**, 1 (1989).
[18] W.J. Marciano, A. Sirlin, *Phys. Rev. Lett.* **46**, 163 (1981); A. Sirlin, *Phys. Lett.* **B232**, 123 (1989); G. Degrassi, S. Fanchiotti, A. Sirlin, *Nucl. Phys.* **B351**, 49 (1991); G. Degrassi, A. Sirlin, *Nucl. Phys.* **B352**, 342 (1991).
[19] G. Passarino, M. Veltman, *Phys. Lett.* **B237**, 537 (1990).
[20] D. Bardin, T. Riemann, preprint DESY 95–167 (1995) [hep-ph/9509341]; E. Boos *et al.*, Moscow State Univ. preprint MGU-89-63/140 (1989); preprint KEK 92-47 (1992); D. Bardin, M. Bilenky, D. Lehner, A. Olchevski, T. Riemann, in: T. Riemann and J. Blümlein (eds.), Proc. of the Zeuthen Workshop on Elementary Particle Theory – Physics at LEP200 and Beyond, Teupitz, Germany, April 10–15, 1994, *Nucl. Phys.* (Proc. Suppl.) **37B**, 148 (1994); F.A. Berends, R. Kleiss, R. Pittau, *Comput. Phys. Commun.* **85**, 437 (1995); *Nucl. Phys.* **B424**, 308 (1994); *Nucl. Phys.* **B426**, 344 (1994); *Nucl.Phys.* **B**, Proc. Suppl. **37B**, 163 (1994); R. Pittau, *Phys. Lett.* **B335**, 490 (1994); F. Caravaglios, M. Moretti, *Phys. Lett.* **B358**, 332 (1995); D. Bardin, M. Bilenky, A. Olchevski, T. Riemann, *Phys. Lett.* **B308**, 403 (1993); E: [hep-ph/9507277]; T. Sjöstrand, *Comput. Phys. Commun.* **82**, 74 (1994); H. Anlauf, J. Biebel, H.

D. Dahmen, A. Himmler, P. Manakos, T. Mannel, W. Schönau, *Comput. Phys. Commun.* **79**, 487 (1994); E. Accomando, A. Ballestrero, Fortran program WPHACT, in preparation; G. Montagna, O. Nicrosini, G. Passarino, F. Piccinini, *Phys. Lett.* **B348**, 178 (1995); G. J. van Oldenborgh, P. J. Franzini, A. Borrelli, *Comput. Phys. Commun.* **83**, 14 (1994); G. Montagna, O. Nicrosini, F. Piccinini, *Comput. Phys. Commun.* **90**, 141 (1995); G. Passarino, Fortran program WTO, in preparation.

[21] E. N. Argyres *et al.*, *Phys. Lett.* **B358**, 3390 (1995).