

VIRTUAL EFFECTS OF HEAVY CHIRAL FERMIONS AT e^+e^- COLLIDERS*, **

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We derive the low-energy electroweak effective Lagrangian for the case of additional heavy, unmixed, sequential fermions. Present LEP1 data still allow the presence of new fermionic doublets (quarks and/or leptons), with masses greater than $M_Z/2$, provided that these multiplets are sufficiently degenerate. Deviations of the effective Lagrangian predictions from full one-loop computation are sizeable only for fermion masses close to the threshold $M_Z/2$. We analyse the contribution of new heavy sequential fermions to the $e^+e^- \rightarrow W^+W^-$ cross-section at the energies of LEP2 and Next Linear Collider (NLC). The signals coming from an additional doublet will be out of the LEP2 observability level. More interesting is the case of NLC where the simultaneous presence of the kinematic enhancement factor p^2/m_W^2 and of the unitarity delay effect gives deviations from the Standard Model of the order of 10–50 per cent, for a wide range of new fermion masses.

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1. Introduction

LEP1 precision data represent a step of paramount relevance in probing extensions of the Standard Model (SM). Through their virtual effects, the electroweak radiative corrections "feel" the presence of new particles running

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in the loops. The level of accuracy on the relevant observables is such that this set of tests is complementary to the traditional probes on virtual effects due to new physics (*i.e.*, highly suppressed or forbidden flavour changing neutral current phenomena) and in some cases represents the only indirect way to search for these new particles.

In this paper we will discuss electroweak radiative effects from extensions of the ordinary fermionic spectrum of the SM. The new fermions are supposed to possess the same colour and electroweak quantum numbers as the ordinary ones and to mix only slightly with the ordinary three generations. The most straightforward realization of such a fermionic extension of the SM is the introduction of a fourth generation. This possibility has been almost entirely jeopardized by the LEP1 bound on the numbers of neutrinos species. Although there still exists the obvious way out of having new fermion generations with heavy neutrinos, we think that these options are too awkward and do not merit further studies. Instead, what we have in mind in tackling this problem are general frames discussing new physics beyond the SM which leads to new quarks and/or leptons classified in the usual chiral way with iso-doublets and iso-singlets for different chiralities. Situations of this kind may be encountered in grand unified schemes where the ordinary fifteen Weyl spinors of each fermionic generation are only part of larger representations or where new fermions (possibly also mirror fermions) are requested by the group or manifold structure of the schemes. Chiral fermions with heavy static masses may also provide a first approximation of virtual effects in technicolor-like schemes when the dynamical behaviour of technifermion self-energies are neglected. Although such effects have been extensively investigated in the literature [1], our presentation focuses mainly on two aspects, which have only been partially touched in the previous analyses: the use of effective Lagrangians for a model-independent treatment of the problem and a discussion of the validity of this approach in comparison with the computation in the full-fledged theory.

While separate tests can be set up for each different extension of the SM, there may be some advantage in realizing this analysis in a model independent framework. The natural theoretical tool of this approach is to use an effective electroweak Lagrangian where, relaxing the renormalizability requirement, all $SU(2)_L \otimes U(1)_Y$ invariant operators up to a given dimension are present with unknown coefficients, which will eventually determined from the experiments (Section 2). Each different model possesses a unique set of coefficients and the effective Lagrangian thus becomes a useful way to discuss and compare several SM extensions. The introduction of the well known S , T and U [2] or ε_i [3] variables was much in the same spirit and the use of an effective Lagrangian represents in a sense the natural extension of these approaches.

From the latest available ε_i values [4] we derive (Section 3) the actual bounds on new chiral doublets, comparing the computations in the effective Lagrangians approximation and in the full one-loop theory. We find sizeable deviations only for fermion masses very close to the $M_Z/2$ threshold.

These bounds also put severe constraints on the trilinear gauge couplings sector, making non-trivial the question if some effects from this kind of new physics can be seen at the next colliders (LEP2 and NLC). We calculate, again in the effective and the full theory, the deviations from the SM $e^+e^- \rightarrow W^+W^-$ differential cross-section due to an extra chiral fermion doublet (Section 4). We find that for a wide range of new fermion masses sizeable deviations from SM predictions are expected.

2. Effective Lagrangian approach

The use of an effective Lagrangian for electroweak physics was originally advocated for the study of the large Higgs mass limit in the SM [5–7]. Subsequent contributions from chiral $SU(2)_L$ doublets have been considered in the degenerate case [8, 9], for small splitting [10] and in the case of infinite splitting [11, 12]. In the present note we will deal with the general case of arbitrary splitting among the fermions in the doublet [13, 14].

For completeness we consider here the standard list of $SU(2)_L \otimes U(1)_Y$ CP conserving operators built out of the gauge vector bosons W_μ^i ($i = 1, 2, 3$), B_μ and the would be Goldstone bosons ξ^i [6], and containing up to four derivatives:

$$\begin{aligned}
 \mathcal{L}_0 &= \frac{v^2}{4} [\text{Tr}(TV_\mu)]^2 \\
 \mathcal{L}_1 &= i \frac{gg'}{2} B_{\mu\nu} \text{Tr}(T\hat{W}^{\mu\nu}) \\
 \mathcal{L}_2 &= i \frac{g'}{2} B_{\mu\nu} \text{Tr}(T[V^\mu, V^\nu]) \\
 \mathcal{L}_3 &= g \text{Tr}(\hat{W}_{\mu\nu}[V^\mu, V^\nu]) \\
 \mathcal{L}_4 &= [\text{Tr}(V_\mu V_\nu)]^2 \\
 \mathcal{L}_5 &= [\text{Tr}(V_\mu V^\mu)]^2 \\
 \mathcal{L}_6 &= \text{Tr}(V_\mu V_\nu) \text{Tr}(TV^\mu) \text{Tr}(TV^\nu) \\
 \mathcal{L}_7 &= \text{Tr}(V_\mu V^\mu) [\text{Tr}(TV^\nu)]^2 \\
 \mathcal{L}_8 &= \frac{g^2}{4} [\text{Tr}(T\hat{W}_{\mu\nu})]^2 \\
 \mathcal{L}_9 &= \frac{g}{2} \text{Tr}(T\hat{W}_{\mu\nu}) \text{Tr}(T[V^\mu, V^\nu])
 \end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{10} &= [\text{Tr}(TV_\mu) \text{Tr}(TV_\nu)]^2 \\
\mathcal{L}_{11} &= \text{Tr}((\mathcal{D}_\mu V^\mu)^2) \\
\mathcal{L}_{12} &= \text{Tr}(T\mathcal{D}_\mu \mathcal{D}_\nu V^\nu) \text{Tr}(TV^\mu) \\
\mathcal{L}_{13} &= \frac{1}{2} [\text{Tr}(T\mathcal{D}_\mu V_\nu)]^2 \\
\mathcal{L}_{14} &= i g \varepsilon^{\mu\nu\rho\sigma} \text{Tr}(\hat{W}_{\mu\nu} V_\rho) \text{Tr}(TV_\sigma). \quad (1)
\end{aligned}$$

If we define the Goldstone boson contribution $U = \exp(i\vec{\xi} \cdot \vec{\tau}/v)$ (so that in the unitary gauge $U = 1$), than:

$$T = U\tau^3 U^\dagger, \quad V_\mu = (D_\mu U)U^\dagger, \quad (2)$$

$$D_\mu U = \partial_\mu U - g\hat{W}_\mu U + g'U\hat{B}_\mu, \quad (3)$$

where \hat{W}_μ , $\hat{W}_{\mu\nu}$ and \hat{B}_μ , $\hat{B}_{\mu\nu}$ are respectively the matrices containing the gauge fields and the corresponding field strengths defined by:

$$\begin{aligned}
\hat{W}_\mu &= \frac{1}{2i} \vec{W}_\mu \cdot \vec{\tau}, & \hat{W}_{\mu\nu} &= \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\mu - g[\hat{W}_\mu, \hat{W}_\nu], \\
\hat{B}_\mu &= \frac{1}{2i} B_\mu \tau^3, & \hat{B}_{\mu\nu} &= \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu. \quad (4)
\end{aligned}$$

Finally the covariant derivative acting on V_μ is given by:

$$\mathcal{D}_\mu V_\nu = \partial_\mu V_\nu - g[\hat{W}_\mu, V_\nu]. \quad (5)$$

The effective electroweak Lagrangian can be written as:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{i=0}^{14} a_i \mathcal{L}_i, \quad (6)$$

where \mathcal{L}_{SM} is the “low-energy” SM Lagrangian, \mathcal{L}_i are the operators listed in Eq. (1) and a_i are the terms containing all the new physics contributions¹.

These coefficients a_i ($i = 0, \dots, 14$) are determined by computing the corresponding full one-loop contribution to a set of n -point gauge boson functions ($n = 2, 3, 4$), in the limit of low external momenta, and by matching the predictions of the full and effective theory. For example, the two-point vector boson functions in the limit $p^2 \ll \Lambda^2$, (Λ generically represents the mass of the new heavy particles running in the loop), can be expanded around $p^2 = 0$:

$$\begin{aligned}
\Pi_{ij}^{\mu\nu}(p) &= g^{\mu\nu} \Pi_{ij}(p^2) + (p^\mu p^\nu \text{ terms}) & (i, j = 0, 1, 2, 3) \\
\Pi_{ij}(p^2) &\equiv A_{ij} + p^2 F_{ij}(p^2) = A_{ij} + p^2 F_{ij}(0) + \dots \quad (7)
\end{aligned}$$

¹ Here we do not need to include the Wess-Zumino term [11].

The neglected terms are suppressed by increasing powers of p^2/Λ^2 and so will do not appear in this effective Lagrangian approach. Denoting with M and m the masses of the upper and lower weak isospin components, and with $r = m^2/M^2$ the square ratio, we obtain (in units of $1/16H^2$) the following expressions:

$$\begin{aligned}
 a_0^q &= \frac{3M^2}{2v^2} \left(\frac{1 - r^2 + 2r \log r}{1 - r} \right), \\
 a_1^q &= \frac{1}{12(-1+r)^3} [3(1 - 15r + 15r^2 - r^3) + 2(1 - 12r - 6r^2 - r^3) \log r], \\
 a_2^q &= \frac{1}{12(-1+r)^3} [3(3 - 7r + 5r^2 - r^3) + 2(1 - r^3) \log r], \\
 a_3^q &= \frac{1}{8(-1+r)^3} [3(-1 + 7r - 7r^2 + r^3) + 6r(1+r) \log r], \\
 a_4^q &= \frac{1}{6(-1+r)^3} [5 - 9r + 9r^2 - 5r^3 + 3(1+r^3) \log r], \\
 a_5^q &= \frac{1}{24(-1+r)^3} [-23 + 45r - 45r^2 + 23r^3 - 12(1+r^3) \log r], \\
 a_6^q &= \frac{1}{24(-1+r)^3} [-23 + 81r - 81r^2 + 23r^3 \\
 &\quad - 6(2 - 3r - 3r^2 + 2r^3) \log r], \\
 a_7^q &= -a_6^q, \\
 a_8^q &= \frac{1}{12(-1+r)^3} [7 - 81r + 81r^2 - 7r^3 + 6(1 - 6r - 6r^2 + r^3) \log r], \\
 a_9^q &= -a_6^q, \\
 a_{10}^q &= 0, \\
 a_{11}^q &= -\frac{1}{2}, \\
 a_{12}^q &= \frac{1}{8(-1+r)^3} [1 + 9r - 9r^2 - r^3 + 6r(1+r) \log r], \\
 a_{13}^q &= 2a_{12}^q, \\
 a_{14}^q &= \frac{3}{8(-1+r)^2} [1 - r^2 + 2r \log r] \quad (8)
 \end{aligned}$$

for quarks, and:

$$\begin{aligned}
 a_i^l &= \frac{1}{3} a_i^q \quad (i = 0, i = 3, \dots, 14), \\
 a_1^l &= \frac{1}{12(-1+r)^3} [1 - 15r + 15r^2 - r^3 - 2(1 + 6r^2 - r^3) \log r],
 \end{aligned}$$

$$a_2^l = \frac{1}{12(-1+r)^3} [-1 - 3r + 9r^2 - 5r^3 - 2(1-r^3) \log r] \quad (9)$$

for leptons.

Indeed the use of an effective Lagrangian in precision tests has its own limitations. One can ask how large M has to be in order to obtain a sensible approximation from the truncation of the full one-loop result, and this we shall consider in the next section.

3. Two point functions

For new chiral fermions which do not mix with the ordinary ones, all the virtual effects measurable at LEP1 are described by operators bilinear in the gauge vector bosons. We describe these effects in the effective (we shall refer to this as the “static approximation”) as well in the full-fledged theory.

3.1. Static approximation

The coefficients a_i of the effective Lagrangian \mathcal{L}_{eff} can be related to measurable parameters. In particular, to make the connection with the full set of LEP1 data, we can use the ε_i parameters defined by [3].

A recent analysis of the available data from LEP1, SLD, low-energy neutrino scatterings and atomic parity violation experiments leads to the following values [4]:

$$\begin{aligned} \varepsilon_1 &= (3.6 \pm 1.5) \cdot 10^{-3}, \\ \varepsilon_2 &= (-5.8 \pm 4.3) \cdot 10^{-3}, \\ \varepsilon_3 &= (3.6 \pm 1.5) \cdot 10^{-3}. \end{aligned} \quad (10)$$

The theoretical expression of the ε_i can be written as sum of the SM and new physics contributions:

$$\varepsilon_i = \varepsilon_i^{\text{SM}} + \Delta\varepsilon_i. \quad (11)$$

The $\varepsilon_i^{\text{SM}}$ were calculated by [3] as functions of the Higgs and top masses. The new physics contributions $\Delta\varepsilon_i$ are model-dependent quantities, and can be related to the effective parameters a_i through the relations:

$$\begin{aligned} \Delta\varepsilon_1 &= 2a_0, \\ \Delta\varepsilon_2 &= -g^2(a_8 + a_{13}), \\ \Delta\varepsilon_3 &= -g^2(a_1 + a_{13}). \end{aligned} \quad (12)$$

From Eqs (8) and (9) one finds the following expressions:

$$\Delta\varepsilon_1^q = 3\Delta\varepsilon_1^l = \frac{3M^2}{8\Pi^2} \frac{G}{\sqrt{2}} \left[\frac{1-r^2+2r\log r}{(1-r)} \right], \quad (13)$$

$$\Delta\varepsilon_2^q = 3\Delta\varepsilon_2^l = \frac{Gm_W^2}{12\Pi^2\sqrt{2}} \times \left[\frac{5-27r+27r^2-5r^3+(3-9r-9r^2+3r^3)\log r}{(1-r)^3} \right], \quad (14)$$

$$\Delta\varepsilon_3^q = \frac{Gm_W^2}{12\Pi^2\sqrt{2}} [3+\log r], \quad (15)$$

$$\Delta\varepsilon_3^l = \frac{Gm_W^2}{12\Pi^2\sqrt{2}} [1-\log r]. \quad (16)$$

It is clear from Eqs (12)–(16) that only $\Delta\varepsilon_1$ can have a contribution proportional to M^2 . However, as is well known, this term vanishes in the limit of degenerate doublet ($r \rightarrow 1$). Consequently the ε_1 analysis can only put a limitation on the mass splitting between the two fermions in the doublet, and do not give an “absolute” statement regarding the number of possible extra doublets. In Fig. 1 we can see that for relatively light masses (*i.e.*, $M = 200$ GeV (dotted line)) a small splitting is still allowed ($0.5 \leq r \leq 1.5$), while for heavier masses (*i.e.*, $M = 1000$ GeV (full line)), the doublet must be practically degenerate ($0.91 \leq r \leq 1.08$).

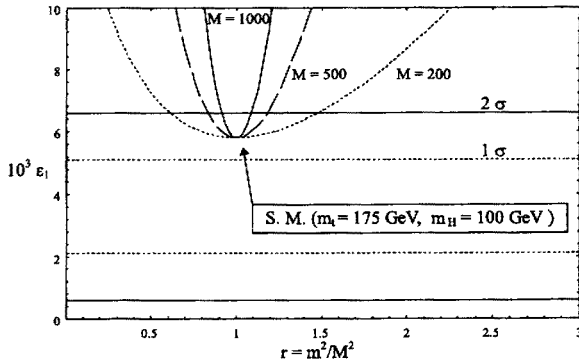


Fig. 1. Predictions for ε_1 from an additional heavy quark doublet for three different masses ($M = 200$ GeV (dotted line), $M = 500$ GeV (dashed line) and $M = 1000$ GeV (full line). The 1σ (dotted horizontal line) and 2σ (full horizontal line) allowed regions are also displayed. The value $\varepsilon_1(r = 1)$ is the SM prediction for $m_t = 175$ GeV and $M_H = 100$ GeV.

The contributions from $\Delta\varepsilon_2$ and $\Delta\varepsilon_3$ can have only r and $\log r$ dependence. Also $\Delta\varepsilon_2$ vanishes for $r = 1$. Only $\Delta\varepsilon_3$ is non-zero in the $r \rightarrow 1$ limit, and one finds:

$$\Delta\varepsilon_3^q = 3\Delta\varepsilon_3^l = \frac{Gm_W^2}{4\Pi^2\sqrt{2}} \simeq 1.3 \cdot 10^{-3}. \tag{17}$$

Thus, as can be noted from Fig. 2, even if we can not have stringent bounds on the mass splitting from ε_3 , we have however an “absolute” constraint on the number of possible extra heavy fermions. We see that at least one quark doublet² (full line) is not completely ruled out.

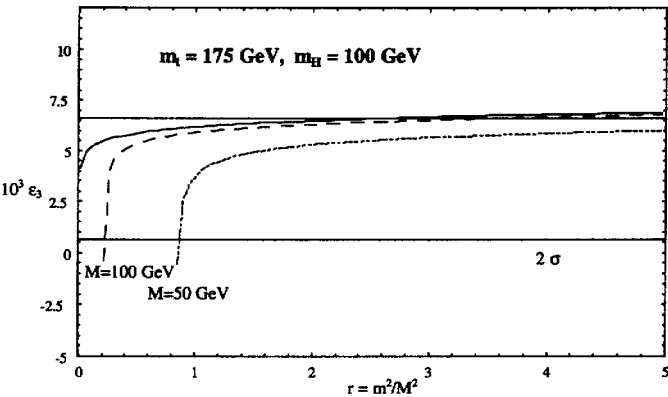


Fig. 2. Comparison between the asymptotic (solid line) and full one-loop (with mass $M = 100$ GeV (dashed lines) and $M = 50$ GeV (dotted lines)) computations of ε_3 versus $r = m^2/M^2$, for an additional quark doublet. The SM contribution is computed assuming $m_t = 175$ GeV and $m_H = 100$ GeV. The 2σ allowed region is also displayed (full horizontal lines).

3.2. Full calculation

We are thus lead to consider the possibility of relatively light (but obviously under production threshold) chiral fermions, both to check the agreement with the present data, and to test the reliability of our effective Lagrangian approach. If the additional fermions are not sufficiently heavy, we do not expect that their one-loop effects are accurately reproduced by the coefficients a_i in Eqs (8) and (9). In this case we have to consider the

² Also one complete extra generation or four extra lepton doublets are still allowed. In fact the leptonic contribution in the $r = 1$ limit is just 1/3 of the hadronic contribution.

full dependence of the Green functions from the external momenta and not just the first two terms of the p^2 expansion given in Eq. (7). We recall that in this case the parameters ε_i are given by [15]:

$$\begin{aligned}\Delta\varepsilon_1 &= e_1 - e_5(m_Z^2), \\ \Delta\varepsilon_2 &= e_2 - \bar{s}^2 e_4 - \bar{c}^2 e_5(m_Z^2), \\ \Delta\varepsilon_3 &= e_3(m_Z^2) + \bar{c}^2 e_4 - \bar{c}^2 e_5(m_Z^2),\end{aligned}\quad (18)$$

where we have kept in account the fact that in our case there are no vertex or box corrections to four-fermion processes. The e_i parameters³ in Eq. (18) expressed in terms of the unrenormalized vector-bosons vacuum polarizations read as:

$$\begin{aligned}e_1 &= \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2}, \\ e_2 &= \Pi'_{WW}(0) - \bar{c}^2 \Pi'_{ZZ}(0) - 2\bar{s}\bar{c} \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} - \bar{s}^2 \Pi'_{\gamma\gamma}(m_Z^2), \\ e_3(p^2) &= \frac{\bar{c}}{\bar{s}} \left\{ \bar{s}\bar{c} [\Pi'_{\gamma\gamma}(m_Z^2) - \Pi'_{ZZ}(0)] + (\bar{c}^2 - \bar{s}^2) \frac{\Pi_{\gamma Z}(p^2)}{p^2} \right\}, \\ e_4 &= \Pi'_{\gamma\gamma}(0) - \Pi'_{\gamma\gamma}(m_Z^2), \\ e_5(p^2) &= \Pi'_{ZZ}(p^2) - \Pi'_{ZZ}(0),\end{aligned}\quad (19)$$

where

$$\Pi'_{VV'}(p^2) = \frac{\Pi_{VV'}(p^2) - \Pi_{VV'}(m_{VV'}^2)}{(p^2 - m_{VV'}^2)}, \quad V = (\gamma, Z, W) \quad (20)$$

with $m_{WW} = m_W$, $m_{ZZ} = m_Z$, $m_{Z\gamma} = m_{\gamma\gamma} = 0$ and the effective sine

$$\bar{s}^2 = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\Pi\alpha_{SM}(p^2)}{\sqrt{2}G_F m_Z^2}}. \quad (21)$$

We introduce also (for later use) the quantities e_6 , $\Delta\alpha(p^2)$, $\Delta k(p^2)$, $\Delta\rho(p^2)$ and Δr_W :

$$\begin{aligned}e_6 &= \Pi'_{WW}(m_W^2) - \Pi'_{WW}(0), \\ \Delta\alpha(p^2) &= \Pi'_{\gamma\gamma}(0) - \Pi'_{\gamma\gamma}(p^2), \\ \Delta k(p^2) &= -\frac{\bar{c}^2}{\bar{c}^2 - \bar{s}^2} (e_1 - e_4) + \frac{1}{\bar{c}^2 - \bar{s}^2} e_3(p^2), \\ \Delta\rho(p^2) &= e_1 - e_5(p^2), \\ \Delta r_W &= -\frac{\bar{c}^2}{\bar{s}^2} e_1 + \frac{\bar{c}^2 - \bar{s}^2}{\bar{s}^2} e_2 + 2 e_3(m_Z^2) + e_4.\end{aligned}\quad (22)$$

³ The expressions for the quantities e_i , in the case of an ordinary quark or lepton doublet can be easily derived from the literature [16].

If $p^2 = m_Z^2$ then $\Delta\alpha(p^2)$, $\Delta k(p^2)$, $\Delta\rho(p^2)$ coincide with the corrections $\Delta\alpha$, Δk and $\Delta\rho$ which characterize the electroweak observables at the Z resonance⁴.

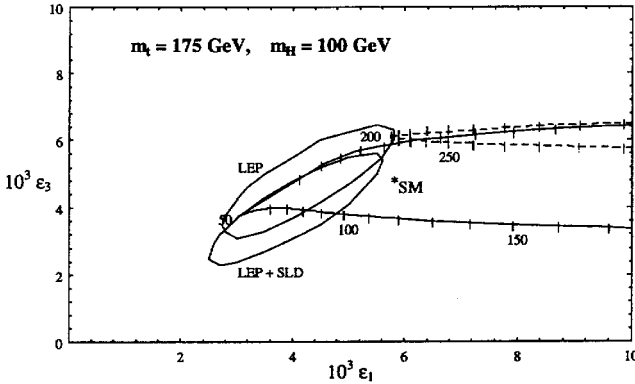


Fig. 3. Predictions for ε_1 , ε_3 from an additional quark doublet. The lower (upper) dashed line represents the case m (M) = 200 GeV, M (m) varying between 200 and 300 GeV, evaluated with \mathcal{L}_{eff} . The lower (upper) full line corresponds to m (M) = 50 GeV, M (m) varying between 50 and 170 GeV, evaluated with a complete one-loop computation. The SM point corresponds to $m_t = 175$ GeV and $m_H = 100$ GeV. The upper (lower) ellipses is the 1σ allowed region, obtained by a fit of the high energy data which excludes (includes) the SLD measurement.

In Fig. 3 we illustrate our full one-loop result in the plane $\varepsilon_1 - \varepsilon_3$. The upper (lower) ellipse represents the 1σ experimentally allowed region obtained by combining all LEP1 (LEP1+SLD) data [4]. The full line plot is obtained when one of the two masses in the doublet is fixed at 50 GeV, and the other runs from 50 to 170 GeV. In this case we have two branches, according to which mass (m or M) has been fixed. If at least one of the two masses is small, this causes a substantial deviation from the asymptotic, effective Lagrangian prediction. In particular, as was observed in [15], a large negative contribution to both ε_1 and ε_3 is now possible, due to a formal divergence of $\Pi'_{ZZ}(m_Z^2) - \Pi'_{ZZ}(0)$ at the threshold which produces a large and positive e_5 . Clearly, this behaviour can not be reproduced by \mathcal{L}_{eff} , which, at the fourth order in derivatives, automatically sets $e_5 = 0$. The dashed lines show the predictions when both the masses are “heavy”. Here we fix one of the two to 200 GeV and let the other vary from 200 to 300

⁴ The new parameter e_6 is added to the five defined in [15] by taking in account the presence of the one-loop correction of the W 's external line in the $e^+e^- \rightarrow W^+W^-$ process.

GeV. As expected, it appears that only a small amount of splitting among the doublet components is allowed. For the chosen value of $m_t = 175$ GeV and $m_H = 100$ GeV the SM prediction already lies outside the 1σ allowed region and additional positive contributions to ε_1 tend to be disfavoured. On the contrary, the positive contribution to ε_3 , almost constant in the chosen range of masses, is still tolerated.

A relevant question is then to ask when the asymptotic regime starts, *i.e.*, how close to M_Z should the masses of the new quarks or leptons be for observing deviations between the full and the effective expressions. A detailed analysis shows that already for new fermion masses above 70 – 80 GeV the difference between the values of the ε_i obtained with the truncated and full expression of $\Pi_{ij}(p^2)$ are as small as 10^{-4} , *i.e.*, below the present experimental level of accuracy. This is illustrated in Fig. 2 where the asymptotic contribution of ε_3 (full line) and the full one-loop one (dashed and dotted) are compared as a function of r .

Beyond the indirect precision tests, the possibility of having new fermions carrying the usual $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers can also be clearly bounded by the direct searches. Concerning the present searches, from LEP1 we have the lower bound of $M_Z/2$ ⁵ which exists independently from any assumption on the decay modes of the new fermions which couple to the Z boson. Much stronger limits on the new quark masses can be inferred from the Tevatron results. However, as we know from the search for the top quark, these latter bounds rely on assumptions concerning the decay modes of the heavy quark. For instance, in the case of the top search it was stressed that if a new decay channel to a b quark and a charged Higgs boson were available, then one could not use the CDF bounds on m_t [17] which were quoted in the years leading up to the actual discovery of the top quark.

Now, it may be conceivable that the new physics related to the presence of extra-fermions can also affect their possible decay channels making the lightest of the new fermions unstable. Indeed, we stated in our assumption that the new fermions do not essentially mix with the ordinary ones; hence one has to invoke new physics if one wants to avoid the formation of stable heavy mesons made out of the lightest stable new fermion and the ordinary fermions of the SM. If the new fermions can decay within the detector, then the bounds on their masses, coming from Tevatron data, must be discussed in a model-dependent way and even the case of new quarks with masses lighter than m_t are not fully ruled out. If on the contrary the lightest new quark is stable, then searches for exotic heavy mesons at CDF have already ruled out the possibility of them being near the threshold $M_Z/2$.

⁵ Recent analysis [18] of data taken during the upgrade from LEP1 to LEP2 increase the lower bound for fermion masses from $M_Z/2$ to $\simeq 60$ GeV.

The existence of a coloured particle with charge ± 1 is strictly constrained to be heavier than 130 GeV from CDF searches [19]. Finally note that for charged leptons the bounds coming from CDF are much less stringent and a new stable charged lepton of mass of 60 – 70 GeV can not be ruled out.

4. Three point functions

If new physics beyond the SM were modeled by additional heavy chiral fermions of the kind we have considered, then we could draw information on the searches at future colliders of anomalous trilinear gauge boson couplings.

We define the kinematics of the WWV vertex (V stands for neutral vector boson) as

$$V(p, v_1) \rightarrow W^-(q, v_2) + W^+(\bar{q}, v_3), \quad (23)$$

where p, q, \bar{q} are respectively the momenta of V, W^-, W^+ , and v_1, v_2, v_3 are their polarization vectors. For simplicity we can demand that the produced W 's are on-shell, so:

$$q^2 = \bar{q}^2 = m_W^2, \quad q \cdot v_2 = \bar{q} \cdot v_3 = 0. \quad (24)$$

Using the definitions of [23], the general CP-conserving coupling of two on-shell charged vector bosons with a neutral vector boson can be derived from the following effective Lagrangian:

$$\begin{aligned} \frac{\mathcal{L}_{WWV}}{g_{WWV}} = & ig_1^V (W_{\mu\nu}^\dagger W^\mu V^\nu - W_\mu^\dagger V^\nu W_{\mu\nu}) + i\kappa_V W_\mu^\dagger W^\nu V^{\nu\mu} \\ & + i\frac{\lambda_V}{\Lambda^2} W_{\lambda\mu}^\dagger W_\nu^\mu V^{\nu\lambda} + g_5^V \varepsilon^{\mu\nu\rho\sigma} (W_\mu^\dagger \overleftrightarrow{\partial}_\rho W_\nu) V_\sigma. \end{aligned} \quad (25)$$

Here V_μ stands for either the photon ($V = \gamma$) or the Z field ($V = Z$), W_μ is the field associated with the W^- , $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and Λ is a mass scale parameter opportunely chosen. By convention $g_{WW\gamma} = -e$ and $g_{WWZ} = -e c/s$.

In the momentum space the corresponding WWV vertex can be decomposed as:

$$\begin{aligned} \Gamma_V^{\alpha\beta\mu}(q, \bar{q}, p) = & f_1^V (q - \bar{q})^\mu g^{\alpha\beta} - \frac{f_2^V}{m_W^2} (q - \bar{q})^\mu p^\alpha p^\beta + f_3^V (p^\alpha g^{\mu\nu} - p^\beta g^{\alpha\mu}) \\ & + if_5^V \varepsilon^{\alpha\beta\mu\sigma} (q - \bar{q})_\sigma. \end{aligned} \quad (26)$$

All the form factors f_i^V are dimensionless functions of p^2 . From Eqs (25) and (26) it is easy to recover the relations:

$$\begin{aligned} f_1^V &= g_1^V + \frac{p^2}{2\Lambda^2} \lambda_V, \\ f_2^V &= \lambda_V, \\ f_3^V &= g_1^V + \kappa_V + \lambda_V, \\ f_5^V &= g_5^V. \end{aligned} \quad (27)$$

Obviously we can add to the effective Lagrangian \mathcal{L}_{WWV} higher dimension operators, replacing V_μ by $\partial^{2n} V_\mu$ (with n arbitrary integers), that will contribute through p^{2n} terms to the right side of Eq. (27). So in general we need only the four form factors of Eq. (26) to parametrize all the new physics (CP-conserving) effects in the trilinear gauge vertex.

This description univocally defines the tree-level trilinear couplings. At higher order one has to then declare exactly which contributions wants to be include in the “form factors” definition. For example let \mathcal{K} be a renormalization dependent quantity. We can redefine the forms factors

$$f_i^V = f_i^{\text{SM}} + \Delta f_i^V \quad \text{with} \quad \Delta f_i^V = \mathcal{K} f_i^{\text{SM}} + \delta f_i^V \quad (i = 1, 2, 3, 5). \quad (28)$$

Here f_i^{SM} are the SM form factors and δf_i^V are the 1PI one-loop trilinear contributions due to new physics virtual effects. The term $\mathcal{K} f_i^{\text{SM}}$ depends on the choice of the overall normalization of the trilinear vertex WWV , usually denoted by g_{WWV} , and on the renormalization scheme adopted. Consequently the parameters Δf_i^V include both two and three-point one-loop contributions.

To avoid this kind of indetermination we prefer to define the trilinear gauge couplings in the physical process of W pair production in e^+e^- annihilation,

$$e^-(k, \sigma) + e^+(\bar{k}, \bar{\sigma}) \rightarrow W^-(q, \lambda) + W^+(\bar{q}, \bar{\lambda}). \quad (29)$$

Following [23], the helicity amplitude for this process is:

$$\mathcal{M} = \sqrt{2} \, e^2 \, \tilde{\mathcal{M}}(\Theta) \, \varepsilon \, d_{\Delta\sigma, \Delta\lambda}^{J_0}(\Theta), \quad (30)$$

where $\varepsilon = (-1)^\lambda \Delta\sigma$ is a sign factor, $\Delta\sigma = \sigma - \bar{\sigma}$, $\Delta\lambda = \lambda - \bar{\lambda}$, $J_0 = \max(|\Delta\sigma|, |\Delta\lambda|)$, Θ is the scattering angle of W^- with respect to e^- in the e^+e^- c.m. frame and $d_{\Delta\sigma, \Delta\lambda}^{J_0}$ are angular functions depending on the helicity of the initial and final states.

After the inclusion of the one-loop corrections and the appropriate counterterms, the reduced amplitude for the process at hand reads:

$$\bullet \Delta\lambda = \pm 2$$

$$\tilde{\mathcal{M}} = -\frac{\sqrt{2}}{\bar{s}^2} \frac{\delta_{\Delta\sigma,-1}}{1 + \beta^2 - 2\beta \cos \Theta} \left[1 - \frac{\bar{s}^2}{\bar{c}^2 - \bar{s}^2} \Delta r_W - e_6 \right] \quad (31)$$

$$\bullet |\Delta\lambda| \leq 1$$

$$\begin{aligned} \tilde{\mathcal{M}}^\gamma &= -\beta \delta_{|\Delta\sigma|,1} [1 + \Delta\alpha(p^2)] \left[A_{\lambda\bar{\lambda}}^{\text{SM}} + \Delta A_{\lambda\bar{\lambda}}^\gamma \right] \\ \tilde{\mathcal{M}}^Z &= \beta \frac{p^2}{p^2 - m_Z^2} \left[\delta_{|\Delta\sigma|,1} - \frac{\delta_{\Delta\sigma,-1}}{2\bar{s}^2(1 + \Delta k(p^2))} \right] \\ &\quad \times \left[1 + \Delta\rho(p^2) + \frac{\bar{c}^2 - \bar{s}^2}{\bar{c}^2} \Delta k(p^2) \right] \left[A_{\lambda\bar{\lambda}}^{\text{SM}} + \Delta A_{\lambda\bar{\lambda}}^Z \right] \\ \tilde{\mathcal{M}}^\nu &= \frac{\delta_{\Delta\sigma,-1}}{2\bar{s}^2 \beta} \left[1 - \frac{\bar{s}^2}{\bar{c}^2 - \bar{s}^2} \Delta r_W - e_6 \right] \left[B_{\lambda\bar{\lambda}}^{\text{SM}} - \frac{1}{1 + \beta^2 - 2\beta \cos \Theta} C_{\lambda\bar{\lambda}}^{\text{SM}} \right] \end{aligned} \quad (32)$$

with $\beta = (1 - 4m_W^2/p^2)^{1/2}$ and $\Delta\rho$, Δr_W , Δk , $\Delta\alpha$ and e_6 defined in Eq. (22). $A_{\lambda\bar{\lambda}}^{\text{SM}}$, $B_{\lambda\bar{\lambda}}^{\text{SM}}$ and $C_{\lambda\bar{\lambda}}^{\text{SM}}$ are the tree-level SM coefficients listed in Table I, whereas $\Delta A_{\lambda\bar{\lambda}}^\gamma$ and $\Delta A_{\lambda\bar{\lambda}}^Z$ are the new fermion contributions expressed in terms of the CP -invariant form factors according to the relations:

$$\begin{aligned} \Delta A_{++}^V &= \Delta A_{--}^V = \Delta f_1^V, \\ \Delta A_{+0}^V &= \Delta A_{0-}^V = \gamma(\Delta f_3^V + \beta \Delta f_5^V), \\ \Delta A_{-0}^V &= \Delta A_{0+}^V = \gamma(\Delta f_3^V - \beta \Delta f_5^V), \\ \Delta A_{00}^V &= \gamma^2 \left[-(1 + \beta^2) \Delta f_1^V + 4\gamma^2 \beta^2 \Delta f_2^V + 2\Delta f_3^V \right]. \end{aligned} \quad (33)$$

TABLE I

Standard Model coefficients expressed in terms of $\gamma^2 = p^2/4m_W^2$.

$\lambda\bar{\lambda}$	$A_{\lambda\bar{\lambda}}^{\text{SM}}$	$B_{\lambda\bar{\lambda}}^{\text{SM}}$	$C_{\lambda\bar{\lambda}}^{\text{SM}}$
$++$, $--$	1	1	$1/\gamma^2$
$+0$, $0-$	2γ	2γ	$2(1 + \beta)/\gamma$
$0+$, -0	2γ	2γ	$2(1 - \beta)/\gamma$
00	$2\gamma^2 + 1$	$2\gamma^2$	$2/\gamma^2$

Our definition of the one-loop form factors consists in taking $\mathcal{K} = -\Pi'_{WW}(m_W^2)$ in Eq. (28), and consequentially $\Delta f_i^V \equiv \delta f_i^V - \Pi'_{WW}(m_W^2) \times f_i^{\text{SM}}$. Therefore Δf_i^V includes both the three-points one-loop corrections to the vertex WWV and the wave-function renormalization of the external W legs, taken on the mass-shell. This makes the terms Δf_i^V finite⁶.

Finally, it's worth making some comments about the unitarity constraints. In the high energy limit, the individual SM amplitudes from photon, Z and ν exchange are proportional to γ^2 when both the W 's are longitudinally polarized (LL), and proportional to γ when one W is longitudinal and the other is transverse (TL). The cancellation of the γ^2 and γ terms in the overall amplitude is guaranteed by the tree-level, asymptotic relation $A_{\lambda\bar{\lambda}}^{\text{SM}} = B_{\lambda\bar{\lambda}}^{\text{SM}}$. When one-loop contributions are included, one has new terms proportional to γ^2 and γ (see $\Delta A_{\lambda\bar{\lambda}}^\gamma$ and $\Delta A_{\lambda\bar{\lambda}}^Z$ in Eq. (33)) and the cancellation of those terms in the high energy limit entails relations among oblique and vertex corrections. For instance, if one omits the gauge boson self-energies such a cancellation does not occur any longer and the resulting amplitudes violate the requirement of perturbative unitarity. These properties show us the relevance of considering both the bilinear and trilinear contributions in the results of Eq. (32).

On the other hand, one of the possibilities to have appreciable deviations in the cross-section is to delay the behaviour required by unitarity. This may happen if in the energy window $m_W \ll \sqrt{p^2} \leq 2M$ (M denotes the mass of the new particles) the above cancellation is less efficient and terms proportional to positive powers of γ survive in the total amplitude [24]. If γ is sufficiently large then a sizeable deviation from the SM prediction is not unconceivable. It is useful to introduce the quantity

$$\Delta R_{AB} = \frac{\left(\frac{d\sigma}{d\cos\Theta}\right)_{AB} - \left(\frac{d\sigma}{d\cos\Theta}\right)_{AB}^{\text{SM}}}{\left(\frac{d\sigma}{d\cos\Theta}\right)_{AB}^{\text{SM}}}$$

with

$$\frac{d\sigma}{d\cos\Theta} = \frac{\beta}{32 \Pi p^2} |\mathcal{M}|^2 \quad (34)$$

representing the relative deviation from SM due to new physics effects in the different helicity channels $AB = \text{LL, TL, TT, tot.}$

⁶ Other choice, for making the form factors finite, is the one used by [24] where $\Delta f_i^V \equiv \delta f_i^V - \frac{1}{s} \Pi'_{W_3Z}(m_W^2) f_i^{\text{SM}}$. Evidently this choice is not consistent with our Eqs (31)–(32) because of the different convention for \mathcal{K} .

4.1. Static approximation

In the limit $m_W^2 \ll p^2 \ll M^2$ we can choose $\Lambda^2 = M^2$ and neglect the term λ_V/Λ^2 in Eq. (25). From the effective Lagrangian of Eqs (6)–(9) the anomalous trilinear couplings can be expressed as combinations of the coefficients a_i and the renormalization dependent quantity \mathcal{K}^V :

$$\begin{aligned}\Delta f_1^\gamma &= \Delta f_5^\gamma = \mathcal{K}^\gamma, \\ \Delta f_3^\gamma &= -g^2(a_1 - a_2 + a_3 - a_8 + a_9) + 2\mathcal{K}^\gamma, \\ \Delta f_1^Z &= -\frac{g^2}{c^2}a_3 + \mathcal{K}^Z, \\ \Delta f_3^Z &= g^2 \left[\frac{s^2}{c^2}(a_1 + a_{13} - a_2) - a_3 + a_8 - a_9 + a_{13} \right] + 2\mathcal{K}^Z, \\ \Delta f_5^Z &= \frac{g^2}{c^2}a_{14}.\end{aligned}\tag{35}$$

In the static approximation $\mathcal{K}^V = -H'(m_W^2) = 0$, and so $\Delta f_i^V = \delta f_i^V$. Obviously other conventions, which give different expressions for the form factors, are possible. For example putting

$$\mathcal{K}^Z = \frac{1}{c^2 - s^2} \left[a_0 + \frac{e^2}{c^2}(a_1 + a_{13}) \right] \quad \text{and} \quad \mathcal{K}^\gamma = 0 \tag{36}$$

we obtain the relations found by [10].

These formulae can readily be evaluated by substituting in Eq. (35) the explicit expressions of the coefficients a_i given in Eqs (8)–(9). If we restrict our analysis in particular to the case of a degenerate quark doublet⁷ we find very small values for the form factors:

$$\begin{aligned}\Delta f_1^\gamma &= \delta f_5^\gamma = 0, \\ \Delta f_3^\gamma &= -\frac{Gm_W^2}{4\Pi^2\sqrt{2}} \sim -1.3 \cdot 10^{-3}, \\ \Delta f_1^Z &= -\frac{Gm_W^2}{4\Pi^2\sqrt{2}} \frac{1}{c^2} \sim -1.7 \cdot 10^{-3}, \\ \Delta f_3^Z &= -\frac{Gm_W^2}{4\Pi^2\sqrt{2}} \frac{1+c^2}{c^2} \sim -3.1 \cdot 10^{-3}, \\ \Delta f_5^Z &= 0.\end{aligned}\tag{37}$$

⁷ Remember that for a leptonic doublet the contributions in the $r = 1$ limit are exactly 1/3 of the hadronic ones.

The analytical expression for ΔR_{LL} , for $\cos\Theta$ not closed to 1, reads⁸:

$$\Delta R_{LL} = \frac{g^2}{16H^2} 4\gamma^2 = \frac{4\sqrt{2}G}{16H^2} p^2. \quad (38)$$

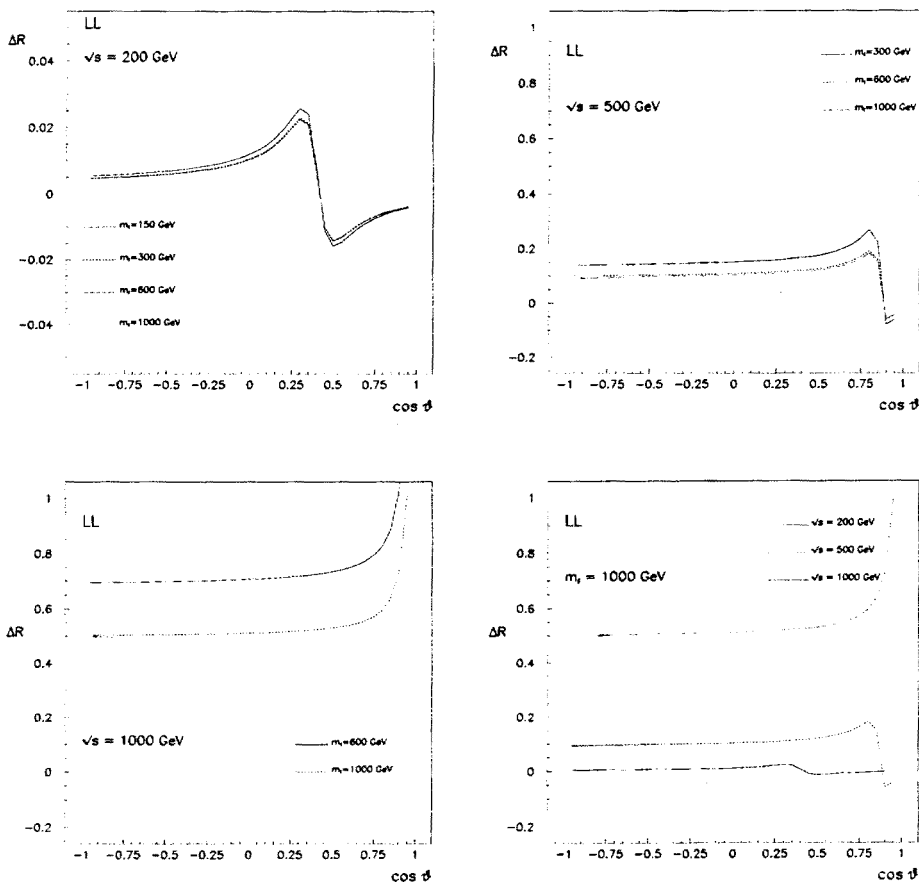


Fig. 4. Relative deviations in the differential cross section due to heavy fermion contributions (LL channel) for different c.m. energies and different masses.

Hence the deviation from the SM grows as p^2 , but this does not disagree with the unitarity requirement because we are in the static region. Even if this is an oversimplified formula we still obtain a realistic indication of the dimension of the effects which we are considering. Putting $\sqrt{p^2} = 500$ GeV

⁸ Only in this region can we safely neglect the contributions from the $C_{\lambda\lambda}^{\text{SM}}$ terms, which Table I shows are of the order $1/\gamma$ or $1/\gamma^2$.

in Eq. (38) one finds $\Delta R_{\text{LL}} \sim 0.10$, which is very close to the exact value of Fig. 4, the latter being calculated at the same energy with $M = 1000$ GeV in the region $\cos \Theta \ll 1$. A similar γ^2 dependence can also be derived for ΔR_{LT} , while the deviations from the SM values for ΔR_{TT} are of the order γ^0 , because neither the SM nor new physics contributions have dependence on positive powers of γ (see Table I and Eq. (33)).

4.2. Full calculation

The results obtained in the previous sub-section, whenever suggestive, are not completely satisfactory for essentially two reasons. Firstly, we learned that in some regions of the phase space (*i.e.*, near the production threshold) the effective Lagrangian becomes unreliable and effects of the order of p^2/M^2 can not be neglected. Secondly, the approximations behind the derivation of Eq. (38) are not applicable in two very important regions:

- for $\sqrt{p^2} = 1000$ GeV (NLC) where the mass of the extra virtual fermion and the energy of the collider can be of the same order⁹,
- for $\sqrt{p^2} = 200$ GeV (LEP2) where m_W is no longer negligible with respect to the c.m. energy.

So we are lead to analyse the full one-loop calculation.

The general expressions for the form factors and for ΔR are rather complicate. It is possible to derive simpler expressions only in the limit $m_W^2 \ll M^2$, p^2 where, for example, the quantity ΔR_{LL} , evaluated at $\cos \Theta \ll 1$, reads

$$\Delta R_{\text{LL}} = \frac{g^2 N_c}{16\pi^2} \frac{4M^2}{p^2} 4\gamma^2 \cdot F\left(\frac{4M^2}{p^2}\right) \quad (39)$$

with the function $F(x)$ given by:

$$F(x) = \left[1 - \sqrt{x-1} \arctan \frac{1}{\sqrt{x-1}} \right]. \quad (40)$$

For $p^2 \gg M^2$, F grows only logarithmically and unitarity is respected. When $M^2 \gg p^2$, $F \simeq p^2/12M^2$ and we obtain again the result of Eq. (38). In the range of energies $m_W \ll \sqrt{p^2} \leq 2M$ ΔR_{LL} is of order $G_F M^2$.

The complete results are plotted in Fig. 4 for the case of a heavy degenerate quark doublet. We show the relative deviation ΔR_{LL} as a function of

⁹ The stability of the Higgs potential puts an upper bound on the chiral fermion mass ($M \leq 1.3$ TeV, see, for example, [11]).

$\cos\Theta$ at $\sqrt{p^2} = 200, 500, 1000$ GeV for several values of fermion masses. At LEP2 energies (top-left graphic) the deviations in the LL channel are of the order of 1 per cent for both relatively light and heavy fermions. Only when one considers particles very close to the production threshold, can deviations of some per cent be achieved. In any case this is well below the LEP2 observability level, because the number of events in the LL (or even the TL) channel will be very small at the foreseen luminosity. On the other hand in the TT channel, where the number of events will be “adequate”, the deviations expected are not enhanced by the γ factor and stay at the order of 0.1 per cent. More interesting is the situation at the higher energies of NLC (top-right and bottom-left graphics). Unitarity delay in the LL (LT) channels, due to the γ enhancement factor, gives deviations from the SM of the order 10–50 per cent for a wide range of new particle masses [24]. The effects at $\sqrt{p^2} = 1000$ GeV soon become very large, and reach the limit of the validity of the perturbation expansion. A similar behaviour is also exhibited in the TL channel. In the TT and tot channels this effect essentially disappears for $\cos\Theta \simeq 1$, where bigger is the number of expected events. This makes it clear that for the observation of the effect under discussion good identification of the W polarization is essential. In the bottom-right graphic of Fig. 4 we display the energy dependence of the effect for a fixed mass $M = 1000$ GeV. Also if the differential cross section in the LL channel is two orders below the TT one, with the energies and the luminosity promised by NLC these effects will be easily seen, as it is showed in Fig. 5.

Here we plot the number of events per bin at $\sqrt{p^2} = 1000$ GeV for channel LL, taking $M = 600$ GeV and assuming a luminosity of 100 fb^{-1} . The error bars refer to the statistical error, the full line denote the SM expectations and the dotted line is the prediction for one extra heavy chiral fermion doublet. We notice a clear signal, fully consistent with present experimental bounds.

Finally we would like to mention that this behaviour is typical only for heavy chiral fermions. We checked explicitly that, in the case of vector-like fermions (*i.e.*, like those found in the MSSM), no unitarity delay takes place at high energies ($\sqrt{p^2} = 500$ GeV or greater). The deviations ΔR remain under the per cent level, making questionable the possibility of observing such effects in the next generation e^+e^- colliders [27].

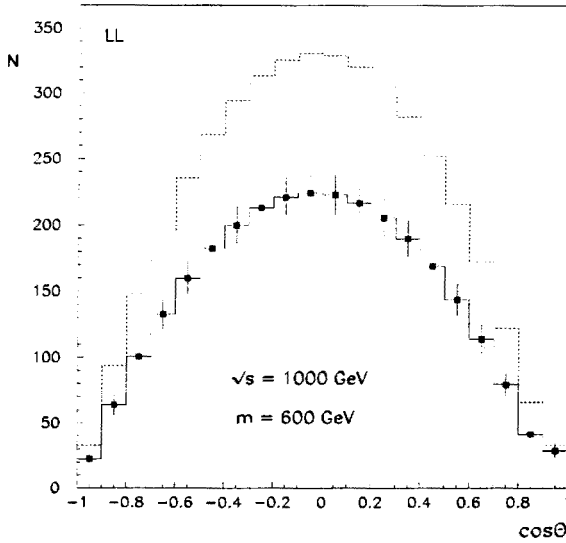


Fig. 5. Number of events predicted from the SM (full line) at energy $\sqrt{p^2} = 1000$ GeV and luminosity $L = 100 \text{ fb}^{-1}$ for one extra doublet of a heavy chiral fermion (dotted line) of mass $M = 600$ GeV. The expected statistical errors are also shown.

5. Conclusion

To conclude, we discussed the impact of the presence of new sequential fermions. We showed that the present data still allow the presence of new quark and/or lepton doublets with masses greater than $M_Z/2$ if they are nearly degenerate. The description obtained from the effective Lagrangian of Eq. (6) is rather satisfactory. Only for new fermions with masses very close to the threshold $M_Z/2$ one finds drastic departures of the effective Lagrangian result from the full one-loop radiative corrections obtained in the SM. However, the presence of new fermions carrying the usual $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers with masses as low as $60 - 80$ GeV is severely limited both by accelerator results and cosmological constraints. The presence of heavier chiral fermions ($M > 200$ GeV), still allowed if degenerate, can produce a huge effect at the energies provided by NLC. The reason rely in a sort of fine tuning between the unitarity delay ($p^2 \leq 4M^2$) and the kinematic enhancement felt in the LL and LT channels. Deviations from the SM are of the order of 10–50 per cent for a wide range of new particles masses. These effects would seem to be easily measured at the NLC luminosity.

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