DCC: ATTRACTIVE IDEA SEEKS SERIOUS CONFIRMATION

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(Received June 4, 1996)

Dedicated to Andrzej Białas in honour of his 60th birthday

The theoretical ideas relevant for the physics of the disorientend chiral condensate (DCC) are reviewed.

PACS numbers: 25.75.-q, 13.85.Hd

1. Introduction

As is well known, quantum chromodynamics (QCD), the theory of strong interactions, has an approximate global SU(2)× SU(2) invariance. This invariance is spontaneously broken and the relevant part of the order parameter is a vector $\phi = (\sigma, \vec{\pi})$ transforming under the O(4) subgroup of SU(2)×SU(2). In the physical vacuum ϕ points in the σ direction. One calls *disoriented chiral condensate* (DCC) a medium where ϕ is *coherently misaligned.* Experimental observation of a signal of transcient DCC formation would be a striking probe of the chiral phase transition.

DCC is now a topical subject. This research has a prehistory [1–6]: Some results have already been found long ago, but forgotten for various

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reasons, mostly because they were apparently lacking theoretical underpinning. They have been rediscovered independently in the new context, with a better motivation. Actualy the true history of DCC research begins in the early 90's, when several points have been simultaneously realized:

- In spite of the fact that confinement and hadronization are quantum phenomena, the production of multiparticle *hadron* states can be described in terms of an *effective* theory for which a classical approximation is meaningful. In practice, one is led to consider classical radiation of soft pions, described by the σ-model, in very high multiplicity events, where this radiation is intense (the most promising applications are in high-energy heavy-ion collisions) [7–9].
- It is rather natural to expect that in some high-energy collisions there could appear a space region, shielded for some time from the outer physical vacuum, where a DCC can develop [8]. In particular, this is what one finds in the solution of the σ-model corresponding to Heisenberg's idealized boundary conditions [9] [1].
- It is a common feature of the non-trivial solutions of non-linear field equations with internal symmetries, that they break the symmetry. In the present case one expect the field configuration to break the O(4) symmetry. Since there is no a priori reason to priviledge any direction in the internal space, all field orientations are expected to be equiprobable. Assuming that the isospin orientation of the pion field is constant throughout a space domain — remember, that we are interested in long wavelength modes — and points in some random direction, one predicts that the ratio

$$f = \frac{N_{\pi^0}}{N_{\pi^0} + N_{\pi^-} + N_{\pi^+}} \tag{1}$$

should be distributed according to the simple law [2]

$$dP(f) = \frac{df}{2\sqrt{f}}\,. \tag{2}$$

Hence, DCC formation has a very distinctive signature, which actually reflects correlations coded in the classical, *i.e.* coherent field. The

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[1] This solution belongs to a general class of solutions proposed earlier in [7].

[2] Apparently this result appeared for the first time in [5], where the distribution of the neutral pion fraction has been calculated for a coherent multipion state of total isospin zero. A simple quasi-classical argument, given first in [9], enables one to find (2) immediately: The intensity of pion radiation is quadratic in the pion field. Hence, one seeks the probability that a unit vector, randomly oriented in isospace, has its 3rd component equal to $\sqrt{f}$. The calculation is elementary.

deviation from the narrow Gaussian centered at $f = \frac{1}{3}$ predicted by statistical arguments is striking.

- Long wavelength modes of the pion field can be dramatically amplified during the out-of-equilibrium cooling of the quark-gluon plasma [10, 11].

We shall develop these points in the following sections. Our aim is to explain some of the main ideas, not to give a comprehensive account of all papers on this rapidly evolving subject. Reader's attention is called to some earlier reviews. Those of Bjorken and collaborators [12–14] are particularly helpful in gaining physical intuition. They also contain a discussion of relevant experimental matters. A comprehensive presentation of the subject can be found in an excellent review by Rajagopal [15].

2. Choice of theoretical framework

For a DCC to be produced, there must be a stage, during the collision process, where chiral symmetry is restored. It is tempting to identify this stage with the formation of a hot quark-gluon plasma. The latter can be directly described in QCD language, and insight into the corresponding physics can be obtained using the perturbative aproach. The later stages of the collision involve eventually soft pions radiation. This can be desribed by an effective theory in which the Lagrangian has the form of a series involving an increasing number of derivatives. The first term, the one with two derivatives, is uniquely determined. It corresponds to the so-called non-linear σ-model and gives account of the physics of the softest modes. An educated guess is needed to figure out what exactly happens at the intermediate stage between these two extremes, *i.e.* to describe the cooling of the plasma leading to its decay into pions.

One commonly uses for the description of this intermediate stage the linear σ-model. There are several reasons for this choice, which we briefly discuss. First of all, the chiral symmetry is spontaneously broken when one crosses the phase transition point. There exist suggestive arguments [16, 10] to the effect that QCD with two flavors of massless quarks belongs to the same *static* universality class as an O(4) Heisenberg ferromagnet. Hence, in the static regime, the long wavelength modes can be described by a Ginzburg-Landau Lagrangian, which in this case is identical to that of the linear σ-model. Integrating out the σ field in the latter one gets the non-linear σ-model Lagrangian as the leading term, plus higher derivative corrections. Thus, the linear σ-model offers a correct description of very long wavelength pion modes. Furthermore, it can be, at least formally, extended to describe the disordered state, where all four components of ϕ are fluctuating independently.

For these reasons, the linear σ-model appears as a natural first choice in the modelling of DCC formation and dynamics [3]. The following caveat should be, however, borne in mind: DCC can only be produced during out-of-equilibrium cooling [4] and an infinity of distinct stochastic processes can have in common the same equilibrium ensemble. Moreover, one is here mostly interested in non-universal parameters, and not in the behaviour of the system in the immediate vicinity of the phase transition.

With the appropriate choice of units, the Lagrangian is

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}(\phi^2 - 1)^2 + H\sigma\,. \tag{3}$$

The small symmetry breaking term is introduced to take into account the effect of small quark masses. Only the case $\lambda \gg H$ is relevant for phenomenology. Since we are interested in the dynamics of the model, a specification of the initial conditions is mandatory. Here, we touch another source of uncertainty: One wishes to start the evolution in a state with unbroken chiral symmetry. But above the chiral phase transition point the use of the σ-model is questionable, and at very high temperature, it presumably makes no sense !

At this point it is customary to introduce the idea of a "quench" [10]. One assumes that the hot plasma is suddenly frozen, and that its subsequent dynamics is correctly given by the (zero temperature) equations of the linear σ-model. In that way the problem becomes mathematically well defined, although the difficulty has been actually only displaced: What is the physical mechanism of the quench? As will be argued later on, it is likely that a rapid expansion of the system produces a damping of fluctuations, which is indeed approximately equivalent to a quench.

Most of the works which have been done so far, with the exception of some attempts to incorporate quantum corrections which we shall discuss in a later section, deal with the *classical* linear σ-model. In particular, the various "scenarios" proposed refer, in fact, to various approximations used to solve the complicated classical dynamics of the model, and to different ways of implementing the initial conditions. The purpose of the next section is to give the reader a qualitative insight into the dynamics of the linear σ-model, through selected illustrative examples.

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[3] One should mention that interesting studies of alternative models have also been presented. We shall not enter here into the discussion of these models [17, 18]. The σ-model will suffice to illustrate all the relevant points.

[4] In an adiabatic process, the domain size is comparable to $m_\pi^{-1} \sim T_c^{-1}$ and there is no way of producing a coherent multipion state [10].

3. Qualitative trends

The field equations read

$$\partial^2 \phi = -\lambda(\phi^2 - 1)\phi + H n_\sigma , \tag{4}$$

where n_σ is the unit vector in the σ direction. In this section, quantum effects are neglected, so that ϕ is to be regarded as a *classical* field. The field ϕ evolves in a potential which, apart from the symmetry breaking term $\propto H$, has the form of a "Mexican hat". We wish to follow the time evolution of ϕ, starting somewhere near the top of the "Mexican hat" and ending at the very bottom. Mathematically, the classical problem is precisely defined once the initial conditions $\phi(\boldsymbol{x}, t)$, $\partial_t \phi(\boldsymbol{x}, t)$ are specified. In its full generality the problem is fairly complicated. We start by adopting an idealization, originally due to Heisenberg [1]. This will enable us to follow analytically all the stages of the time evolution of ϕ, taking into account the expansion of the system, at the expense of dramatically reducing the number of degrees of freedom.

Time evolution during expansion [19]

Assume that initially, at time $t = 0$, the whole energy of the collision is localized within an infinitesimally thin slab with infinite transverse extent. The symmetry of the problem then implies that ϕ is a function of the *proper* time $\tau = \sqrt{t^2 - x^2}$ only. The field equations become ordinary differential equations and can be solved analytically. We shall not enter here into the algebra, concentrating on the significance of the results.

Of course, viewed in the laboratory, the system expands: at time t it extends from $x = -t$ to $x = t$. Notice, that $\partial^2 \phi \equiv \ddot{\phi} + \dot{\phi}/\tau$, where the dot denotes the derivative with respect to τ. Hence, the equations of motion involve not only the acceleration term but also a friction term. This "friction" reflects the decrease of the energy in a covolume, due to expansion, and turns out to be very important .

From Eqs (4) one obtains easily

$$\vec{\pi} \times \dot{\vec{\pi}} = \frac{\vec{a}}{\tau} \tag{5}$$

and

$$\vec{\pi}\dot{\sigma} - \sigma\dot{\vec{\pi}} = \frac{\vec{b}}{\tau} + \frac{H}{\tau} \int^{\tau} \vec{\pi}\tau d\tau . \tag{6}$$

Eq. (5) is a consequence of the conservation of the isovector current, while Eq. (6) reflects the partial conservation of the iso-axial-vector current. The isovectors $\vec{a}$ and $\vec{b}$ are integration constants. The lengths of these vectors measure the initial strength of the respective current. It is easy to see that the component of $\vec{\pi}$ along $\vec{a}$ vanishes, $\pi_a = 0$.

One can show that the second term on the RHS of Eq. (6) is irrelevant as long as $\tau \ll b/\sqrt{H}$ and $a \ll b$. With $H = 0$ one can write

$$\pi_b = -r \sin\theta\,, \tag{7}$$

$$\pi_c = \frac{a}{\sqrt{a^2+b^2}}\, r\cos\theta\,, \tag{8}$$

$$\sigma = \frac{b}{\sqrt{a^2+b^2}}\, r\cos\theta\,, \tag{9}$$

where $\vec{c} = \vec{a} \times \vec{b}$. Thus the motion is planar. It remains approximately so even at later time provided the condition $a \ll b$ is satisfied. The component π_c is then always very small and the pion field oscillates along the (random) direction defined by the isovector $\vec{b}$. We concentrate our attention on this particularly simple and interesting case. Assuming that at $\tau = \tau_0$ the distributions of ϕ and $\dot{\phi}$ are Gaussian, with variances σ_f and σ_g respectively, one can calculate the probability that b takes a given value:

$$\frac{dP(b)}{db} \propto \frac{\mathrm{e}^{-b/b_0}}{\sqrt{b}}\,,\quad 0 < a < A \ll b \tag{10}$$

with $b_0 = \sigma_f \sigma_g \tau_0$. This is an important result, because it turns out that the initial strength b of the axial current controls the time evolution (see below).

The motion can be described by two variables, the radial variable r and the angular variable θ. Solving the corresponding equations of motion one can identify several stages in the *proper* time evolution of this simple dynamical system:

- By assumption, for $\tau < \tau_0$ the model does not apply. (A plausible value of τ_0 is 1 fm/c.)
- For $\tau_0 < \tau \lesssim b/\sqrt{2\lambda}$ the radial and the angular motion are strongly coupled. Both are damped by friction, which, as we have already explained, is another facet of the expansion.
- For $b/\sqrt{2\lambda} \lesssim \tau b/\sqrt{H}$ the radial motion corresponds to high frequency damped oscillations about the equilibrium position $r = 1$. For large enough τ the solution takes the particularly transparent form

$$r = 1 + \frac{C\cos(\tau\sqrt{2\lambda} + \delta)}{\sqrt{\tau\sqrt{2\lambda}}}\,, \tag{11}$$

  where C, δ are constants sensitive to the initial conditions. Remember, that in this context $\sqrt{2\lambda}$ should be regarded as a large parameter. Setting $r \to \langle r\rangle = 1$ in the equation of motion for θ, the latter becomes that of a damped pendulum. Provided b is large enough the motion of the pendulum is circular:

$$\theta = \ln \frac{\tau}{\tau_0} . \tag{12}$$

The regime just described corresponds to the non-linear σ-model (we recover the solution found in [9]).

• Near $\tau \approx b/\sqrt{H}$ the damping produces a cross-over from the circular to the oscillatory motion and one finds

$$\pi_b \approx -\theta \approx \frac{\sqrt{b}\cos\left(\tau\sqrt{H} + \delta'\right)}{\sqrt{\tau\sqrt{H}}} . \tag{13}$$

The RHS of (13) is a solution of the Klein-Gordon equation, describing free propagation of pions with mass $\sqrt{H}$. These are the DCC decay products one hopes to observe.

It is elementary to find the Fourier transform (with respect to x) of the RHS of (13) and to calculate the energy radiated at large t. One finds a rapidity plateau of height b. The plateau is a consequence of the boost invariant boundary conditions. The relevant result is that *the energy released in the decay of DCC is proportional to the strength of the initial iso-axial-vector current.* This result together with (10) suggests that the probability to release large energy via DCC decay strongly depends on the initial conditions (via the parameter b) and that it is damped exponentially. Thus, *observable* DCC are likely to be rare events. If this conclusion is correct, then the calculation of *average* DCC characteristics is of little phenomenological interest.

Amplification of long wavelength modes [11, 20–22]

With Heisenberg's boundary conditions the problem is eventually reducible to that of a dynamical system in $0+1$ dimensions. But the true problem is $1+3$ dimensional and its solution requires using a computer. However, the most salient conclusions reached via numerical simulations can be qualitatively understood within an approximate framework. This is what we are going to explain now.

Let us separate the field $\phi(\boldsymbol{x}, t)$ into its spatial average $\langle\phi(t)\rangle$ and a space dependent fluctuating part $\delta\phi(\boldsymbol{x}, t)$:

$$\phi(\boldsymbol{x}, t) = \langle\phi(t)\rangle + \delta\phi(\boldsymbol{x}, t) . \tag{14}$$

We have

$$\langle\phi(t)\rangle = \frac{1}{\Omega}\int \mathrm{d}^3x \, \phi(\boldsymbol{x}, t) , \tag{15}$$

where Ω is the volume of the system and, by definition, the spatial average of $\delta\phi(\boldsymbol{x},t)$ vanishes: $\langle\delta\phi\rangle = 0$. By taking the spatial average of the equation of motion, we get:

$$\partial_t^2 \langle\phi(t)\rangle = -\,\lambda\left(\langle\phi^2(\boldsymbol{x},t)\phi(\boldsymbol{x},t)\rangle - \langle\phi(t)\rangle\right) + H n_\sigma\,, \tag{16}$$

where we have assumed that the spatial derivative of the field vanishes at the boundary of the reference volume. By replacing, on the RHS of this equation $\phi(\boldsymbol{x},t)$ by its decomposition (14), one obtains an equation which involves spatial averages of products of fluctuations, that is, the equation for $\langle\phi(t)\rangle$ is not a closed one. It needs to be complemented by the equations of motion for the fluctuations.

At this stage, we introduce some *approximations* in order to treat the fluctuations.

(i) Given a product of fluctuation fields, one replaces all the pair products $\delta\phi_j\delta\phi_k$ by $\langle\delta\phi_j\delta\phi_k\rangle$, and one adds terms corresponding to different contraction schemes, much in the same way as in writing the well known Wick theorem in field theory. Consequently the Gaussian average of the initial product of fields and that of the final approximate expression are identical. Notice that, with this aproximation, the average of a product of an odd number of fluctuation fields is zero.

(ii) We assume that the 4×4 tensor $\langle\delta\phi_j\delta\phi_k\rangle$ is diagonal in the (moving) orthogonal frame whose one axis points along $\langle\phi\rangle$.

As a further simplification, we shall also use the formal large N limit (of the O(N) σ-model), neglecting terms like $\langle\delta\phi_j^2\rangle$, of the order O(1), as compared to $\langle\delta\phi^2\rangle$, which is of the order O(N) [5].

The equation for $\langle\phi\rangle$ is immediately found from (16). The equations for the fluctuations are obtained from the exact (classical) field equations, by subtracting the equation of motion for $\langle\phi\rangle$. Introduce the notation:

$$\omega_\perp^2(k,t) = k^2 + \lambda\left(\langle\phi(t)\rangle^2 + \langle\delta\phi^2(t)\rangle - 1\right)\,, \tag{17}$$

$$\omega_\parallel^2(k,t) = k^2 + \lambda\left(3\langle\phi(t)\rangle^2 + \langle\delta\phi^2(t)\rangle - 1\right)\,. \tag{18}$$

One easily finds

$$\partial_t^2 \langle\phi(t)\rangle = -\,\omega_\perp^2(0,t)\langle\phi(t)\rangle + H n_\sigma\,, \tag{19}$$

and

$$\partial_t^2 \delta\phi_\parallel(\boldsymbol{k},t) = -\,\omega_\parallel^2(k,t)\delta\phi_\parallel(\boldsymbol{k},t)\,, \tag{20}$$

$$\partial_t^2 \delta\phi_\perp(\boldsymbol{k},t) = -\,\omega_\perp^2(k,t)\delta\phi_\perp(\boldsymbol{k},t)\,. \tag{21}$$

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[5] This approximation is necessary to fulfill Goldstone theorem when $H = 0$. Otherwise, the Goldstone bosons acquire a non-vanishing mass, of order $1/N$.

Here $\delta\phi_{\parallel}$ ($\delta\phi_{\perp}$) denotes the fluctuation parallel (perpendicular) to $\langle\phi\rangle$. The approximation which we have just done is usually refered to as the Hartree approximation. One encounters a similar structure when discussing quantum corrections (see the next section). In the present section, the Hartree approximation is introduced for purely pedagogical purposes: it helps to understand the main features of the non linear classical dynamics. Solving numerically eqs. (19)-(21) is not really simpler than solving the exact classical equations (4).

It is obvious from Eqs (19)–(21) that $\delta\phi$ will be amplified during the time evolution whenever the corresponding ω^2 becomes negative. The following points should be clear from the inspection of the formulae for ω:

- The amplification can only occur if k^2 is small enough. In other words, only long wavelength modes are amplified.
- The amplification of $\delta\phi_{\perp}$ stops when $\langle\phi\rangle$ approaches unity. The amplification of $\delta\phi_{\parallel}$ is not expected to be significant (notice the factor of 3 in front of $\langle\phi\rangle^2$ in (18)).
- Large fluctuations prevent the amplification. Thus one does not expect DCC domains to form when the energy density (temperature) is too high.

One could derive Hartree equations taking into account expansion, as in the earlier example. Assuming that the fields depend on time via $\tau = (t^2 - \sum_1^D x_i^2)^{1/2}$ one finds a friction force $-(D/\tau)\partial_\tau\phi$. Since friction damps fluctuations, we arrive to the conclusion:

- Introducing expansion favors the formation od DCC domains. The larger is D, the stronger is the effect.

4. Quantitative estimates

More realistic calculations require the use of a computer. First simulations [11] have been carried out in the static set-up, *i.e.* neglecting expansion. Classical equations of motion have been used, assuming (ad hoc) that initially ϕ and $\partial_t\phi$ are Gaussian random variables living on a discrete grid of points. A dramatic amplification of long wavelength modes have been observed: the larger the wavelength, the stronger the amplification. These results have been confirmed by other people [20]. Notice that the amplified soft modes are not spatially separated from the hard ones. One should remember this point when refering to “DCC domains”. Moreover, the amplification of soft modes is a transcient phenomenon: The non-linearity of the equation of motion leads eventually to the equipartition of the energy in a static set-up. Of course, this does not mean that DCC signal will not be observed, since, in particular, the expansion may prevent the system to reach this regime .

The expansion can be introduced as explained in the preceding section. Upgrading the simple model presented there by assuming that ϕ depends not only on the proper time τ but also on the rapidity variable $\eta = (1/2)\ln(t-x)/(t+x)$ it is found [23] that DCC is localized in a rapidity interval 2 to 3 units long. In Ref. [24] the same group has assumed invariance under longitudinal boosts, allowing the system to expand transversally. The results are very encouraging. With appropriate initial conditions the domains of DCC with 4-5 fm in size have been observed at $\tau = 5$ fm. But, the emergence of DCC strongly depends on the choice of the initial conditions (see also [25]).

A systematic method of sampling initial field configurations from an equilibrium ensemble at a given temperature has been devised in Ref. [22]. In Ref. [26] the same author has studied the dynamical trajectories generated by the classical equations of motion (4) starting from initial configurations generated using the sampling method quoted above. The trajectories are drawn on the $(\langle\phi\rangle, \langle\delta\phi^2\rangle)$ plane, where the region of instability is also exhibited. When the system is prepared at the temperature $T = 400$ MeV the incursion into the unstable region (*i.e.* amplification of soft modes) only occurs for $D > 1$. At $D = 3$ instabilities occur for starting temperatures ranging from 200 MeV to 500 MeV (at least). What happens, is that at the initial stage of the evolution the fluctuations fall rapidly, while $\langle\phi\rangle$, small initially, changes little so that the system enters the instability region. Then, $\langle\phi\rangle$ increases steadily and eventually the instability is shut-off. At much higher temperatures the fluctuations do not have time to decrease enough before a substantial increase of $\langle\phi\rangle$ and the instability conditions are never met.

Although DCC is essentially a quasi-classical phenomenon, several quantum effects could play a role. First, the classical evolution cannot continue forever: when the energy density becomes low enough, the description of the system of particles in terms of a classical field becomes meaningless. There is also another effect [27], less trivial and usually disregarded in the DCC context: DCC evolves as an open system. The interaction with other nuclear debris, acting as a "bath", is the source of decoherence. There are also quantum corrections to the dynamics of the condensate.

Attempts to deal with the last problem have been made by several groups. The authors of Ref. [28] have considered a static set-up, analogous to the one of Ref. [11], quantizing conventionally on the hypersurfaces $t =$ const. In Refs [29, 30] the expansion is taken into account and the system is quantized on the hypersurfaces $\tau =$ const. Thus the two definitions of the final state wave functions are not equivalent. Particle production is calculated by following the adiabatic vacuum of the fluctuation and measuring particle production with respect to this vacuum. In order to proceed both groups are led to make approximations and their final equations, those used in the actual numerical work, have exactly the structure of the Hartree equations written in the preceding section. There are obvious modifications: $\langle \ldots \rangle$ becomes the average over thermal and quantum fluctuations and one has to define the product of fluctuation fields at coincident points, which requires regularization and renormalization, in the standard fashion. On the whole it appears that the quantum corrections do not change qualitatively the picture, and that the conclusions of these studies are not in variance with expectations drawn from the study of the classical equations. Starting with thermal fluctuations one can hardly produce any amplification without expansion. And the results are very sensitive to initial conditions. Let us mention that an enhancement at low transverse momentum, correlated with the DCC formation has been reported in [30].

5. Lessons from numerical simulations

The numerical simulations provide an underpinning to the qualitative discusion presented in Sect. 3. The main thing that has been learned is that in more or less realistic set-ups the initial fluctuations can get amplified during the evolution of the system, giving rise to the emergence of a DCC-like phenomenon. Thus, it is plausible, but not certain, that the effect will be occasionally strong enough to be observable. The other important lesson is that the results depend strongly on the initial conditions. And, as we have already emphasized, in the present state of the art the choice of initial conditions is the Achilles's tendon of the theory.

Nobody has been able to estimate the most relevant parameter: the probability of producing an observable DCC signal. We are fully aware of the fact that the calculations carried out so far are theorist's games. Their purpose is to gauge an idea, not to produce numbers to be directly compared with experiment. However, even in a theory that is not fully realistic one would be pleased to learn whether the effect is expected to occur at the level of one per cent or one per billion. A result analogous to (10), but valid in a more realistic set-up would be most welcome. One can associate a measure with the thermal configurations of the σ-model [22]. This measure can, at least in principle, be used to estimate the probability we are talking about. Although, as already mentioned, the use of the σ-model is quaestionable above the critical temperature, such a phase-space measure may be a good guide. It is not uncommon in multiparticle production phenomenology to obtain reasonable estimates from statistical arguments, also in instances where, strictly speaking, thermal equilibrium arguments do not apply.

6. Experimental signatures

There is no doubt that the predicted large fluctuations of the neutral pion fraction are the most striking signature of DCC formation. There are, however, at least two problems in identifying such a signal. First, one can wonder to what extent the simple law (2) is distorted by secondary effects. Second, assuming that the distortion is not very significant, one faces the problem of extracting the DCC signal from the background.

Various corrections to (2) have been studied in the literature. Eq. (2) could be modified due to the coupling of the DCC isospin to the isospin of other collision debris. The authors of Ref. [31] find that the effect is insignificant, at least for $0.1 < f < 0.9$, as long as the isospin of DCC is less than 30% of the pion multiplicity [6]. Another source of distortion could come from final state pion-pion interactions with charge exchange. But, the mean free path of soft pions is estimated to be much larger than the DCC domain size, so that the final state interactions should not be very important [33]. We suspect, that blurring of (2) will mostly come from the pion field orientation in isospace being not exactly constant all over the DCC domain.

Concerning the problem of DCC signal identification, we would like to report about an interesting suggestion put forward in Ref. [34]. The authors propose to use the modern technique of signal processing, the so-called multiresolution wavelet analysis, to study the distribution of f on the lego plot. Let us briefly explain the idea in one dimension.

Consider a histogram H^0 with bin size 1 (in appropriate units). Use it to produce a smoother histogram H^1, doubling the bin size. Of course, $H^0 = H^1 + R^1$ and there are fine structures in R^1 over distance 1. Repeat the operation to get $H^0 = H^n + R^1 + \dots R^n$, $n = 1, \dots N$. By definition N is such that H^N is structureless. The result of these manipulations is a series of increasingly coarse-grained histograms: H^n is living on bins of size 2^n and the associated R^n has fine structures at scale 2^{n-1} only. The beauty of the story is that R^n can be written as a superposition of the so-called wavelet functions $W_j^n(x)$, which form a set orthonormal with respect to both indices. Furthermore all $W_j^n(x)$ can be obtained by rescaling and shifting a "mother" function $W(x)$:

$$W_j^n(x) = 2^{-n/2} W(2^{-n}x - j) \,. \tag{22}$$

Various mother functions have been explicitly constructed. There exist computer programs performing the decomposition [35]. The extension to

[6] It has been argued, however, that isospin non-conserving effects could be amplified due to coherence, altering the standard expectations [32].

more than one dimension is straightforward. Wavelets are local in space and scale and are therefore, contrary to trigonometric functions, particularly suitable to uncover localized structures and find the associated scales.

The authors of [34] have applied this technique to analyse rapidity distributions of f generated by the classical evolution of the 1+1 dimensional linear σ-model, in the conditions leading to DCC formation. They compared these "DCC data" with "random noise data" and found very significative diferences. In particular, the power spectrum associated with a given scale

$$P^n = \sum_j \mid (H^0, W_j^n) \mid^2 \tag{23}$$

has a dramatic scale dependence for "DCC data", while it is scale independent in the random sample. It will be, of course, interesting to see how the method performs in more realistic cases.

Secondary signatures of DCC formation have also been suggested. They include specific pion pair correlations [36, 37] and anomalies in electromagnetic decays of resonances [38].

7. Conclusions

There exist a few cosmic ray events, the so-called Centauros (see [39]), where one observes jets consisting of as many as 100 charged pions and no neutrals. Are Centauros an evidence for DCC formation? We would not risk any definite answer. For the moment, the search for more Centauro-like events in cosmic ray interactions and in accelerator data has been unsuccessful. An experiment at the Tevatron [40] has been designed to look for the phenomenon. Other experiments, with heavy ions, are being planned and should be encouraged. Until the idea does not receive a firm experimental confirmation it will continue having the status of a smart speculation. However, this speculation has led people to think more about non-equilibrium processes in high-energy nuclear collisions and this is certainly a very positive development. Whatever will be the future of this idea, we have already learned quite a lot on the theory side. What is amusing, is that the discovery of Centauro events has been met with widespread scepticism: how can one seriously claim that it is possible to produce a multipion state with so small a fraction of neutrals? Now, as we have a plausible mechanism for the effect, experimenters should be prepared to hear the opposite blame: the phenomenon is so natural, how can it be that you do not see it?

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