BIN–BIN CORRELATION MEASUREMENT BY THE BUNCHING-PARAMETER METHOD

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A new method for the experimental study of bin-bin correlations is proposed. It is shown that this method is able to reveal important additional information on bin-bin correlations, beyond that of factorial-correlator measurements.

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1. Introduction

In order to obtain a comprehensive knowledge of the dynamics of particle production in high-energy reactions, two aspects of multiplicity fluctuations need to be studied:

1. the dependence of the multiplicity distribution (or its characteristics) on the size of the phase-space interval;
2. the dynamical correlations between two or more bins where this dependence is investigated.

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The first point corresponds to the measurement of the local fluctuations, the second one to a simultaneous measurement of the local characteristics in two (or more) different bins in order to reveal correlations between these local fluctuations. If no correlations exist between fluctuations in different bins, then complete information on an experimental sample can be obtained from local fluctuation measurements.

Dynamical information on fluctuations in a system with an infinite number of particles per event can be obtained from the multivariate density probability distribution $P(\rho_1, \rho_2, ..\rho_M)$, where ρ_m is the particle density in bin m ($m = 1, \ldots, M$). This distribution can be studied by constructing the multivariate moments $\langle \rho_1^{q_1} \rho_2^{q_2} \ldots \rho_M^{q_M} \rangle$. Due to the very complex structure of this quantity, however, one usually resorts to the study of only two moments: $\langle \rho_m^q \rangle$ and $\langle \rho_m^q \rho_{m'}^{q'} \rangle$, which contain a small fraction of the information on dynamical fluctuations in a system. The bivariate moment $\langle \rho_m^{q_m} \rho_{m'}^{q'} \rangle$ contains the information on bin-bin correlations.

In practice, bin-bin correlations always exist, *i.e.*, $\langle \rho_m^q \rho_{m'}^{q'} \rangle \neq \langle \rho_m^q \rangle \langle \rho_{m'}^{q'} \rangle$, since final-state particles are not produced independently of each other. The production of a particle at high energy usually enhances the probability of producing other particles. The number of particles observed in a given phase-space bin, therefore, is always affected by the number of particles found in other bins. Moreover, there are more trivial (statistical) reasons for the observation of correlations in a system of finite fixed final-state multiplicity: for such a system, finding a particle in a single bin is less probable if another particle has already been counted in another bin. The latter case has no dynamical reason, but can influence the correlations observed in such a system.

In [1], Białas and Peschanski have adapted the method of normalized factorial moments to the measurement of dynamical bin-bin correlations by means of factorial correlators. The use of these quantities, as well as of the normalized factorial moments, has mainly been motivated by the Poissonian-noise suppression [2], thereby opening the possibility of modelling intermittency phenomena and bin-bin correlations by means of continuous densities.

In this paper we propose another experimental tool to measure bin-bin correlations by means of the bunching-parameter approach [3–6]. In the following, we shall discuss the experimental advantages of using such a method (Section 2). As an illustration, the bin-bin correlation measurement by the lowest-order bunching correlator is given in Section 3.

2. Bunching correlators

One of the characteristic features of any local multiplicity fluctuations in high-energy physics is the existence of bin-bin correlations. If we have two non-overlapping bins, m and m' of size δ, then the discrete two-dimensional multiplicity distribution $P^{m,m'}_{n,n'}(\delta)$ having n and n' particles in bins m and m', respectively, cannot be factorized, having

$$P^{m,m'}_{n,n'}(\delta) \neq P^{m}_{n}(\delta) P^{m'}_{n'}(\delta)\,, \tag{1}$$

due to the existence of a bin-bin correlation between the bins m and m' [1].

A procedure for investigating such bin-bin correlations is to measure so-called factorial correlators [1, 7–9], (for a review see [10]). In terms of $P^{m,m'}_{n,n'}(\delta)$, P^{m}_{n}, and $P^{m'}_{n'}$, the factorial correlators for two bins of equal size δ can be written as

$$F^{m,m'}_{q,q'}(\delta) = \frac{\sum^{\infty}_{n,n'} P^{m,m'}_{n,n'}(\delta) n^{[q]}\, n'^{[q']}}{\left(\sum^{\infty}_{n=1} P^{m}_{n}(\delta) n^{[q]}\right)\left(\sum^{\infty}_{n'=1} P^{m'}_{n'}(\delta) n'^{[q']}\right)}, \quad q', q > 1\,, \tag{2}$$

where $n^{[q]} = n(n-1)\ldots(n-q+1)$. The quantity in the numerator is called the bivariate factorial moment. In contrast to the usual (univariate) factorial moment $\langle n^{[q]}\rangle = \sum^{\infty}_{n=1} P^{m}_{n}(\delta) n^{[q]}$, which characterizes only the local fluctuations in a single phase-space bin m, the bivariate factorial moment contains information on correlation between the local fluctuations in the two bins, m and m'.

If no correlation exists between bins m and m', we get $F^{m,m'}_{q,q'}(\delta) = 1$ due to factorization of the multiplicity distribution in the numerator of (2).

To increase the statistics, one can assume translational invariance and average (2) over all bin combinations with the same bin-bin distance, D. After symmetrization, one has

$$F_{q,q'}(D) = \frac{1}{2(M-k)} \sum^{M-k}_{m=1} \left(F^{m,m+k}_{q,q'}(\delta) + F^{m,m+k}_{q',q}(\delta)\right)\,, \tag{3}$$

where $M = \Delta/\delta$, Δ is a full phase-space interval, and $k = D/\delta$.

Correlators similar to (2) have also been proposed in [11]. In this approach, the bin of size δ is divided into two parts. If n_{L} and n_{R} are the

[1] Strictly speaking, any statistical dependence between these bins can lead to property (1).

number of particles in the left part and the right part of the bin, respectively, then one can define [11]

$$F_2(M) = \frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_{\rm L} n_{\rm R}\rangle}{\langle n_{\rm L}\rangle\langle n_{\rm R}\rangle}\,. \tag{4}$$

As is the case for the usual univariate factorial moment, the multivariate factorial moments presented above are sensitive to the "tail" of the multivariate multiplicity distribution obtained in an experiment. The limited statistics of an experiment reduce fluctuations measured by means of the high-order factorial moments because of the truncation of the multiplicity distribution [12–14]. This can exert a negative influence on the behavior of the factorial correlators.

We note another shortcoming of the factorial correlators. As the usual factorial moments, the multivariate definition selects only "spikes". Dynamical information from "dips", therefore, is completely lost. This means that we lose important information on bin-bin correlations. As an example, correlations should exist between different bins that contain no particles, *i.e.*,

$$P^{m,m'}_{0,0}(\delta) \neq P^m_0(\delta)P^{m'}_0(\delta)\,. \tag{5}$$

According to the definition, the factorial correlator is not able to measure such correlations.

The complete information on bin-bin correlations can be obtained, without the bias arising from restricted statistics of an experiment, if one formulates the problem in terms of the bunching parameters [3–6]. The univariate bunching parameters for bin m are defined in terms of the probabilities $P^m_n(\delta)$ as

$$\eta^m_q(\delta) = \frac{q}{q-1}\frac{P^m_q(\delta)P^m_{q-2}(\delta)}{(P^m_{q-1}(\delta))^2}\,. \tag{6}$$

Accordingly, it is possible to construct bivariate bunching parameters in the same way as that done for bivariate factorial moments,

$$\eta^{m,m'}_{q,q'}(\delta) = \frac{qq'}{(q-1)(q'-1)}\frac{P^{m,m'}_{q,q'}(\delta)P^{m,m'}_{(q-2),(q'-2)}(\delta)}{\left(P^{m,m'}_{(q-1),(q'-1)}(\delta)\right)^2},\quad q,q'>1\,. \tag{7}$$

The relation of BPs with usual moments have been found in [3–5]. For bivariate BPs, such a kind of relation can be written as

$$\eta_{q,q'}^{m,m'}(\delta) \simeq \frac{\langle \rho_{m,m'}^{q,q'} \rangle \langle \rho_{m,m'}^{q-2,q'-2} \rangle}{\langle \rho_{m,m'}^{q-1,q'-1} \rangle^2}, \qquad \delta \to 0 \tag{8}$$

due to the suppression of Poissonian noise in the limit of small δ.

As is the case for multi-dimensional probabilities, these quantities can be expressed as

$$\eta_{q,q'}^{m,m'}(\delta) = \eta_q^m(\delta)\eta_{q'/q}^{m'}(\delta) = \eta_{q'}^{m'}(\delta)\eta_{q/q'}^{m}(\delta)\,, \tag{9}$$

where $\eta_q^m(\delta)$ is the usual univariate bunching parameter and $\eta_{q'/q}^{m'}(\delta)$ represents a conditional bunching parameter for bin m' constructed from conditional probabilities, *i.e.*, the probability to observe q' particles in bin m' under the condition that q particles have been found in another bin m. Then, the conditional BPs have the form

$$\eta_{q'/q}^{m'}(\delta) = \frac{q'}{(q'-1)} \frac{P_{q'/q}^{m'}(\delta) P_{(q'-2)/(q-2)}^{m'}(\delta)}{\left(P_{(q'-1)/(q-1)}^{m'}(\delta)\right)^2}, \qquad q, q' > 1\,. \tag{10}$$

If the two bins are statistically independent, then the bivariate bunching parameters factorize:

$$\eta_{q,q'}^{m,m'}(\delta) = \eta_q^m(\delta)\eta_{q'}^{m'}(\delta)\,. \tag{11}$$

By analogy with the factorial correlators, the bunching correlators can, therefore, be defined as

$$\breve{\eta}_{q,q'}^{m,m'}(\delta) = \frac{\eta_{q,q'}^{m,m'}(\delta)}{\eta_q^m(\delta)\eta_{q'}^{m'}(\delta)}\,. \tag{12}$$

As is the case for (2), this definition grants unity if the cells m and m' are statistically independent.

The bunching correlators, in general, are not symmetric in q and q'. As is performed in (3), we can symmetrize this definition:

$$[\breve{\eta}_{q,q'}^{m,m'}(\delta)]_{\mathrm{S}} = \tfrac{1}{2}(\breve{\eta}_{q,q'}^{m,m'}(\delta) + \breve{\eta}_{q',q}^{m,m'}(\delta))\,. \tag{13}$$

Defining the distance D between two bins, the bunching correlators can further be averaged over many pairs of equidistant bins. In analogy to

(3), the problem of bin-bin correlations can be formulated in terms of the bunching correlators

$$\eta_{q,q'}(D) = \frac{1}{(M-k)} \sum_{m=1}^{M-k} [\breve{\eta}_{q,q'}^{m,m+k}(\delta)]_S \tag{14}$$

and their behavior in the limit $D \to 0$.

According to the above definition of bunching correlators, the second-order bunching correlator contains important extra information *on empty bin-bin correlation* that cannot be extracted by means of factorial correlators. Indeed, if such correlations exist, then, due to (5), one obtains

$$\breve{\eta}_{q,q'}^{m,m'}(\delta) \neq 1 \tag{15}$$

for any combination such as $\{2,2\}$, $\{2,3\}$, $\{3,2\}$ *etc.* For the symmetrized and averaged bunching correlators, this leads to

$$\eta_{q,q'}(D) \neq 1, \qquad q = 2, \quad q' = 2, 3 \ldots . \tag{16}$$

On the other hand, if only such (hypothetical) correlations exist, the factorial correlators are equal to one for any higher rank.

3. The lowest-order bunching correlator and its behavior

The value of $\eta_{2,2'}(D)$ is affected by events having no particles in both bins and, hence, it incorporates the empty bin-bin correlations that cannot be measured by means of factorial correlators. In this section we shall illustrate the dependence of this quantity on the distance D between the two bins.

For our numerical calculations, we can rewrite the definition of $\eta_{2,2'}(D)$ as follows:

$$\eta_{2,2'}(D) = \frac{1}{M-k} \sum_{m=1}^{M-k} \breve{\eta}_{2,2'}^{m,m+k}(\delta), \tag{17}$$

$$\breve{\eta}_{2,2'}^{m,m'}(\delta) = \frac{\eta_{2,2'}^{m,m'}(\delta)}{\eta_2^m(\delta)\eta_{2'}^{m'}(\delta)}. \tag{18}$$

To define bivariate and univariate BPs, we introduce the following expression as an indicator for the presence of a given spike configuration for a given experimental event t:

$$W_q(m,m',t) = \begin{cases} 1, & \text{if both bins } m \text{ and } m' \text{ contain } q \text{ particles,} \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

Then, we have

$$\eta_2^m(\delta) = 2\frac{\overline{W}_2(m,m)\overline{W}_0(m,m)}{\overline{W}_1^2(m,m)}\,, \tag{20}$$

$$\eta_{2,2'}^{m,m'}(\delta) = 4\frac{\overline{W}_2(m,m')\overline{W}_0(m,m')}{\overline{W}_1^2(m,m')}\,, \tag{21}$$

where $\overline{W}_q(m,m')$ is the average of $W_q(m,m',t)$ over N_{ev} experimental events

$$\overline{W}_q(m,m') = \frac{\sum_{t=1}^{N_{\mathrm{ev}}} W_q(m,m',t)}{N_{\mathrm{ev}}}\,. \tag{22}$$

An exact calculation of the statistical error (standard deviation) is always a complex task and requires special attention to any local measurement. Below, we give a sketch of propagation of the standard deviation for (17).

The square of the standard deviation for $\overline{W}_q(m,m')$ is given by

$$S_q^2(m,m') = \frac{1}{N_{\mathrm{ev}}(N_{\mathrm{ev}}-1)}\left[\sum_{t=1}^{N_{\mathrm{ev}}} W_q^2(m,m',t) - N_{\mathrm{ev}}\overline{W}_q^2(m,m')\right]\,. \tag{23}$$

The square of the standard deviation for second-order BPs is given by

$$V_2^2(m,m') = \frac{\overline{W}_0^2}{\overline{W}_1^4}s_2^2 + \frac{4\overline{W}_2^2\overline{W}_0^2}{\overline{W}_1^6}s_1^2 + \frac{\overline{W}_2^2}{\overline{W}_1^4}s_0^2\,. \tag{24}$$

This expression gives us the square of the standard deviation for univariate BPs if

$$\overline{W}_q = \overline{W}_q(m,m), \qquad s_q^2 = 4S_q^2(m,m)\,. \tag{25}$$

The square of the standard deviation for bivariate BPs can be found from (24) if

$$\overline{W}_q = \overline{W}_q(m,m'), \qquad s_q^2 = 16S_q^2(m,m')\,. \tag{26}$$

The total statistical error for (17) can be found by combining the standard deviations for the univariate and bivariate BPs and averaging the results over all bin pairs.

In Fig. 1a, the behavior of $\eta_{2,2'}(D)$ is shown for the case of purely statistical phase-space fluctuations. For our numerical calculations, we simulate the phase-space distribution by a pseudo-random number generator in the "phase space" $0 < x < 1$. The total number of events is 30,000. In this figure we consider the cases in which a total number of particles N in

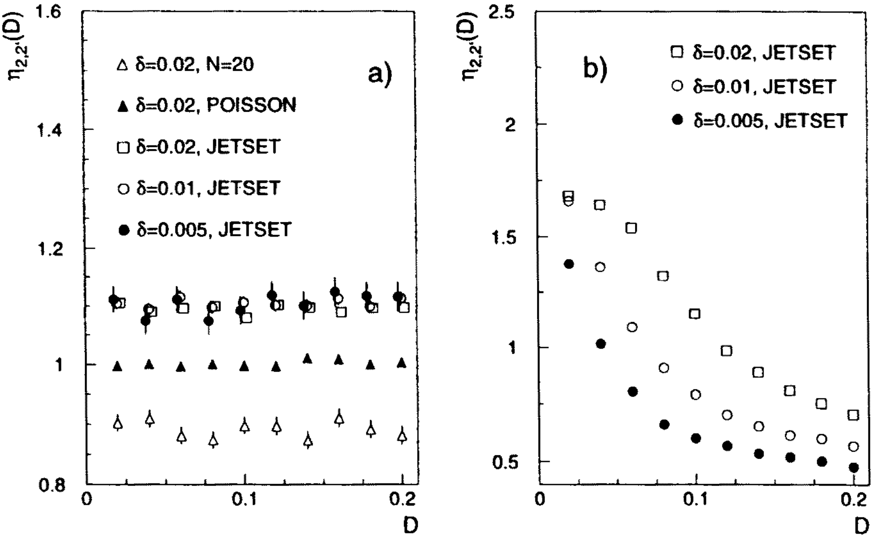

Fig. 1. Value of $\eta_{2,2'}(D)$ as a function of distance D between bins. (a) — the behavior in the case of purely statistical fluctuations for different distributions of particles in full phase space. (b) — the behavior for the case of dynamical fluctuations (phase-space distribution in azimuthal variable) simulated by the JETSET 7.4 PS model.

full phase space fluctuates according to full-phase-space fluctuations. We considered the following cases:

1. N is fixed for all events (N=20);
2. N is distributed according to a Poissonian law with mean $\overline{N} = 20$;
3. N is distributed according to the JETSET 7.4 PS model [15] simulating e^+e^--annihilation at a c.m. energy of 91.2 GeV. Such a distribution is similar to a negative binomial. For this case, we also consider different values of bin size δ.

As expected, the value of the bunching correlator is equal to 1 for the Poisson distribution. We have verified that this result is independent of the mean of the Poisson distribution and of the bin size δ.

For the sample with fixed multiplicity ($N = 20$), there is a negative correlation, since $\eta_{2,2'}(D) < 1$. This kind of correlation is due to the trivial effect that the probability of finding a particle in a bin is always less if another particle has already been found in another bin. In the case of no dynamical phase-space correlations, such a negative (pseudo) correlation leads to a D-independent bunching correlator of value smaller than unity.

If particles are distributed according to a distribution broader than Poisson, one should expect a positive correlation. For the case of no phase-space correlations, this again leads to a D-independent bunching correlator, but with a value of $\eta_{2,2'}(D) > 1$.

In Fig. 1b we present $\eta_{2,2'}(D)$ for a more realistic situation. Here, N again fluctuates according to JETSET 7.4 PS, but the phase-space distribution is defined in the azimuthal angle with respect to the e^+e^- collision axis. To compare the results with the previous cases, this variable (with full phase-space range 2π) has been transformed to a new variable with unit range. Due to the jet structure of single events, the phase-space distribution in this variable contains dynamical fluctuations. As can be seen from Fig. 1b, such fluctuations lead to a bin-bin correlation. The correlation increases for decreasing distance D, from $\eta_{2,2'}(D) < 1$ for large D to $\eta_{2,2'}(D) > 1$ for small D. Moreover, in contrast to Fig. 1a, the value of $\eta_{2,2'}(D)$ is affected by the value of the bin size δ.

4. Conclusions

In this paper, the bunching-parameter method has been extended to measure bin-bin correlations. This application of the bunching-parameter method has been achieved by considering bunching correlators in analogy to factorial correlators. The method not only allows one to study fluctuations inside a phase-space bin without experimental bias from finite statistics, but also to study correlations between bins separated in phase-space.

One of the remarkable features of the bin-bin correlation study is that the main properties of local fluctuations inside bins, and correlations between the bins can be formulated in a unified manner. Based on our analysis of second-order bunching correlations and on [5], we conclude:

1. For purely statistical phase-space fluctuations, the values of the univariate bunching parameters and those of the bunching correlators are independent of bin size and bin-bin distance. These values are affected by event-to-event multiplicity fluctuations, but are equal to unity for Poisson-distributed particle multiplicity in full phase space;
2. For dynamical phase-space fluctuations, the values of univariate bunching parameters, and bunching correlators increase for decreasing bin size δ or distance D between two bins.

Such a similarity in the behavior of these quantities is the result of an intrinsic relation between fluctuation and correlation properties of the local fluctuations.

Finally, from our study, let us note that no universal scaling relation between the local fluctuations and correlations is observed for the azimuthal-angle distribution in JETSET 7.4 PS model, as it follows from the random-cascade model [1, 2], for which the factorial correlators are δ-independent. The analysis of bin-bin correlations based on the bunching correlators clearly shows that the behavior of the second-order correlator is affected by the bin size δ. In fact, this means that realistic intermittent fluctuations cannot be fully described by the scaling indices of the univariate normalized moments as is the case for the random-cascade model. For this reason, the experimental measurement of the correlators is an important complementary part of the fluctuation analysis, which, therefore, cannot be reduced to the investigation of the scaling indices of the local quantities only.

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