THE SOFT POMERON AND HERA

A. DONNACHIE

Department of Physics and Astronomy, University of Manchester
Manchester, M13 9PL, UK

AND

P.V. LANDSHOFF

DAMTP, University of Cambridge
Cambridge CB3 9EW, UK

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Dedicated to Andrzej Białas in honour of his 60th birthday

The standard phenomenology of the soft pomeron in hadron-hadron interactions is recalled briefly. The model is confronted with the HERA data for the total photoproduction cross section, deep inelastic scattering, diffractive vector meson photoproduction and diffractive electroproduction of vector mesons. Although much of the data can be explained by the model, there are some aspects of the HERA data which require a more rapid variation with energy than can be incorporated. It is argued that the perturbative (BFKL) pomeron cannot give a sufficiently large contribution to explain these observations. Possible nonperturbative solutions to this problem are indicated.

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Regge theory [1] tells us that the amplitude for a high-energy hadronic elastic scattering process at a centre-of-mass energy $\sqrt{s}$ and momentum transfer t is a sum of terms of the form $T(s,t) \sim \beta(t)\xi_\alpha(t)s^{\alpha(t)}$. Here $\beta(t)$ is an unknown real function of t, and $\xi_\alpha(t)$ is a definite phase calculated from the Regge trajectory $\alpha(t)$. The contribution of each term to the total cross section is given by $\sigma_T = (1/s)\mathrm{Im}\, T_\alpha(s, t = 0) \sim s^{\alpha(0)-1}$. There are two principal contributions to the total cross section. One is from the ρ,

ω, f, a families with $\alpha_R(0) \sim \frac{1}{2}$ so that their contribution to σ_T has a behaviour close to $s^{-\frac{1}{2}}$. The other is the soft pomeron, with a trajectory $\alpha_P(t) = 1 + \varepsilon + \alpha' t$ and with $\varepsilon > 0$, giving a contribution to $\sigma_T \sim s^{\varepsilon}$.

The precise value of $\alpha_R(0)$ is 0.5475 and fitting hadronic total cross sections with

$$\sigma_T = X s^{\alpha_R(0)-1} + Y s^{\varepsilon} \tag{1}$$

yields $\varepsilon = 0.0808$ [2]. The form of Eq.(1) with *universal* constants is applicable to *all* hadronic total cross sections. Examples are shown in Fig. 1. Note that the fits to the total cross sections are *not* unique but that fixing $\alpha_R(0)$ does strongly constrain ε, assuming it to be essentially constant.

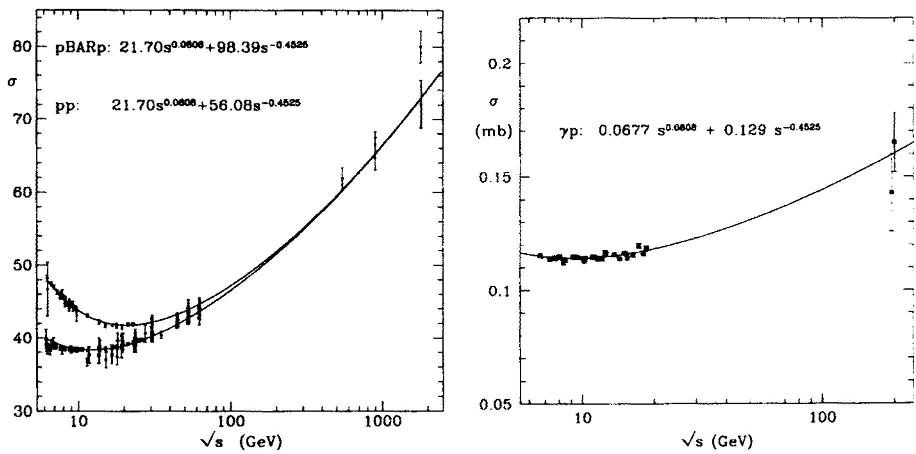

Fig. 1. Total cross sections, with fits of the form (1).

It should also be noted that ε represents an *effective* power. It is not sufficient to consider the exchange of a single pomeron: multiple exchanges should be taken into account. Their magnitude is *a priori* incalculable, but phenomenologically their contribution to the total cross section appears to be small [3]. Specifically, we make the two-pomeron-exchange amplitude differ from the eikonal approximation by the introduction of a real multiplicative parameter λ less than 1. The value of λ was chosen so as to make the imaginary part of the amplitude vanish at the position of the dip observed in pp scattering at $\sqrt{s} = 31$ GeV, which gives $\lambda = 0.43$, and a two pomeron exchange contribution to the forward amplitude of only a few percent at Tevatron energies. This "weak absorption" conclusion is compatible with the use of $\alpha_R(0)$ in Eq. (1): if absorption were strong, then this would be significantly modified.

The fit to all the hadronic total cross section data is excellent, with one exception. This is the upper of the two Tevatron data points for the $\bar{p}p$ cross section, shown in Fig. 1. The CDF [4] value is $\sigma_T = 80.03 \pm 2.24$ mb, compared to the predicted value of 73 mb (which does agree with the earlier E710 result [5] also shown in the figure). Multiple exchanges (absorption) produce an effective ε which *decreases* with increasing energy, and so the model presented here cannot be readily reconciled with the CDF result.

Pomeron exchange is relevant for many aspects of HERA data. The most straightforward is the application in Fig. 1 of (1) to the total photoproduction cross section. The HERA data on the total γp cross section [6], which appeared subsequent to the fits at fixed target energies, were predicted correctly by the model. The model prediction of 157μb is to be compared with the most recent H1 value of $165 \pm 2 \pm 11\mu$b.

Pomeron exchange is equally applicable to high energy forward differential cross sections in both hadron and photon interactions. For example, a calculation of diffractive ρ^0 photoproduction can be obtained by assuming vector meson dominance [7] and quark additivity [8]. The simplest version of vector meson dominance tells us that the forward cross section for $\gamma p \to \rho p$ is proportional to that for elastic $\rho^0 p$ scattering, the constant of proportionality being $4\pi\alpha/\gamma_\rho^2$ where $4\pi/\gamma_\rho^2$ is the ρ-photon coupling as found from the e^+e^- decay width of the ρ. According to quark additivity the forward amplitude for $\rho^0 p \to \rho^0 p$ is simply the average of the forward amplitudes for $\pi^+ p \to \pi^+ p$ and $\pi^- p \to \pi^- p$, which are known from the total cross section fits and the defined Regge phases. The cross section predicted [9] in this parameter-free approach has the correct energy dependence but the normalisation is too high: a multiplicative factor of 0.84 is required, when it then is compatible with the data from $\sqrt{s} = 4$ GeV to $\sqrt{s} = 70$ GeV, where it is in excellent accord with the HERA measurements [10]. Reasons for this normalisation difference are not hard to find: finite-width corrections to the $\rho \to e^+e^-$ decay rate [11]; deviations from exact quark additivity which is only accurate, in the pomeron dominated sector, to about 5-10% [2]; and the intrinsic uncertainties in the application of naive VMD.

The same approach can be applied to ϕ photoproduction, which is pomeron dominated even at low energies because of Zweig's rule. The effective coupling of the pomeron to the strange quark is weaker than to the light quarks, only 73% on the basis of quark additivity and the smaller pomeron-exchange contribution to the $K^\pm p$ total cross section compared to the $\pi^\pm p$ cross section. The resulting factor of 0.53 in the ϕ photoproduction cross section still leaves the predicted cross section twice the observed one. So instead of the correction factor of 0.84 which is required for the ρ^0, the required factor for ϕ photoproduction is close to $\frac{1}{2}$ [12]. Invoking $\omega - \phi$ mixing through the 3π channel [13] decreases the cross section only by about 12% and so alternative explanations have to be found. One is that vector dominance becomes less reliable as the mass of the vector meson increases. Another is that the problem is due to specific wave-function effects which are associated with the ϕ having a small radius [14] and which should disappear at large Q^2. The energy dependence of the ϕ photoproduction cross section is entirely compatible with the canonical $s^{2\epsilon}$ expected from soft pomeron exchange, from $\sqrt{s} = 2.0$ GeV to HERA energies [15].

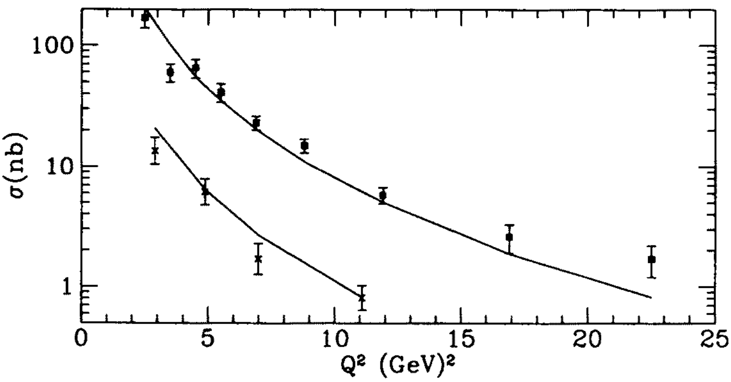

Fig. 2. NMC data [18] for $\gamma^* p \to \rho p$ and ϕp with predictions from reference [17].

For photons at large Q^2 vector meson dominance is no longer relevant, but one can still apply the same basic concepts of pomeron exchange [24] to exclusive vector meson production. The virtual photon dissociates into a $q\bar{q}$ pair, one on-shell and one off-shell, and the latter scatters diffractively thereby going on-shell and allowing the two quarks to recombine to form a vector meson. For ρ electroproduction [17] this works remarkably well at fixed-target energies [18] giving the correct normalisation, the correct Q^2-dependence, the correct t-dependence and the correct ρ-alignment. The calculation of elastic ϕ electroproduction is equally successful at fixed target energies, allowing only for the smaller effective coupling of the pomeron to the strange quark *i.e.*the factor 0.53 discussed above for ϕ photoproduction is retained. However, the additional factor of $\frac{1}{2}$ required for ϕ photoproduction is no longer necessary. Once again the normalisation and the Q^2 dependence are predicted correctly. The absence of wave-function effects is even more dramatic in the case of the J/ψ. The model [17] predicts that at sufficiently large Q^2 the J/ψ electroproduction cross section should be comparable to that of the ρ, which seems to be in accord with EMC data [19].

One can also apply pomeron exchange to deep inelastic scattering. At small x and small Q^2 the structure function νW_2 is governed by Regge theory giving a sum of terms $(1/x)^{\alpha(0)-1}$ [20]. The NMC data [21] at moderate Q^2 and not-too-small x show that νW_2 contains such Regge terms at small x: a slowly varying term from soft pomeron exchange and close to $\sqrt{s}$ from f, a exchange. In the spirit of the fits to the total cross section data, the simplest fit to the small-x structure function analogous to Eq. (1) is:

$$\nu W_2 = X x^{-0.0808} \left[\frac{Q^2}{Q^2+a}\right]^{1.0808} + Y x^{0.4525} \left[\frac{Q^2}{Q^2+b}\right]^{0.5475} \tag{2}$$

the Q^2 dependent terms ensuring that νW_2 vanishes linearly with Q^2 at $Q^2 = 0$.

The original fit [22] was made to the small-x NMC data [21] up to $Q^2 = 10$ GeV^2. It is only a two-parameter fit, as for each choice of X and Y the parameters a and b are determined so as to reproduce the fit to the γp total cross section. The predictions of the fit are in remarkably good accord with the subsequent E665 data [23] over the same Q^2 range but at smaller values of x than used in the original fit.

The process of diffractive dissociation in hadron-hadron collisions is one in which one of the incoming hadrons emerges with only a very small change of momentum. a small fraction ξ of its initial momentum is carried off by a pomeron, which collides with the other hadron, so effectively one can study pomeron-hadron collisions. The cross section for diffractive dissociation in p-p collisions is of the form [24].

$$\frac{d^2\sigma}{dt\,dx_P} = F_{P/p}(t, x_P)\sigma^{Pp}\,, \tag{3}$$

where t is the momentum transfer from the initial to the final fast proton, x_P is the fraction of the proton's momentum taken by the pomeron and $F_{P/p}(t, x_P)$ is the amplitude for the proton to have "radiated" a pomeron:

$$F_{P/p} = \frac{9\beta^2}{4\pi^2}[F_1(t)]^2 x_P^{1-2\alpha(t)}\,, \tag{4}$$

with $\beta^2 = 3.5$ GeV^2 and $F_1(t)$ the electromagnetic form-factor of the proton. The quantity σ^{Pp} is the cross section for the pomeron interacting with the other proton, and the invariant mass squared of the pomeron-proton system is $M_X^2 = x_P s$.

As the soft pomeron is described by a regge trajectory it is in some sense particle-like, and Ingelman and Schlein introduced the concept of the pomeron structure function and hard diffraction [25] and suggested that the partonic structure of the pomeron could be established by studying the pomeron-proton system for evidence of hard scattering.This was verified by the UA8 experiment at the CERN collider, which found [26] events in which the proton (or antiproton) scattered diffractively containing jets with $p_T > 8$ GeV/c. The relevant process is one in which a parton from the pomeron participates in the hard scattering that produces the high-p_T jets and the other is a longitudinal spectator. The UA8 data cannot determine whether the pomeron structure function is quark or gluon (or a combination of both) but they do indicate that the structure function is hard.

The quark structure function of the pomeron, $F_{q/P}$, can be measured directly in deep inelastic scattering from the subset of events in which the proton scatters diffractively. There it is defined by the relation

$$x\frac{d^3q}{dtdx_PdQ^2} = F_{q/P}(t,\beta,Q^2)F_{P/p}(t,x_P)\,, \tag{5}$$

where $F_{P/p}$ is the pomeron flux factor of (4) and $\beta = x/x_P$. Away from small x, the quark structure function of the soft pomeron, for each light quark or antiquark, is found to be [24].

$$\beta q_P(\beta) \sim 0.075\pi\beta(1-\beta)\,. \tag{6}$$

There are additional t-dependent terms, which however are important only at small x [24]. Neglecting these t-dependent terms in the structure function, (5) can be integrated over t to obtain the x_P dependence. To a good approximation $\alpha(t)$ in (4) may be replaced by $\alpha_{t=0}$ because of the peripheral nature of diffraction. The dependence on x_P is thus approximately $1/x_P^{1.16}$

Initial data from H1 [27] and ZEUS [28] are in qualitative accord with these predictions. The data are consistent with factorisation, compatible with the form of (5) for the pomeron structure function apart from an additional component at small x, and have a $1/\xi$ dependence close to that expected. Specifically for the latter H1 [27] find a power $1.19 \pm 0.06 \pm 0.07$ and ZEUS [28] find $1.30 \pm 0.08 \pm 0.11$. The soft-pomeron approach is, unfortunately, not unique in its compatibility with these data and using perturbative QCD [29] or treating the pomeron as if it were genuinely a hadron [30] are equally successful.

Although the soft pomeron can explain a great variety of data there are some with which there is a significant discrepancy, and a more rapid variation with energy is found. These are data for the small-x behaviour of F_2, which behaves [31] more like $f(Q^2)(W^2)^{0.3}$ than $f(Q^2)(W^2)^{0.08}$, and exclusive J/ψ photoproduction and electroproduction [32], which again behave more like $[f(Q^2)(W^2)^{0.3}]^2$. The same may be true of both ρ and ϕ

electroproduction, though the two HERA experiments are not in agreement here [33]. To these must be added the possible faster rise than $s^{0.08}$ in the total $p\bar{p}$ cross section [4] referred to earlier, where again there is disagreement between the two experiments [4, 5]. It is not yet clear what is the cause of this more violent variation with energy. Are these deviations from straightforward soft pomeron phenomenology pointing to the precocious emergence of aspects of perturbative QCD or do they have a non-perturbative explanation *e.g.* in different manifestations of absorptive effects or as a consequence of the effects of generalised vector dominance?

Certainly a candidate explanation for these more rapid variations with energy is that the perturbative BFKL pomeron is responsible [34]. However it does appear [35] that while the power of s predicted by the BFKL equation can fit the observed behaviour, the magnitude of the constant that multiplies it is almost certainly much too small.

An unambiguous calculation of this magnitude is not possible because one cannot cleanly separate the perturbative and nonperturbative effects. This problem arises already in lowest order of simple two-gluon exchange. At large s, with perturbative gluons, this gives a constant cross-section:

$$\sigma_0 = \frac{8}{9} \int d^2 k_T \, \frac{\alpha_s^2}{k_T^4} \, . \tag{7}$$

Here k_T is the transverse momentum of the internal quark lines, which correspond to final-state jets. It is unsafe to use this perturbative formula for the production of quark jets with too low a transverse momentum, $k_T^2 < \mu^2$, with μ expected to be of the order of a GeV, as then the integral extends into the region where it is nonperturbative. Excluding this nonperturbative region from the integration in (7) gives a quark-quark cross-section of $1.1\alpha_s$ mb, which for any reasonable perturbative value of α_s is considerably less than the observed cross-section of a few mb. Thus if the lowest-order calculation is a good guide most of the cross-section comes from the nonperturbative region. Cudell, Donnachie and Landshoff [35] (CDL) argue that a similar result holds when higher orders are included, and that it applies not only to purely soft processes like total hadron-hadron cross sections, but also to semihard ones such as exclusive vector production or the small-x behaviour of structure functions.

In its simplest form, the BFKL equation describes asymptotically large energies, where energy conservation constraints have become unimportant. Hitherto attempts [36] to impose energy conservation have been unsatisfactory in two respects. The BFKL equation takes an input amplitude Im $T_0(s)$ and modifies it by real and virtual gluon corrections. These two types of correction need to be handled differently, since only the real gluons are constrained by energy conservation. Also, energy conservation restricts the *sum* of the transverse energies of all the real gluons to be less than $\sqrt{s}$, while previous work has imposed this constraint on just their individual energies.

Energy conservation imposes a cut-off at the high-momentum end of the loop integrations in the BFKL equation. The low-momentum end also needs attention, since the BFKL equation works with perturbative gluon propagators. Because of confinement effects, at small k^2 the gluon propagator receives very significant nonperturbative corrections [37] so that, even though the BFKL equation has a finite solution with a purely perturbative propagator, this solution makes no physical sense. There have been several attempts to take this into account, none of them very satisfactory [36, 38]. They either simply exclude the low-k^2 part of the loop integration, or they try to use a nonperturbative propagator at low k^2 (so raising the question whether the BFKL equation itself, and not just the gluon propagator, must not also be modified, and also encountering major gauge invariance problems), or they use a nonperturbative input amplitude T_0 (which can be only part of the solution).

CDL initially impose a lower cut-off μ on the transverse momenta of the real gluons. That is, at first they calculate only a small part $\sigma(K_T > \mu)$ of the total cross-section for quark-quark scattering, arising from events where the final state consists only of any number of minijets of transverse momentum greater than μ. They impose the conditions that each parton has transverse momentum at least equal to μ and that the total transverse energy of the partons is no more than $\sqrt{s}$. There is also a limit on the total energy of the real partons but of course this cannot be applied to the virtual insertions so the real and virtual contributions have to be treated separately. It turns out that this is also necessary for practical computational reasons and it introduces a parameter $C_{\rm eff}$ which is a measure of the effect of the virtual insertions. The result for that part of the cross section where the final state contains only partons with transverse momentum greater than μ is

$$\sigma(s|K_T > \mu) = \frac{i\pi}{81}\bar{s}^{-C_{\rm eff}} \int dc d^2b \frac{e^{-ic\sqrt{s}} - 1}{c} [I(b,c)]^2 \bar{s}^{I(b,c)} , \qquad (8)$$

with

$$I(b,c) = \frac{6}{\pi} \int_\mu dK \frac{\alpha_s(K) J_0(bK)}{K} e^{icK} . \qquad (9)$$

The quantities $C_{\rm eff}$ and μ are effectively two parameters in the calculation. $C_{\rm eff}$ is constrained by the theory to be less than 1.44, and on the basis of rather general physics arguments CDL work with $C_{\rm eff} = 1$. Any higher value would, of course, reduce the output.

The resulting cross-section for quark-quark scattering is shown in Fig. 3. According to the additive-quark rule, it must be multiplied by 9 to give the contribution to the pp or $\bar{p}p$ total cross-section. Account has also to be taken

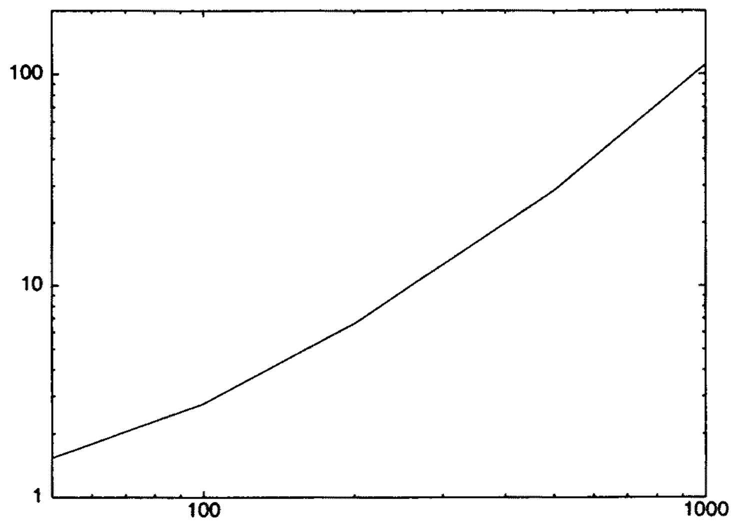

Fig. 3. $\sigma(s|K_T > \mu)$ from (8) with $C_{\rm eff} = 1$ and $\mu = 2$ GeV.

of the fact that it is extremely rare that *all* the partons will have $K_T > \mu$. In a general event the final-state partons may be grouped according to their rapidities. As there is no transverse-momentum ordering, their transverse momentum is not correlated with their rapidity. So as the rapidity range is scanned groups of partons will be found all having transverse momentum greater than μ, with each such group separated by a group in which none of the partons have transverse momentum greater than μ. Summing over all partons in a group with $K_T < \mu$ gives the hard (BFKL) pomeron. Summing over all partons in a group with $K_T < \mu$ gives a soft exchange. Summing over all final states then gives a sum $S + H + S \otimes H + H \otimes S + S \otimes H \otimes S +$ ...... ; see Fig. 4. The soft contribution can be approximated by soft-pomeron exchange as the bare hard contribution has to be small to be consistent with data. This mixing in of contributions from soft interactions does provide some enhancement of the contribution from the hard ones, but it is at the very most an order of magnitude. By comparing their calculation with pp and $p\bar{p}$ total cross section data CDL show that it is unsafe to take μ to be less than 2 GeV.

While one does not expect that a "soft" process such as the total cross section for quark-quark scattering should receive much contribution from states containing only minijets having transverse momentum greater than 2 GeV, for semihard processes things might be different [39]. However CDL argue that, although it may be true that soft contributions are relatively suppressed in such processes, the hard contribution is still tiny. That is, the fact that the hard contribution may be essentially the only contribution does not make it any larger than when it is not. As an explicit example they consider ρ electroproduction, where their conjecture is substantiated by explicit calculation. Even in a purely hard process, such as $\gamma^*\gamma^* \to \rho\rho$, the perturbative contribution is found to be appreciably smaller than the soft contribution until quite large values of Q^2 are reached, although the enhanced sum does become comparable at significantly lower values of Q^2.

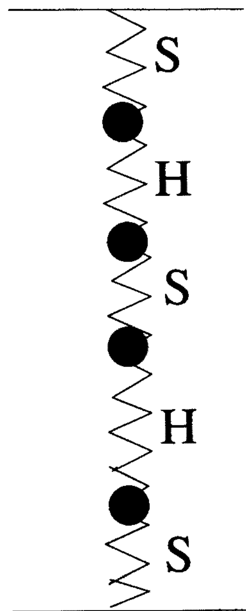

Fig. 4. Alternating soft and hard exchanges: the term $S \otimes H \otimes S \otimes H \otimes S$.

In applying the analysis to semihard and hard processes, CDL used experimental information from soft processes, namely that the pp and $p\bar{p}$ total cross sections have at most only a very small very-rapidly-rising component. Although nobody expects the perturbative contribution to such soft processes to form a significant fraction of the total, it should nevertheless be present and it is important to use experimental information to set limits on it and thereby reduce the uncertainties about the corresponding contributions to semihard and hard processes. The pp and $p\bar{p}$ total cross section data show that any hard pomeron contribution is no more than 10% at $\sqrt{s} = 1800$ GeV and beause it falls so rapidly as $\sqrt{s}$ is decerased it becomes negligibly small at HERA energies, even in semihard and hard processes.

CDL consider the BFKL pomeron only at $t = 0$. It remains possible that at large t the situation will be different since then, even if the BFKL contribution is small, there is much less competition from nonperturbative mechanisms.

As the discussion of the BFKL pomeron inevitably requires consideration of nonperturbative contributions, it must involve considerable uncertainty. The CDL approach, when faced with decisions where there is theoretical uncertainty, is to maximise the cross section within the constraints coming from HERA and the Tevatron. One issue is the value chosen for $C_{\mathbf{eff}}$, which was taken to be 1.0. With this choice the perturbative contribution $\gamma^* q \to \rho q$ is totally negligible at HERA energies, even allowing for the enhanced corrections. To make it comparable to with the soft contribution (and hence with data) requires that $C_{\mathbf{eff}}$ be reduced to 0. This is not reasonable as it corresponds to an absence of virtual corrections. Further its x dependence would be totally wrong. The choice $C_{\mathbf{eff}} = 1$ gives an effective power $(1/x)^{0.3}$; changing $C_{\mathbf{eff}}$ to 0 would make this $(1/x)^{1.3}$.

CDL argue that the BFKL pomeron is not detectable, at least at $t = 0$. This means that some other explanation must be found for the rapid rise in the HERA data for J/ψ photo and electroproduction and for F_2 at small x, and this explanation must be nonperturbative in origin.

One possibility is to suggest that the weak rescattering assumption is incorrect. In a strong absorption model, it is possible to choose $\varepsilon \sim 0.3$ for the bare pomeron and reduce this to the effective ~ 0.1 by multiple rescattering, although for this to be convincing would require a successful reworking of all soft-pomeron phenomenology. In deep inelastic scattering, as Q^2 increases the interaction time decreases and so does the absorption, so the effective ε can increase from ~ 0.08 to ~ 0.3 as indicated by the data [40, 41]. An alternative non-perturbative approach is that of generalised vector dominance [42]. It is possible to describe the striking behaviour of F_2 at small x observed at HERA with contributions from heavy long-lived fluctuations of the incoming photon.

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