GRAVITATION, THE QUANTUM, AND COSMOLOGICAL CONSTANT

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Dedicated to Andrzej Białas in honour of his 60th birthday

The arguments of statistical nature for the existence of constituents of active gravitational masses are presented. The present paper proposes a basis for microscopic theory of universal gravitation. Questions like the relation of cosmological constant and quantum theory, black hole radiance and the nature of inertia are addressed.

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It is well known that the classical gravitating systems behave in the way foreign to statistical quantum mechanics. The negative specific heat of those systems and the phenomenon of gravitational collapse are different facets of the same reality. The difficulties arise which necessitate that the complete atomic description of gravitation and space-time be sought after. The departure from Einstein's General Relativity Theory compatible with the statistical basis of the Universal Second Law of Thermodynamics takes the form of the mass-energy quantization condition. In this paper we show that a gravitating mass M in thermal equilibrium, sometimes called a black hole, behaves statistically like a system of some number N of harmonic oscillators [1, 2] whose zero-point energy $\varepsilon/2$ depends on N universally in such a way that

$$N\varepsilon^2 \sim \frac{hc^5}{G}\,, \tag{1}$$

where G is the Newton constant, c is the velocity of light in vacuum, and h is the Planck constant. The sum over all oscillators of the squares of zero-point energies is fixed and independent of the number of those oscillators.

Similarly, the sum over the fourth power of zero-point energies gives the vacuum energy density, the so-called cosmological constant λ. The cosmological constant is identified by the order of magnitude with the number of oscillators N in the Universe

$$\lambda \sim N\varepsilon^4 (hc)^{-3} \sim \frac{c^7}{G^2 hN} . \tag{2}$$

The lower bound for this total number of oscillators in the observed Universe is obtained from the present upper bound on λ, $N \sim 10^{122}$.

The system of N correlated oscillators [2, 3, 15], known macroscopically as a gravitating mass, at the temperature of the cosmic relic radiation $T \sim 2.7K^\circ$ has a total mass-energy exceeding the critical mass-energy of the observed Universe by $\sim$ 30 orders of magnitude. This suggests that the observed Universe is extremely 'young' and its age is only 10^{-15} times that of the time(-space) scale available to the whole Universe out there. Consider a system of N *gravitational atoms* or *parcels of mass-energy* [2, 3, 12, 15]. When N is very large we must apply statistical reasoning to describe the state of such a large system. The statistical quantum theory is a statement about the objective reality based on the *Atomic Hypothesis* [2, 3, 12, 15]. The statistical reasoning applied to the *simplest* properties of *gravitational atoms* leads to thermal properties of the mass-energy, which in the limit $N \to \infty$ converge to those already proposed earlier on the basis of the General Relativity Theory [4] and the Quantum Theory [5]. The enigmatic Bekenstein entropy of black holes [4] has not yet been derived on the basis of microscopic theory. Our work should be considered as a first step in the direction of establishing such a basis. The problem with previous approaches has been the silent assumption that the total entropy of black holes *must* be given by the Bekenstein formula [4],

$$S_{bh} = 4k\pi M^2 , \tag{3}$$

where S_{bh} is the entropy, k is the Boltzmann constant, and M is the mass-energy in the Planck units. The postulate of gravitational oscillators leads to Bekenstein's formula [4] only after a part of mass-energy fluctuations is neglected. We will show that neglecting $1/N$ terms in the mass-energy fluctuation formula of Einstein [1], when applied to the system consisting of N *gravitational atoms*, leads to the thermodynamics of black holes [4, 5, 3].

The principal conclusion from the *Atomic Hypothesis* is that the thermodynamical limit of infinitely large N at fixed mass-energy eliminates the additional source of mass-energy fluctuations which is responsible for the total positive heat capacity of gravitating masses. Neglecting this new kind of mass-energy fluctuations leads to one aspect of phenomena known macroscopically as the universal gravitation. This is much like neglecting the famous first term in the energy fluctuation formula for the black body radiation which was discovered by Einstein [1]. The true meaning of the first term, as we know today, is the corpuscular nature of electromagnetic phenomena. The second term in the fluctuation formula for black body radiation is the well known Rayleigh–Jeans term. Therefore, neglecting the first term in the energy fluctuation formula for the black body radiation amounts to an incomplete description of radiation as waves. *The direct implication of neglecting a* $1/N$ *term in the formula for the gravitational mass-energy fluctuations is that the specific heat is negative for gravitating systems.* This also means that *the gravitational collapse*, with its typical property of negative specific heat (the latter one is governing formation of stars, galaxies, clusters and superclusters of galaxies *etc.*), *can proceed only to the extent that the new source of mass-energy fluctuations is neglected completely.* If, however, the circumstances occur that those new mass-energy fluctuations cannot be neglected, then a completely new physical reality opens up. The collapse is avoided. We should expect a plethora of new phenomena where those new mass-energy fluctuations shall demonstrate themselves.

In this paper we shall describe some predictions about phenomena following from physical reasoning based on *Atomic Hypothesis.* One of them concerns the cosmological constant [6]. The paradox of large cosmological constant was noticed pretty late [7, 8]. It appears to be a paradox because the present theoretical models seem to have a limited scope of application. There is no doubt that the nonvanishing cosmological constant λ must be very small. The present upper bound on the value of this constant is an extremely small number in natural units. We now know [9] that λ cannot be larger than $10^{-122}\mu^4$, $\mu = \sqrt{\hbar c/G} = 5.46\ 10^{-5}$g.

The smallness of the cosmological constant in natural Planck units is a result of an almost perfect thermodynamical limit. This is to say that the smallness of the cosmological constant is an effect due to an enormous number N of hypothetical *gravitational atoms.* The present upper bound on the 'cosmological constant' λ allows us to draw the conclusion about the lower bound on a number of *gravitational atoms* in the observed Universe, $N \sim 10^{122}$. The large numbers' coincidences noticed long time ago [10, 11] have something in it after all. We have only one reservation to add: elementary particles are not the same as the hypothetical *gravitational atoms*, and, therefore, have little to do with the cosmological constant. The existence of the latter must be inferred indirectly from phenomena as it was previously done for atoms and elementary particles. We find an unusual coordination between the gravitational atomistic aspect of physical reality in the regime usually called infrared, or large distance scale, and the regime usually called ultraviolet, or short distance scale [17].

It is now time to turn to the elementary physical considerations which have led the present author to the general *Atomic Hypothesis* [2, 3, 12, 15, 18]. Consider N identical harmonic oscillators, characterized by the zero-point energy $\frac{1}{2}\varepsilon$. We assume that ε depends on N in such a way that when $N \to \infty$ then $\varepsilon(N) \to 0$. This assumption is logically simple, and also necessary because otherwise the generalized correspondence principle is violated [2, 3]. The thermal properties of a large gravitating mass would be in disagreement with the predictions following from General Relativity Theory and Quantum Mechanics [4, 5] unless the $N = \infty$ limit is taken in the formulas for the specific heat capacity with a fixed mass-energy. We assume that for large N the following formula is valid:

$$N\varepsilon^2 = b\mu^2 c^4 , \tag{4}$$

where $\mu = \sqrt{hc/G}$ is the Planck mass-energy. b is a numerical constant to be determined later from the correspondence principle considerations. This formula ought to be justified by the results and predictions following from it, in the same way the hypothesis of the universal law of gravitation was justified by the derivation of the Kepler laws. The mathematical theory of new wave equations for gravitating particles [2, 3, 12, 15, 18] leads to mass-energy quantization of the type assumed in this paper.

Consider now the partition function of N correlated harmonic oscillators

$$Z(N,\beta) = \left(2\sinh\frac{\beta\varepsilon'(N)}{2}\right)^{-N} , \tag{5}$$

where $\varepsilon' = \varepsilon - \chi$, and χ is the chemical potential.

In the following we assume $\varepsilon' > 0$. The average mass-energy of this system is

$$\overline{E} = U = N\varepsilon\left(\frac{1}{2} + \frac{1}{e^{\beta\varepsilon'} - 1}\right) , \tag{6}$$

$\beta = 1/kT$.

Now, the Einstein energy fluctuation formula [1],

$$\overline{(\Delta E)^2} = -\frac{\partial U}{\partial \beta} , \tag{7}$$

gives for our system the following expression

$$\overline{(\Delta E)^2} = N\varepsilon^2\left(\frac{1}{e^{\beta\varepsilon'} - 1} + \frac{1}{(e^{\beta\varepsilon'} - 1)^2}\right) . \tag{8}$$

The average mass-energy at zero temperature is

$$U_0 = \frac{N\varepsilon}{2} . \tag{9}$$

Defining the deviation of an average mass-energy U at a given temperature from its zero temperature value U_0 by

$$\Delta U = U - U_0 = 2U_0(e^{\beta\varepsilon'} - 1)^{-1}, \tag{10}$$

we see that the energy fluctuations are of two types: there are terms linear and quadratic in ΔU. The linear term is corresponding to the corpuscular character of quanta (it is of an order of $N^{-1/2}$), while the quadratic term corresponds to their wave character (it is of the order of N^{-1}). This formula has been well known for the last 90 years [1]. The gravitational mass-energy displays not only the corpuscular characteristic, as described by the General Theory of Relativity, but also the wave-like behaviour typical of wave mechanics. The conclusions we can draw from the first principles are twofold. The fact that the gravitational mass-energy coordinates all three fundamental categories of existence of an objective reality, time, space, and matter, leads us to suggest the following: In addition to *'geometric optics'*, or corpuscular, behaviour predicted by the General Theory of Relativity there exists a new *'wave optics'* regime of behaviour of space-time-matter. *Not only the mass-energy displays the wave-like property but also the space and time seem to behave in this way, as far as the statistical properties of gravitating systems are concerned.* How this fact coordinates with the observed properties of gravitating masses?

The most unusual character of the gravitational mass-energy oscillators is that they somehow manage, via the quadratic sum rule defining the Newton constant G,

$$\sum_i \varepsilon_i{}^2 = b\frac{hc^5}{G}, \tag{11}$$

to reduce their zero-point energy when the number N of them grows. This also means that a cold large gravitational mass $M \sim \mu\sqrt{N}$ consists of N *constituents.* The formula

$$M^2 = \frac{1}{2\pi}\mu^2 N \tag{12}$$

was derived long time ago by the present author [12]. The physical meaning of the 'phenomenological' entropy of Bekenstein [4] is that it is the measure of the number N of *constituents* making up a very cold large body. The more massive is a gravitating mass the softer are the *constituents* or *gravitational quanta.* Otherwise, as usual with oscillators, there are two sources of statistical fluctuations of mass-energy corresponding to the corpuscular and wave aspect of quanta. We calculate the mass-energy fluctuations of the system in terms of the average energy U,

$$\overline{(\Delta E)^2} = N^{-1}U^2 - \frac{1}{4}N\varepsilon^2 = N^{-1}U^2 - \frac{1}{4}b\mu^2c^4. \tag{13}$$

Neglecting the $\frac{1}{N}$ term in this formula, when U is fixed, we obtain the expression for statistical fluctuations typical of gravitating systems:

$$\overline{(\Delta E)^2}_{bh} = -\frac{1}{4} b\mu^2 c^4 \,. \tag{14}$$

The well known relation between the mass-energy fluctuations and the behaviour of entropy near the state of thermal equilibrium,

$$\overline{(\Delta E)^2} = -k\left(\frac{\partial^2 S}{\partial U^2}\right)^{-1}, \tag{15}$$

leads to the entropy of such a *truncated system*:

$$\frac{\partial^2 S_{bh}}{\partial U^2} = 4kb^{-1}\mu^{-2}c^{-4}. \tag{16}$$

Integrating this last equation gives the inverse temperature

$$\beta_{bh} = 4b^{-1}\mu^{-2}c^{-4}U \,, \tag{17}$$

where an arbitrary integration constant is fixed to be zero by demanding that a very massive body is also very cold [4, 5]. The entropy is given by the 'phenomenological' entropy formula of Bekenstein [4]:

$$S_{bh} = 2kb^{-1}\mu^{-2}c^{-4}U^2 + \text{const} \,. \tag{18}$$

The model calculation of Hawking [5] leads to a numerical value of the constant b, $b = \frac{1}{4\pi^2}$. Quite independently of the actual value of the numerical constant b the entropy S_{bh} has a lower bound

$$S_0 = 2kb^{-1}\mu^{-2}c^{-4}{U_0}^2 \,, \tag{19}$$

which depends only on N. This follows from the fact that the total mass-energy U is bounded from below by the zero temperature value U_0, $U \geq U_0$. Now,

$${U_0}^2 = \frac{1}{4} b\mu^2 c^4 N \,, \tag{20}$$

and, therefore, the lower bound on the entropy does not depend on b at all,

$$S_0 = k\frac{N}{2} \,. \tag{21}$$

It is quite natural for the entropy to be bounded from below by the number $N/2$ of *constituents*.

We have seen the emergence of the Bekenstein formula [4] for the black hole entropy from the hypothesis about the microscopic nature of gravitational phenomena. It should be noticed that exactly in the same way as the light quanta [1] have emerged from the Wien black body radiation formula, the necessity of introduction of the *gravitational mass-energy quanta* is forced upon us by the Universal Second Law of Thermodynamics and the mass-energy fluctuation formula following from it. The adiabatic invariance arguments for the irreducible mass $M_{\rm ir}$ of the Kerr black hole due to Christodoulou [13] have led to the concept of the black hole entropy of Bekenstein [4]. Bekenstein has proposed the Generalized Second Law of Thermodynamics [4], which states that the total entropy of a black hole and its exterior cannot decrease. The Universal Second Law of Thermodynamics [15] allows one to draw conclusions about the behaviour of a general system near its state of thermal equilibrium. In particular, the total entropy *must be* a *maximum* at the state of thermal equilibrium. The Boltzmann formula,

$$S = k \ln W\,, \tag{22}$$

coordinating the relation between the thermodynamical property of a system, the entropy S, and the *thermodynamic probability* W allows us to draw conclusions about the total combinatorial factors defining W in terms of statistics of *atoms* or *quanta*. The positivity of $\overline{(\Delta E)^2}$ is strictly implied by the maximal value of the Boltzmann thermodynamic probability W at the state of thermal equilibrium. We have applied this idea to *gravitational atoms*. The thermodynamic probability W calculated on the basis of our hypothesis poses the following question: What kind of statistics leads to this W? I will report on this question later.

If the Bekenstein entropy were the whole thing, as far as the thermal properties of gravitating masses are concerned, then the World would be always in a state of the lowest thermodynamic probability. This conclusion would lead then to the statement that the behaviour of a visible Universe is determined by the condition that it is in a state of the lowest statistical weight. Considering an ensemble of such Universes, regarded as local thermal phenomena in a sense suggested in the introduction, we would be persuaded to conclude that our Universe is the least probable one. The Universe must be regarded as a very typical one in the *statistical ensemble of Universes*, which is also the statement of the maximal thermodynamic probability W of Boltzmann. The Universal Second Law of Thermodynamics with its property of positive mass-energy fluctuations which we have consistently used in our arguments for the *Atomic Hypothesis* [2, 3, 12, 15] must be considered as the basic notion underlying the Law of Universal Gravitation. It should be noticed that the notion of a statistical ensemble for the observable Universe is justified only after we identify *atoms* whose existence is underlying the totality of phenomena.

We have given an independent statistical arguments for the existence of *gravitational atoms* elsewhere [3,15]. The arguments advanced, when taken at the face value, mean that the observed negative specific heat of gravitating systems is a result of a coarse grained description of the phenomena. Apparently there exist statistical fluctuations in the mass-energy whose role is to compensate in some regime the negative contribution coming from the large scale part of the fluctuation spectrum, with the latter observed on a macroscopic scale. The fact that the other, compensating, part of the spectrum of fluctuations is not observed at large scale does not mean that it is not inherent in phenomena when inspected closely.

One of the most obvious implications of the atomistic nature of gravitation is that large massive objects believed to be formed in the gravitational collapse will not display the total negative specific heat property. *In fact, in contrary to the prediction about the nature of an almost thermal radiation emitted by black holes* [5], *those objects do not get hotter in the process of losing energy.* Quite opposite behaviour takes place:

objects initially very hot become cooler and cooler as they emit energy.

In particular, *we should not expect to observe mini-black hole explosions* [5, 14] *at all. In fact, there exist at least two ways to understand the lack of observational evidence for primordial black hole explosions. One of them is that such black holes were not produced copiously in a very early hot periods of existence of our Universe, which is also a hypothesis of small probability. The second one is that very dense objects with some spectrum of masses were produced copiously in an early Universe* [16, 14] *but they became cooler by losing enormous amounts of energy to the surrounding space. It is, therefore, not unlikely that the most distant quasars may give us information on the dynamics of energy production and its emission mechanisms which will be compatible with the hypothesis presented here. The younger the quasars the hotter they should appear.* This means that *for the highest redshift Δz quasars we should expect on average the highest power of emission of energy (the highest total luminosity).* Obviously we need more detailed models built on the basis of *Atomic Hypothesis* about the nature of gravitation in order to be able to give more quantitative predictions.

We came to the realization that a gravitational mass-energy consists of a number of *constituents* [2, 3, 12, 15]. This situation we find analogous to that one of a container of gas. Now, a gas container is considered isolated if the total number of gas molecules is sustained constant over the period of time. As far as the *mass-energy parcels* are concerned a 'container' consisting of a given number N of *gravitational atoms* has a total mass-energy $E \sim \mu\sqrt{N}$ (at zero temperature).

A 'container' of *gravitational atoms* is considered isolated if the number of atoms N, and, therefore, its mass-energy E is constant. The change in N is the measure of motion. Now, we can formulate the First Newton's Law of Inertia in the following way:

The physical system called an inertial mass-energy exists in a state characterized by a constant number of gravitational atoms. When this number of atoms is changing the state of motion is changing. Under such circumstances we say that there are forces acting on an inertial mass-energy.

We need the proper formal language of new difference wave equations [2, 3, 15] for gravitating particles which would allow us a more detailed knowledge of microscopic processes underlying gravitation, and, therefore, space-time. We hope to report on the progress in this direction in due time.

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