ON THE ELECTRON STRUCTURE FUNCTION*

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Dedicated to Andrzej Białas in honour of his 60th birthday

The collinear QCD structure of the electron is studied within the Standard Model. The electron structure function is defined and calculated in leading logarithmic approximation. It shows important contribution from the interference of the intermediate electroweak bosons. The problem of momentum scales is extensively discussed. The master equations for the QCD parton densities inside the electron are constructed and solved numerically in the asymptotic region. Significant corrections to the naive evolution procedure are found. Phenomenological applications at present and future momentum scales are discussed.

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1. Introduction

It is the nature of the Quantum Chromodynamics (QCD) and its running coupling constant which cause point-like objects of the electroweak theory to develop quark and gluon structure. This QCD component was first calculated [1] in the case of the photon. In the forward direction, when (nearly) massless quarks are emitted collinearly from the photon, a consecutive QCD cascade developes practically 'at no cost'. The presence of such a 'resolved' photon has been observed in many experiments [2] allowing now for a precise study of its dependence on the momentum fraction x and the momentum scale P^2. Analogous calculation extended to the case of the weak bosons W and Z has also been performed [3]. It shows several differences as

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compared to the photon case. Due to the nature of weak couplings the QCD parton densities depend strongly on the quark flavour and in most cases the polarized densities turn out to be nonzero. The longitudinal bosons show much weaker structure and in the leading logarithmic approximation can be neglected. Phenomenological applications of the 'resolved' W and Z bosons require momentum scales much higher than their masses. This is because only in this region the approximations used are valid, moreover, only there the photon contribution does not dominate entirely the weak bosons' effects.

In order to study the boson structure functions we need a source of real electroweak bosons. But even in the case of the photon, 'real' means in most cases only 'nearly on-shell'. The situation is even more approximate with the weak bosons. Here 'nearly real' means in practice that their momentum squared is negative and close to zero. This is because the best known source of high energy electroweak bosons is a leptonic beam. The procedure applied usually is to replace the incoming lepton with the spectrum of equivalent bosons [5, 6], $F_B^{e^-}$, and to convolute it with the parton distributions, F_h^B, inside the boson, which results in the following QCD parton h distribution inside the electron:

$$F_h^{e^-}(z, \hat{Q}^2, P^2) = \sum_{B=\gamma, \mathrm{W}, \mathrm{Z}} \int dx \, dy \, \delta(z - xy) \, F_h^B(x, P^2) F_B^{e^-}(y, \hat{Q}^2), \quad (1.1)$$

where P^2 is the hard process scale, $\hat{Q}^2$ is the maximum allowed virtuality of the boson (usually taken to be proportional to P^2) and z is the momentum fraction of the parton h with respect to the electron (detailed definitions follow).

The procedure presented above has, however, several unclear points. It does not answer the questions: what is the relation between the two entering momenta scales $\hat{Q}^2$ and P^2, how far off-shell can the electroweak bosons be, are there any interference effects between the intermediate bosons (is the sum actually diagonal in B and/or in the boson polarizations). One may also ask the question: when is the convolution Eq. (1.1) justified and what are the corrections? The best way to clarify these doubts is to address a direct question: what is the QCD content of the incoming electron? In this paper we demonstrate how the concept of the electron structure function allows for a detailed study of the QCD cascade originating from an electron.

The paper is organized as follows. In Section 2 we discuss the general structure of the QCD master equations in the system of leptons and electroweak bosons. The leading order calculation of splitting functions of an electron into a quark (antiquark) is demonstrated in Section 3. The problem of momentum scales and interference effects are discussed there in detail. The constructed master equations for the parton densities inside the electron are solved in the asymptotic region in Section 4. We demonstrate there the

flavour and spin dependence of the asymptotic distributions. Comments on phenomenological applications, summary and conclusions close the paper.

2. Master equations

In order to study the QCD content of a lepton ℓ we look for partons produced by a highly virtual gluon G^* scattering off ℓ:

$$G^* + \ell \to \ell' + h + \text{anything}, \qquad (2.1)$$

where h = quark q, antiquark $\bar{q}$ or gluon G and ℓ' is the outgoing lepton. We have chosen the gluon G^* as a probe because it does not couple directly to the lepton. Our process, Eq. (2.1), is a part of many physical reactions, the far off-shell gluon may originate from an incoming hadron or another lepton. The virtualness P^2 of this gluon sets the scale for the process.

Before going to a detailed calculation we present a heuristic derivation of the master equations for parton densities involved in the process (2.1). We point out which assumptions usually made in this scheme are not fullfilled in a general case. The standard master equation for any 'parton' $\mathcal{A}$ density inside ℓ, $F_{\mathcal{A}}^{\ell}$, reads[4]

$$\frac{d}{dt} F_{\mathcal{A}}^{\ell}(z,t) = \sum_{\mathcal{B}} \int dx\, dy\, \delta(z - xy)\, \mathcal{P}_{\mathcal{A}}^{\mathcal{B}}(x,t)\, F_{\mathcal{B}}^{\ell}(y,t) \qquad (2.2)$$

or, using a shorthand notation for the convolution

$$\frac{d}{dt} F_{\mathcal{A}}^{\ell}(t) = \sum_{\mathcal{B}} \mathcal{P}_{\mathcal{A}}^{\mathcal{B}}(t) \otimes F_{\mathcal{B}}^{\ell}(t) . \qquad (2.3)$$

In general $t = \log(P^2/P_0^2)$, where P_0 is some momentum scale. We discuss the choice of P_0 in the next section. The 'splitting function' $\mathcal{P}_{\mathcal{A}}^{\mathcal{B}}(x,t)$ is the probability density per unit t to find $\mathcal{A}$ carrying fraction x of $\mathcal{B}$'s momentum in a collinear decay of $\mathcal{B}$. The above master equations are quite general within the approximation that the probability to produce a parton in a fusion process is negligible. It is assumed here that this probabilistic approach holds for a physical process *i.e.* the contributions from different 'intermediate' partons $\mathcal{B}$ do not mix. As we will show in the next section, this is not the case for the electroweak sector.

Since there is no direct lepton-gluon coupling the process (2.1) must be mediated by electroweak bosons and we have to consider following types of 'partons' inside lepton ℓ: q, $\bar{q}$, G, ℓ, γ, $W^{\pm}$, Z^0. Assuming for the moment

that the probabilistic approach holds, Eq. (2.3) takes the form

$$\frac{d}{dt}F_{\mathcal{A}}^{\ell}(t) = \sum_{\ell'} \mathcal{P}_{\mathcal{A}}^{\ell'}(t) \otimes F_{\ell'}^{\ell}(t) + \sum_{B=\gamma,\mathrm{W},\mathrm{Z}} \mathcal{P}_{\mathcal{A}}^{B}(t) \otimes F_{B}^{\ell}(t)$$
$$+ \sum_{h=\mathrm{q},\bar{\mathrm{q}},\mathrm{G}} \mathcal{P}_{\mathcal{A}}^{h}(t) \otimes F_{h}^{\ell}(t) \,. \tag{2.4}$$

Within the approximation of the lowest order in the electromagnetic coupling α and leading-log order in the strong coupling $\alpha_{\mathrm{s}}(t)$

$$F_{\ell'}^{\ell}(x,t) = \delta_{\ell}^{\ell'}\,\delta(1-x) + \mathcal{O}(\alpha^2)\,, \tag{2.5}$$

$$\mathcal{P}_{B}^{\ell}(x,t) = \frac{\alpha}{2\pi}P_{B}^{\ell}(x)\,, \tag{2.6}$$

$$\mathcal{P}_{h}^{B}(x,t) = \frac{\alpha}{2\pi}P_{h}^{B}(x)\,, \tag{2.7}$$

$$\mathcal{P}_{h'}^{h}(x,t) = \frac{\alpha_{\mathrm{s}}(t)}{2\pi}P_{h'}^{h}(x)\,, \tag{2.8}$$

and all other $\mathcal{P}_{\mathcal{A}}^{B} = 0$.

Inserting above relations into Eq. (2.4) we arrive at

$$\frac{d}{dt}F_{B}^{\ell}(t) = \frac{\alpha}{2\pi}P_{B}^{\ell}\,, \tag{2.9a}$$

$$\frac{d}{dt}F_{h}^{\ell}(t) = \frac{\alpha}{2\pi}\sum_{B=\gamma,\mathrm{W},\mathrm{Z}} P_{h}^{B} \otimes F_{B}^{\ell}(t) + \frac{\alpha_{\mathrm{s}}(t)}{2\pi}\sum_{h'=\mathrm{q},\bar{\mathrm{q}},\mathrm{G}} P_{h}^{h'} \otimes F_{h'}^{\ell}(t)\,. \tag{2.9b}$$

The solution to Eq. (2.9a) is known as the density of 'equivalent bosons' in the lepton ℓ [6]. The splitting functions P_{h}^{B} vanish for $h = \mathrm{G}$. P_{q}^{B} and $P_{\bar{\mathrm{q}}}^{B}$ have been calculated for $B = \gamma$ in Ref. [1] and for $B = \mathrm{W}, \mathrm{Z}$ in Ref. [3]. $P_{h}^{h'}$ are the standard Altarelli-Parisi splitting functions [4]. Inserting equivalent boson density F_{B}^{ℓ} into Eq. (2.9b) one can solve it for F_{h}^{ℓ}. Eqs.(2.9) suggest also that there is one common momentum scale governing the whole system. This is not true in general as will be demonstrated below.

As already stated the above reasoning neglects interference effects, here possible between γ and Z bosons. Because of different masses and different couplings of these bosons to leptons this interference is non-trivial, *i.e.* cannot be removed by any kind of 'diagonalization' procedure[1]. Thus, instead of F_{B}^{ℓ}, one should rather use a density matrix, F_{AB}^{ℓ}, of equivalent bosons

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[1] The Standard Model gauge group is not simple as opposed to QCD where all colour interferences are taken into account by a group-theoretical weight factor independent of the couplings and kinematical variables.

inside the lepton ℓ. In the following section we perform an explicit calculation which shows how this density matrix arises and how to generalize the first term of Eq. (2.9b) in order to take the γ-Z interference into account.

The use of the density matrix implies — at the first sight — that the probabilistic approach expressed in terms of the master equations breaks down. If, however, instead of looking at the contributions from different electroweak bosons we introduce the notion of the hadronic content of the lepton, the probabilistic interpretation is recovered and the master equations read

$$\frac{d}{dt}F_h^\ell(t) = \frac{\alpha}{2\pi}P_h^\ell(t) + \frac{\alpha_s(t)}{2\pi}\sum_{h'=q,\bar{q},G} P_h^{h'} \otimes F_{h'}^\ell(t)\,, \qquad (2.10)$$

where $P_h^\ell(t)$ is 'lepton to hadron splitting function'. In the next Section we define and calculate this function.

3. QCD current of electron

The hadronic structure of a lepton can be analyzed by looking at any process where the lepton couples to the QCD current. In the following we will calculate the matrix elements squared of this current in the case of $\ell = e^-$.

In the lowest order in the electromagnetic and strong coupling constants (α and α_s) the current matrix element squared for polarized particles reads:

$$\mathcal{J}_{\mu\nu}^{\lambda\eta}(l,p) = \frac{1}{4\pi}\int d\Gamma_{l'}d\Gamma_k d\Gamma_{k'}(2\pi)^4\delta_4(k+k'+l'-p-l)$$
$$\times \sum_{\ell'=e,\nu}\sum_{\lambda'=\pm} \langle e_\lambda^-|J_\mu^\dagger(0)|\ell_{\lambda'}'\, q_\eta\, \bar{q}_{-\eta}\rangle\langle\ell_{\lambda'}'\, q_\eta\, \bar{q}_{-\eta}|J_\nu(0)|e_\lambda^-\rangle\,, \qquad (3.1)$$

where

$$d\Gamma_k = \frac{d^4k}{(2\pi)^4}2\pi\delta(k^2) \qquad (3.2)$$

and indeces λ, λ', η denote helicities of corresponding particles. We do not sum over quark flavours and helicities because we are interested in the densities of polarized partons in the electron. In other words we choose a single flavour of particular helicity in the final state.

The Feynman graphs contributing to our process are shown in figure 1. The incoming electron e^- carries 4-momentum l and the off-shell gluon G^* of 4-momentum p with large $P^2 \equiv -p^2$, supplies the QCD current. In the final state we have massless quark q and antiquark $\bar{q}$ of 4-momenta k and k' and lepton ℓ' (electron or neutrino) of 4-momentum l'. The lepton interacts
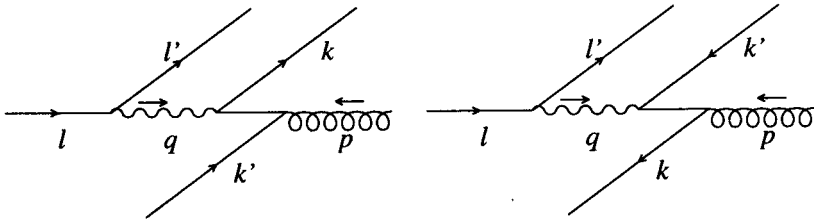
Fig. 1. Lowest order graphs contributing to the process: $e + G^* \to \ell' + q + \bar{q}$.

with the quark exchanging an electroweak boson $B = \gamma, Z, W$ which carries 4-momentum q ($Q^2 \equiv -q^2$). For massless quarks $\mathcal{J}_{\mu\nu}^{\lambda\eta}(l, p)$ is known to have only 3 independent components for any given polarizations λ, η. They can be conveniently paramatrized by the helicities of the incoming gluon. Let us chose a reference frame where the gluon momentum is parallel to the z-axis, $p^\mu = (p_0, 0, 0, p_z)$. Its polarization vectors $e_{(\sigma)}^\mu(p)$ read

$$e_\pm^\mu = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0) \,, \tag{3.3a}$$

$$e_0^\mu(p) = \frac{1}{P}(p_z, 0, 0, p_0) \,, \tag{3.3b}$$

where $P = \sqrt{|p^2|}$.

The three helicity components of $\mathcal{J}$

$$\mathcal{J}_\sigma^{\lambda\eta}(l, p) = e_{(\sigma)}^{\mu*}(p) \, \mathcal{J}_{\mu\nu}^{\lambda\eta}(l, p) \, e_{(\sigma)}^\nu(p) \tag{3.4}$$

are proportional to the cross-section for the (virtual) process

$$e_\lambda^- + G_\sigma^* \to \ell_{\lambda'}' + q_\eta + \bar{q}_{-\eta} \,. \tag{3.5}$$

We will proceed with the calculation of $\mathcal{J}_\sigma^{\lambda\eta}$ in the reference frame where the B boson momentum $\vec{q} = -\vec{p}$, i.e. in the center-of-mass of B and G^*, which is also the rest frame of the $q\bar{q}$ pair. We choose z-axis in the direction of $\vec{q}$, the transverse directions are measured then in the xy plane. In this frame the momenta have following components:

$$\begin{aligned}
q &= (q_0, 0, q_z) \,, \\
p &= (p_0, 0, -q_z) \,, \\
l &= (l_0, \vec{l}_\perp, l_z) \,, \\
l' &= (l_0', -\vec{l}_\perp, l_z') \,, \\
k &= (k_0, \vec{k}_\perp, k_z) \,, \\
k' &= (k_0, -\vec{k}_\perp, -k_z) \,, \tag{3.6}
\end{aligned}$$

where we have used the fact that the quarks are massless.

Introducing the momentum of the $q\bar{q}$ pair, $K = k + k'$ we can decompose the 3-body phase space of Eq. (3.1) as follows

$$d\Gamma_{l'}d\Gamma_k d\Gamma_{k'}(2\pi)^4\delta_4(k + k' + l' - p - l) = \frac{1}{2\pi}dW^2 d\Phi_D d\Phi_2 , \qquad (3.7)$$

where $W^2 = K^2$ is the invariant mass squared of the $q\bar{q}$ pair, $d\Phi_2$ is the 2-body phase space of $l + p \to l' + K$ and $d\Phi_D$ is the phase space of $K \to k + k'$ decay. The explicit expressions read

$$d\Phi_D = \frac{d\cos\omega}{16\pi}\frac{d\chi}{2\pi} , \qquad (3.8)$$

where ω and χ are the polar and azimuthal angles of $\vec{k}$ and

$$d\Phi_2 = \frac{dQ^2}{8\pi\Lambda(s,p^2,m^2)}\frac{d\varphi}{2\pi} , \qquad (3.9)$$

where φ is the azimuthal angle of $\vec{l'}$ The function Λ is defined as $\Lambda(a,b,c) = \sqrt{(a+b-c)^2 - 4ab}$ and s is the total energy squared:

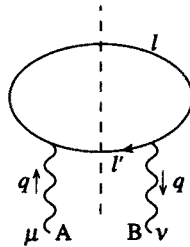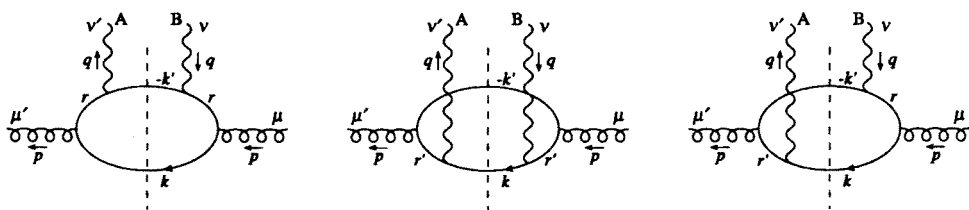$$s = (p + l)^2 . \qquad (3.10)$$

Fig. 2. Lowest order diagram for $\mathcal{L}^{\mu\nu}$.

For given final lepton ℓ' (e^- or ν_e) the current matrix element squared (without phase space factors) can be written as

$$\sum_{\substack{A,B=\\ \gamma,W,Z}} \mathcal{L}^{\mu'\mu}(\lambda,\lambda';A,B)\,\mathcal{H}^{\nu'\nu}(\sigma,\eta;A,B)D_{\nu'\mu'}(A)D_{\nu\mu}(B) . \qquad (3.11)$$

Fig. 3. Lowest order diagrams for $\mathcal{H}^{\nu'\nu}$.

Here, $D_{\mu\nu}$ denote the propagators of electro-weak bosons. The leptonic part $\mathcal{L}^{\mu\nu}(\lambda, \lambda'; A, B)$ is given by the Feynman diagram of Fig. 2 and the hadronic one reads

$$\mathcal{H}^{\nu'\nu}(\sigma, \eta; A, B) = e^{*\mu'}_{(\sigma)}(p)\, \mathcal{H}^{\nu'\nu}_{\mu'\mu}(\eta; A, B)\, e^{\mu}_{(\sigma)}(p) \tag{3.12}$$

with $\mathcal{H}^{\nu'\nu}_{\mu'\mu}(\eta; A, B)$ given by the sum of diagrams shown in Fig. 3 for the outgoing quark of helicty η.

For massless quarks the hadronic tensor satisfies

$$q^{\mu}\mathcal{H}_{\mu\nu}(\sigma, \eta; A, B) = q^{\nu}\mathcal{H}_{\mu\nu}(\sigma, \eta; A, B) = 0 \tag{3.13}$$

which allows us to take

$$D_{\mu\nu}(A) = \frac{1}{Q^2 + M_A^2}\, g_{\mu\nu}\,. \tag{3.14}$$

These propagators can be expressed by the polarization vectors by means of the following identity:

$$e^{*\mu}_+(q)\, e^{\nu}_+(q) + e^{*\mu}_-(q)\, e^{\nu}_-(q) + \frac{q^2}{|q^2|}e^{*\mu}_0(q)\, e^{\nu}_0(q) = \frac{q^{\mu}q^{\nu}}{|q^2|} - g^{\mu\nu}\,. \tag{3.15}$$

Using again equations (3.13) we find that there is no interference between different polarizations of the exchanged bosons. Thus we obtain the formula for $\mathcal{J}_{\sigma}^{\lambda\eta}$ where the contributions from the leptonic and hadronic subprocesses are clearly separated:

$$\mathcal{J}_{\sigma}^{\lambda\eta} = \frac{1}{2^{10}\pi^4}\frac{1}{\Lambda(s, p^2, m^2)} \sum_{\substack{A,B=\\ \gamma,\mathrm{W},\mathrm{Z}}} \int \frac{dQ^2\, dW^2}{(Q^2 + M_A^2)(Q^2 + M_B^2)}$$

$$\times \sum_{\rho} L_{\lambda\rho}(A, B)\, H_{\sigma\eta\rho}(A, B)\,, \tag{3.16}$$

where

$$L_{\lambda\rho}(A, B) = \sum_{\lambda'} \int \frac{d\varphi}{2\pi} \, e_{(\rho)\mu}(q) \, \mathcal{L}^{\mu\nu}(\lambda, \lambda'; A, B) e^*_{(\rho)\nu}(q) \,, \qquad (3.17)$$

$$H_{\sigma\eta\rho}(A, B) = \int \frac{d\chi}{2\pi} \, d\cos\omega \, e^*_{(\rho)\mu}(q) \, \mathcal{H}^{\mu\nu}(\sigma, \eta; A, B) e_{(\rho)\nu}(q) \,. \qquad (3.18)$$

The calculation of the leptonic part, $L_{\lambda\rho}(A, B)$ is straightforward and the result is given by

$$L_{\lambda\rho}(A, B) = \sum_{\lambda'} \int \frac{d\varphi}{2\pi} \, g^{A\ell}_{\lambda'} g^{B\ell}_{\lambda} \left\{ \mathrm{Tr}[\slashed{l}\slashed{e}_{(\rho)} P_{\lambda'} \slashed{l}' \slashed{e}^*_{(\rho)} P_\lambda] + mm' \, \mathrm{Tr}[\slashed{e}_{(\rho)} P_{\lambda'} \slashed{e}^*_{(\rho)} P_\lambda] \right\}$$

$$\equiv \sum_{\lambda'} g^{A\ell}_{\lambda'} g^{B\ell}_{\lambda} L_{\lambda'\lambda\rho} \,, \qquad (3.19)$$

where $e_{(\rho)} \equiv e_{(\rho)}(q)$, $g^{B\ell}_{\lambda}$ denotes the coupling of the electro-weak boson B to the lepton ℓ of helicity λ, m and m' are the masses of initial and final lepton, respectively, and P_λ are projection operators:

$$P_\pm = \tfrac{1}{2}(1 \pm \gamma_5) \,. \qquad (3.20)$$

The explicit expressions for $L_{\lambda'\lambda\rho}$ read

$$L_{\mathrm{LL}-} = L_{\mathrm{RR}+} = 2Q^2 Y_+(y) \,, \qquad (3.21)$$

$$L_{\mathrm{RR}-} = L_{\mathrm{LL}+} = 2Q^2 Y_-(y) \,, \qquad (3.22)$$

$$L_{\mathrm{LL}0} = L_{\mathrm{RR}0} = 2Q^2 Y_0(y) - 2mm' \,, \qquad (3.23)$$

$$L_{\mathrm{LR}+} = L_{\mathrm{RL}+} = L_{\mathrm{LR}-} = L_{\mathrm{RL}-} = -2mm' \,, \qquad (3.24)$$

$$L_{\mathrm{LR}0} = L_{\mathrm{RL}0} = 2mm' \,, \qquad (3.25)$$

where

$$Y_+(y) = \frac{1}{y^2} \,, \qquad (3.26)$$

$$Y_-(y) = \frac{(1 - y)^2}{y^2} \,, \qquad (3.27)$$

$$Y_0(y) = 2\frac{1 - y}{y^2} \,, \qquad (3.28)$$

and y is defined as

$$y = \frac{q_0 + q_z}{l_0 + l_z} \,. \qquad (3.29)$$

In the case of large energy scales we can neglect the electron mass m which results in $L_{\lambda'\lambda\rho}$ being diagonal in the lepton helicities. Thus for $L_{\lambda\dot\rho}$, Eq. (3.19), we obtain

$$
\begin{aligned}
L_{\lambda\rho}(y, Q^2; A, B) &= 4\pi\alpha \hat{g}_\lambda^{A\ell}\hat{g}_\lambda^{B\ell} L_{\lambda\lambda\rho}(y, Q^2) \\
&= 8Q^2\pi\alpha \hat{g}_\lambda^{A\ell}\hat{g}_\lambda^{B\ell}
\begin{cases}
Y_+(y) & \text{for } \rho = \lambda \\
Y_-(y) & \text{for } \rho = -\lambda \\
Y_0(y) & \text{for } \rho = 0
\end{cases} .
\end{aligned}
\tag{3.30}
$$

We have factored out the explicit dependence on α by defining the coupling constants scaled by the positron charge $\hat{g}_\lambda^{A\ell} = g_\lambda^{A\ell}/e$.

Let us proceed now with the hadronic part. First we calculate the integrand of Eq. (3.18). It is given by the Feynman graphs of Fig. 3:

$$
e_{(\rho)\mu}^*(q)\, \mathcal{H}^{\mu\nu}(\sigma, \eta; A, B) e_{(\rho)\nu}(q) = G_{11} + G_{22} + 2\mathrm{Re}\, G_{12}.
\tag{3.31}
$$

Explicit expressions for G_{ik} read

$$
G_{11} = -f_c\, g_s^2 g_\eta^{Aq} g_\eta^{Bq'} \frac{1}{r^4} \mathrm{Tr}[\not{k}'\not{\epsilon}_{(\rho)}^*(q)\not{\epsilon}_{(\rho)}^*(p) P_\eta \not{k}\not{\epsilon}_{(\rho)}(p)\not{\epsilon}_{(\rho)}(q)],
$$

$$
G_{22} = -f_c\, g_s^2 g_\eta^{Aq} g_\eta^{Bq'} \frac{1}{r'^4} \mathrm{Tr}[\not{k}'\not{\epsilon}_{(\rho)}^*(p)\not{t}'\not{\epsilon}_{(\rho)}^*(q) P_\eta \not{k}\not{\epsilon}_{(\rho)}(q)\not{t}'\not{\epsilon}_{(\rho)}(p)],
$$

$$
G_{12} = -f_c\, g_s^2 g_\eta^{Aq} g_\eta^{Bq'} \frac{1}{r^2 r'^2} \mathrm{Tr}[\not{k}'\not{\epsilon}_{(\rho)}^*(p)\not{t}'\not{\epsilon}_{(\rho)}^*(q) P_\eta \not{k}\not{\epsilon}_{(\rho)}(p)\not{t}'\not{\epsilon}_{(\rho)}(q)],
\tag{3.32}
$$

where f_c is the colour factor, g_s — the strong coupling constant and

$$
r = k - p, \quad r' = p - k'.
\tag{3.33}
$$

f_c is the same for all diagrams and independent of the weak boson type. In our case (SU(N) colour group and fixed final quark flavour)

$$
f_c = \tfrac{1}{2}.
\tag{3.34}
$$

On substituting $P_\eta = (1 + \eta\gamma_5)/2$ into Eq. (3.32) we see that Eq. (3.31) becomes the sum of two parts — with and without the γ_5 matrix under the traces. In order to perform the integration of Eq. (3.18) we notice that the denominators of quark propagators depend only on ω and read

$$
r^2 = (k-p)^2 = p^2 - 2pk = -\frac{W^2 + Q^2 + P^2 + \Lambda(W^2, q^2, p^2)\cos\omega}{2},
$$

$$
r'^2 = (k'-p)^2 = p^2 - 2pk' = -\frac{W^2 + Q^2 + P^2 - \Lambda(W^2, q^2, p^2)\cos\omega}{2}.
$$

The 'axial' part (containing γ_5) vanishes upon integration over the angles and we obtain the following result for the hadronic part

$$H_{\rho\sigma\eta} = 8 f_c \, g_s^2 g_\eta^{Aq} g_\eta^{Bq'} h_{\rho\sigma}(x, \tau) \,, \tag{3.35}$$

which depends on two dimensionless kinematic variables x and τ:

$$x = -\frac{p_0 + p_z}{q_0 + q_z} \tag{3.36}$$

and

$$\tau = 1 - \frac{\Lambda(W^2, q^2, p^2)}{W^2 + P^2 + Q^2} \,. \tag{3.37}$$

The explicit expressions for $h_{\rho\sigma}(x, \tau)$ are given in Appendix A. In the following considerations we will need only their leading terms for $P^2 \gg Q^2$.

Before quoting these approximate results let us look at the exact formula for the current matrix element squared, Eq. (3.16). Substituting the results from Eq. (3.30) and Eq. (3.35) we get

$$\mathcal{J}_\sigma^{\lambda\eta} = \frac{\alpha^2 \alpha_s}{2\pi} \int\limits_0^{W^2_{\max}} \frac{dW^2}{\Lambda(s, p^2, m^2)} \sum_{\substack{A,B= \\ \gamma, W, Z}} \int\limits_{Q^2_{\min}}^{Q^2_{\max}} \frac{Q^2 dQ^2}{(Q^2 + M_A^2)(Q^2 + M_B^2)}$$

$$\times \sum_\rho \frac{1}{y} P_{A_\rho B_\rho}^{\ell_\lambda}(y) \, \hat{g}_\eta^{Aq} \hat{g}_\eta^{Bq'} h_{\rho\sigma}(x, \tau) \,, \tag{3.38}$$

where, by definition,

$$P_{A_\rho B_\rho}^{\ell_\lambda}(y) \equiv \frac{y}{8\pi\alpha Q^2} L_{\lambda\rho}(y, Q^2; A, B) \,. \tag{3.39}$$

τ, x, y can be expressed in terms of the integration variables, W^2 and Q^2. τ is given by Eq. (3.37) while x and y, defined by Eqs. (3.36) and (3.29), read

$$x = \frac{\Lambda + P^2 - Q^2 - W^2}{\Lambda + P^2 - Q^2 + W^2} \,, \tag{3.40}$$

$$y = \frac{2\Lambda}{2(s - m^2) + \Lambda + P^2 - Q^2 - W^2} \,, \tag{3.41}$$

where $\Lambda \equiv \Lambda(W^2, q^2, p^2)$.

The integration limits defined by kinematics read:

$$W^2_{\max} = (\sqrt{s} - m)^2 \,, \tag{3.42}$$

$$Q^2_{\substack{\min \\ \max}} = -2m^2 + \frac{1}{2s}(s + m^2 + P^2)(s + m^2 - W^2)$$

$$\mp \frac{1}{2s} \Lambda(s, m^2, p^2) \Lambda(s, m^2, W^2) \,. \tag{3.43}$$

At this stage we are ready to discuss the 'equivalent bosons' approximation for $s > P^2 \gg M_A^2, M_B^2, m^2$. To this end we first observe that for given P^2 the hadronic part $h_{\rho\sigma}(x, \tau)$ is logarithmically dominated by low Q^2 values:

$$h_{\pm\mp}(x, \tau) \simeq x^2 \log \frac{P^2}{Q^2}, \tag{3.44}$$

$$h_{\pm\pm}(x, \tau) \simeq (1 - x)^2 \log \frac{P^2}{Q^2}, \tag{3.45}$$

with other components finite for $Q^2/P^2 \to 0$. Thus we will be interested in the region $Q^2 \ll P^2$ corresponding to the leading logarithmic approximation. In this limit x gains the meaning of the Bjorken variable of the hadronic subprocess:

$$x = \frac{P^2}{2pq}, \tag{3.46}$$

while y becomes the electroweak boson momentum fraction with respect to l:

$$y = \frac{pq}{pl} = \frac{W^2 + P^2}{s + P^2}. \tag{3.47}$$

The product of x and y is independent of the integration variables and reads

$$z \equiv xy = \frac{P^2}{2pl}. \tag{3.48}$$

As shown in Ref.[3] the leading terms of $h_{\rho\sigma}$ are in fact the products of the logarithmic factors and the splitting functions $P_{q\eta}^{\rho}(x)$ and $P_{\bar{q}\eta}^{\rho}(x)$ of a spin-1 boson of helicity ρ into a quark (antiquark) of helicity η:

$$h_{\rho\sigma}(x, Q^2) = \frac{1}{6}[P_{q-\sigma}^{\rho}(x) + P_{\bar{q}-\sigma}^{\rho}(x)] \log \frac{P^2}{Q^2}, \tag{3.49}$$

with

$$P_{q\pm}^{\pm}(x) = P_{\bar{q}\pm}^{\pm}(x) = 3x^2, \tag{3.50a}$$

$$P_{q\pm}^{\mp}(x) = P_{\bar{q}\pm}^{\mp}(x) = 3(1 - x)^2. \tag{3.50b}$$

Changing the integration variable W^2 to y we obtain the formula for $\mathcal{J}_{\sigma}^{\lambda\eta}$ which allows for the discussion of the equivalent boson approximation:

$$\mathcal{J}_{\sigma}^{\lambda\eta} = \frac{\alpha_s \alpha}{6} \sum_{\substack{A,B= \\ \gamma, W, Z}} \sum_{\rho=\pm} \int\limits_z^{y_{max}} \frac{dy}{y} \frac{\alpha}{2\pi} P_{A_\rho B_\rho}^{\ell_\lambda}(y) \, \hat{g}_\eta^{Aq} \hat{g}_\eta^{Bq} \left[P_{q-\sigma}^{\rho} \left(\frac{z}{y} \right) + P_{\bar{q}-\sigma}^{\rho} \left(\frac{z}{y} \right) \right]$$

$$\times \int\limits_{Q^2_{\min}}^{Q^2_{\max}} \frac{Q^2 dQ^2}{(Q^2 + M_A^2)(Q^2 + M_B^2)} \log \frac{P^2}{Q^2} , \qquad (3.51)$$

where

$$y_{\max} = 1 - \mathcal{O}(m^2/P^2) , \qquad (3.52)$$

$$Q^2_{\min} = m^2 \frac{y^2}{1 - y} , \qquad (3.53)$$

$$Q^2_{\max} = P^2 \frac{z + y - zy}{z} . \qquad (3.54)$$

$P^{\ell_\lambda}_{A_\rho B_\rho}$ in the above equation is a generalization of the electron-boson splitting function, which takes into account the interference effects.

Let us note that the minimal value of Q^2 is the only place where the electron mass must be kept finite to regularize collinear divergencies in the case of photon exchange. However, in order to remain within the leading logarithmic approximation we have to pay particular attention to the upper limit of the Q^2 integration. In general it is a function of P^2, however integration up to the maximum kinematically allowed value $Q^2_{\max}$ would violate the condition $Q^2/P^2 \ll 1$. For our approximation to work we must integrate over Q^2 up to some $\hat{Q}^2_{\max} \ll P^2$. Integrating Eq. (3.51) over Q^2 within such limits and keeping only leading-logarithmic terms leads to

$$\mathcal{J}_\sigma^{\lambda \eta} = \frac{\alpha_s \alpha}{6} \sum_{A,B,\rho} \hat{g}_\eta^{Aq} \hat{g}_\eta^{Bq} F_{A_\rho B_\rho}^{\ell_\lambda}(\hat{Q}^2_{\max}) \otimes [P_{qn}^\rho \delta_{\eta,-\sigma} + P_{\bar{q}-\eta}^\rho \delta_{\eta,\sigma}] \log \frac{P^2}{\bar{Q}^2} . \quad (3.55)$$

where $F_{A_\rho B_\rho}^{\ell_\lambda}(y, \hat{Q}^2_{\max})$ is the density matrix of polarized bosons inside the lepton and $\bar{Q}^2$ is an average Q^2 value. (Exact result from Eq. (3.51) is $\bar{Q}^2 = \sqrt{\hat{Q}^2_{\max} Q^2_{\min}}$ but within the leading-log approximation only the order of magnitude is relevant.)

The transverse components of $F_{A_\rho B_\rho}^{e^-}$ in the case of unpolarized electron read (formulae for polarized electron are given in the Appendix B)

$$F_{\gamma_\pm \gamma_\pm}^{e^-}(y, Q^2) = \frac{\alpha}{2\pi} \frac{(1 - y)^2 + 1}{2y} \log \frac{Q^2}{m_e^2} , \qquad (3.56a)$$

$$F_{Z_+ Z_+}^{e^-}(y, Q^2) = \frac{\alpha}{2\pi} \tan^2 \theta_W \frac{\rho_W^2 (1 - y)^2 + 1}{2y} \log \frac{Q^2 + M_Z^2}{M_Z^2} , \quad (3.56b)$$

$$F_{Z_- Z_-}^{e^-}(y, Q^2) = \frac{\alpha}{2\pi} \tan^2 \theta_W \frac{\rho_W^2 + (1 - y)^2}{2y} \log \frac{Q^2 + M_Z^2}{M_Z^2} , \quad (3.56c)$$

$$F_{\gamma_+ Z_+}^{e^-}(y, Q^2) = \frac{\alpha}{2\pi} \tan\theta_W \frac{\rho_W(1-y)^2 - 1}{2y} \log\frac{Q^2 + M_Z^2}{M_Z^2}, \quad (3.56d)$$

$$F_{\gamma_- Z_-}^{e^-}(y, Q^2) = \frac{\alpha}{2\pi} \tan\theta_W \frac{\rho_W - (1-y)^2}{2y} \log\frac{Q^2 + M_Z^2}{M_Z^2}, \quad (3.56e)$$

$$F_{W_+ W_+}^{e^-}(y, Q^2) = \frac{\alpha}{2\pi} \frac{1}{4\sin^2\theta_W} \frac{(1-y)^2}{y} \log\frac{Q^2 + M_W^2}{M_W^2}, \quad (3.56f)$$

$$F_{W_- W_-}^{e^-}(y, Q^2) = \frac{\alpha}{2\pi} \frac{1}{4\sin^2\theta_W} \frac{1}{y} \log\frac{Q^2 + M_W^2}{M_W^2}, \quad (3.56g)$$

where θ_W is the Weinberg angle and

$$\rho_W = \frac{1}{2\sin^2\theta_W} - 1. \quad (3.57)$$

All other density matrix elements (containing at least one longitudinal boson) do not contribute at the leading-logarithmic level.

The density matrix elements $F_{A_\rho B_\rho}^{e^-}(y, Q^2)$ should be considered as a generalized solution to the Eq. (2.9a). At this point we are able to define the splitting functions of an electron into a quark at the momentum scale Q^2 as

$$P_{q_\eta}^{e^-}(Q^2) = \sum_{AB} \hat{g}_\eta^{Aq} \hat{g}_\eta^{Bq} \sum_\rho F_{A_\rho B_\rho}^{e^-}(Q^2) \otimes P_{q_\eta}^\rho. \quad (3.58)$$

This is the generalization of the first sum of Eq. (2.9b). The explicit expressions for quarks read (see Appendix B for the polarized electron case)

$$
\begin{aligned}
P_{q_+}^{e^-}(z, Q^2) \quad &= \frac{3\alpha}{4\pi} \left\{ e_q^2 [\Phi_+(z) + \Phi_-(z)] \log\frac{Q^2}{m_e^2} \right. \\
&+ e_q^2 \quad \tan^4\theta_W [\Phi_+(z) + \rho_W^2 \Phi_-(z)] \log\frac{Q^2 + M_Z^2}{M_Z^2} \\
&\left. - 2e_q^2 \quad \tan^2\theta_W [-\Phi_+(z) + \rho_W \Phi_-(z)] \log\frac{Q^2 + M_Z^2}{M_Z^2} \right\}, \quad (3.59a)
\end{aligned}
$$

$$
\begin{aligned}
P_{q_-}^{e^-}(z, Q^2) \quad &= \frac{3\alpha}{4\pi} \left\{ e_q^2 [\Phi_+(z) + \Phi_-(z)] \log\frac{Q^2}{m_e^2} \right. \\
&+ z_q^2 \quad \tan^4\theta_W [\Phi_-(z) + \rho_W^2 \Phi_+(z)] \log\frac{Q^2 + M_Z^2}{M_Z^2} \\
&+ 2e_q z_q \quad \tan^2\theta_W [-\Phi_-(z) + \rho_W \Phi_+(z)] \log\frac{Q^2 + M_Z^2}{M_Z^2} \\
&\left. + \quad (1 + \rho_W)^2 \Phi_+(z) \delta_{qd} \log\frac{Q^2 + M_W^2}{M_W^2} \right\}, \quad (3.59b)
\end{aligned}
$$

where

$$\Phi_+(z) = \frac{1-z}{3z}(2 + 11z + 2z^2) + 2(1+z)\log z \,, \tag{3.60}$$

$$\Phi_-(z) = \frac{2(1-z)^3}{3z} \,, \tag{3.61}$$

and

$$z_q = \frac{T_3^q}{\sin^2\theta_{\mathrm{W}}} - e_q \,, \tag{3.62}$$

with e_q and T_3^q being the quark charge and 3-rd weak isospin component, respectively. The splitting functions for an antiquark of the opposite helicity can be obtained from Eq. (3.59) by interchanging Φ_+ with Φ_-.

These new splitting functions depend on the scale $\hat{Q}_{\mathrm{max}}^2$. This scale can be in principle fixed by experimental cuts, in such case the final lepton must be tagged. A condition commonly used in phenomenological applications [8] is $\hat{Q}_{\mathrm{max}}^2 = \epsilon P^2$, where $\epsilon \ll 1$ and in general depends on y and zIn this case the electron splitting functions depend on $\log P^2$ and hence on t. In the following we will concentrate on the case with the maximum virtuality being P^2-dependent. In this case the master equations Eq. (2.9b) become

$$\frac{d}{dt}F_{q_\eta/\bar{q}_\eta}^{e_\lambda^-}(t) = \frac{\alpha}{2\pi}P_{q_\eta/\bar{q}_\eta}^{e_\lambda^-}(t) + \frac{\alpha_{\mathrm{s}}(t)}{2\pi}\sum_{\substack{h'= \\ q,\bar{q},G}}\sum_{\rho=\pm 1}P_{q_\eta/\bar{q}_\eta}^{h'_\rho} \otimes F_{h'_\rho}^{e_\lambda^-}(t)\,, \tag{3.63a}$$

$$\frac{d}{dt}F_{G_\eta}^{e_\lambda^-}(t) = \frac{\alpha_{\mathrm{s}}(t)}{2\pi}\sum_{\substack{h'= \\ q,\bar{q},G}}\sum_{\rho=\pm 1}P_{G_\eta}^{h'_\rho} \otimes F_{h'_\rho}^{e_\lambda^-}(t)\,. \tag{3.63b}$$

Note that the convolution of the equivalent boson distributions and boson-quark splitting functions, Eq. (3.58), occurs at the level of splitting functions. It is not equivalent to the usually performed convolution of the distribution functions because of the P^2 dependence of the boson densities Eq. (3.56). Only in the case when the upper limit of integration $\hat{Q}_{\mathrm{max}}^2$ is kept fixed (P^2-independent), are the convolutions equivalent at both levels.

To summarize, our splitting functions show two new features. The first one, already mentioned before, is the contribution from the interference of electroweak bosons (γ and Z only). The second one is their P^2 dependence, which results from the upper integration limit $\hat{Q}_{\mathrm{max}}^2$.

4. Asymptotic solutions to the master equations

The master equations Eq. (3.63), with spin dependent functions, are most conveniently solved in terms of polarized and unpolarized distributions [3]:

$$F_h^{e_\lambda^-} = F_{h_+}^{e_\lambda^-} + F_{h_-}^{e_\lambda^-} \,, \tag{4.1a}$$

$$\Delta F_h^{e_\lambda^-} = F_{h_+}^{e_\lambda^-} - F_{h_-}^{e_\lambda^-} \,, \tag{4.1b}$$

$$P_h^{e_\lambda^-} = P_{h_+}^{e_\lambda^-} + P_{h_-}^{e_\lambda^-} \,, \tag{4.2a}$$

$$\Delta P_h^{e_\lambda^-} = P_{h_+}^{e_\lambda^-} - P_{h_-}^{e_\lambda^-} \tag{4.2b}$$

and

$$P_h^{h'} = P_{h_+}^{h'_+} + P_{h_-}^{h'_+} \,, \tag{4.3a}$$

$$\Delta P_h^{h'} = P_{h_+}^{h'_+} - P_{h_-}^{h'_+} \,. \tag{4.3b}$$

Upon substitution of the above definitions into Eq. (3.63) we obtain two sets of master equations where unpolarized and polarized functions do not mix:

$$\frac{d}{dt} F_h^{e_\lambda^-}(t) = \frac{\alpha}{2\pi} P_h^{e_\lambda^-}(t) + \frac{\alpha_s(t)}{2\pi} \sum_{\substack{h'= \\ q,\bar{q},G}} P_h^{h'} \otimes F_{h'}^{e_\lambda^-}(t) \,, \tag{4.4a}$$

$$\frac{d}{dt} \Delta F_h^{e_\lambda^-}(t) = \frac{\alpha}{2\pi} \Delta P_h^{e_\lambda^-}(t) + \frac{\alpha_s(t)}{2\pi} \sum_{\substack{h'= \\ q,\bar{q},G}} \Delta P_h^{h'} \otimes \Delta F_{h'}^{e_\lambda^-}(t) \,, \tag{4.4b}$$

In the following we will present the asymptotic solutions to these equations for the case of unpolarized electron, $F_h^{e^-}$ and $\Delta F_h^{e^-}$.

The equations Eq. (3.63) have been derived within the leading logarithmic approximation which is justified for asymtotically large P^2 values. In this region all the logarithms of P^2 over any finite scale are approximately equal and in the QCD evolution we choose them to be scaled by Λ_{QCD}:

$$t = \log \frac{P^2}{\Lambda_{QCD}^2} \,. \tag{4.5}$$

We present below the solution to the master equations in this asymptotic (large t) region.

The strong coupling constant is approximated by

$$\alpha_{\mathrm{s}}(t) \simeq \frac{2\pi}{bt}\,, \tag{4.6}$$

with $b = 11/2 - n_{\mathrm{f}}/3$ for n_{f} flavours. Parametrizing the asymptotic solution to Eqs.(4.4) for the QCD parton h in the unpolarized electron as

$$F_h^{\mathrm{e}^-}(z,t) \simeq \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 f_h^{\mathrm{as}}(z)\,t^2\,, \tag{4.7a}$$

$$\Delta F_h^{\mathrm{e}^-}(z,t) \simeq \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 \Delta f_h^{\mathrm{as}}(z)\,t^2\,, \tag{4.7b}$$

we obtain purely integral equations

$$f_h^{\mathrm{as}} = \hat{P}_h^{\mathrm{e}^-} + \frac{1}{2b}\sum_{h'} P_h^{h'} \otimes f_{h'}^{\mathrm{as}}\,, \tag{4.8a}$$

$$\Delta f_h^{\mathrm{as}} = \Delta\hat{P}_h^{\mathrm{e}^-} + \frac{1}{2b}\sum_{h'} \Delta P_h^{h'} \otimes \Delta f_{h'}^{\mathrm{as}}\,, \tag{4.8b}$$

where $\hat{P}_h^{\mathrm{e}^-}(z)$ and $\Delta\hat{P}_h^{\mathrm{e}^-}(z)$ are equal to $P_h^{\mathrm{e}^-}(z)$ and $\Delta P_h^{\mathrm{e}^-}(z)$, respectively, with all logs put to 1.

We solve the above equations by a numerical procedure described in Ref. [3]. We take number of flavours $n_{\mathrm{f}} = 5$. In the asymptotic region, where the quark masses can be neglected, we are left with only two different quark distributions: up-type and down-type.

In figures 4, 5 and 6 we present the unpolarized quark, anti-quark and gluon distributions. The polarized distributions are depicted in figures 7 and 8. The density $z\Delta f_{\mathrm{u}}^{\mathrm{as}}(z)$, not shown here, is more than 10 times smaller than $z\Delta f_{\mathrm{d}}^{\mathrm{as}}(z)$. One notices significant contribution from the W intermediate state in the density of d-type and ū-type quarks. The most surprising however is the γ-Z interference contribution which cannot be neglected, as it is comparable to the Z term. It violates the standard probabilistic approach where only diagonal terms are taken into account. This also stresses the necessity of introducing the concept of electron structure function in which all contributions from intermediate bosons are properly summed up. Due to the nature of weak couplings they turn out to be nonzero, even in the case of gluon distributions. Again the γ-Z interference term is important and the W contribution dominates in the asymptotic region.

One should keep in mind that at finite t the logarithms multiplying the photon contribution differ from the remaining ones (*cf.* Eq. (3.56a)). Being scaled by m_e, they lead to the photon domination at presently available P^2. The importance of the interference term remains constant relative to the Z contribution, as they are both governed by the same logarithm.
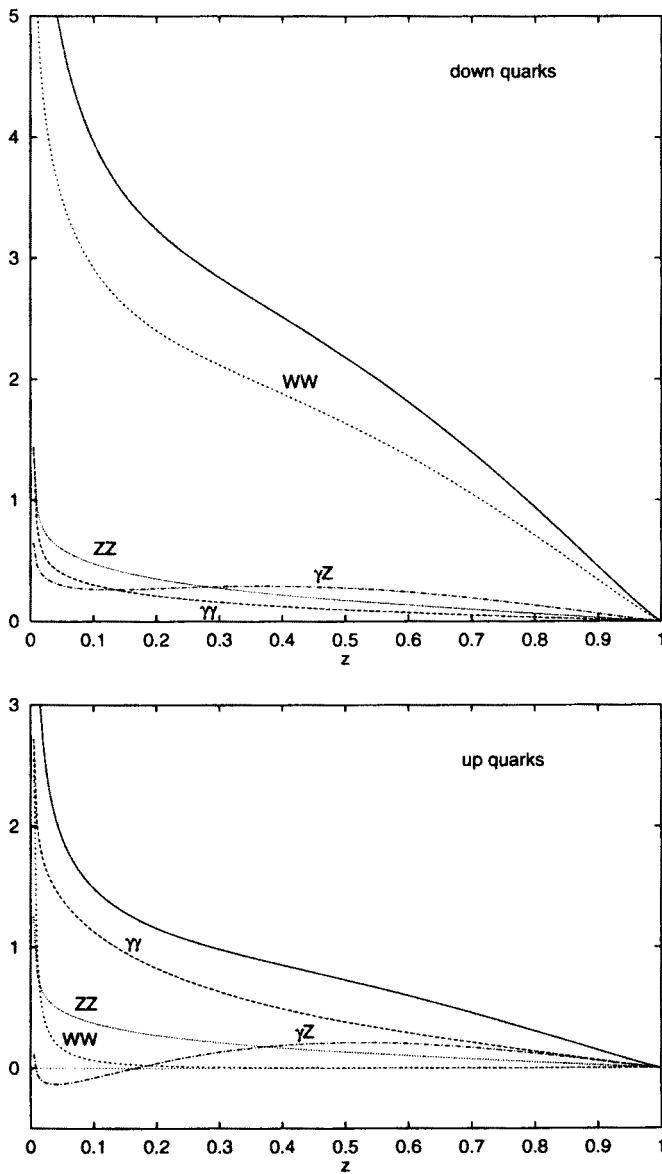
Fig. 4. Unpolarized quark distributions $z f_{u/d}^{as}(z)$ — solid line. The other lines show contributions from different electroweak bosons.
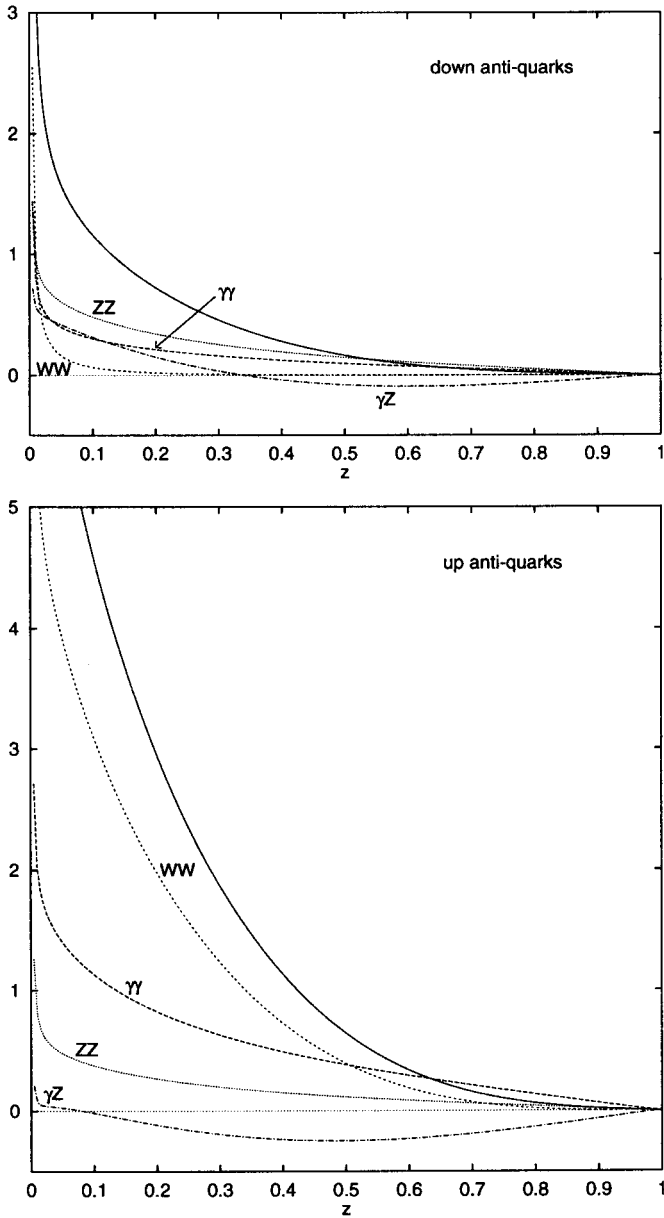
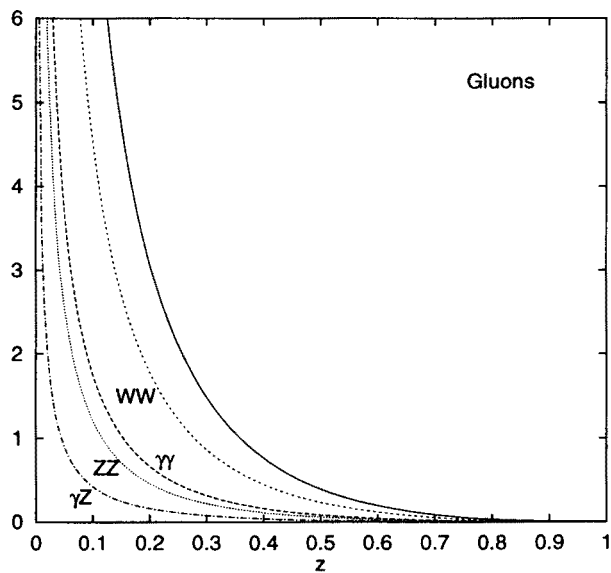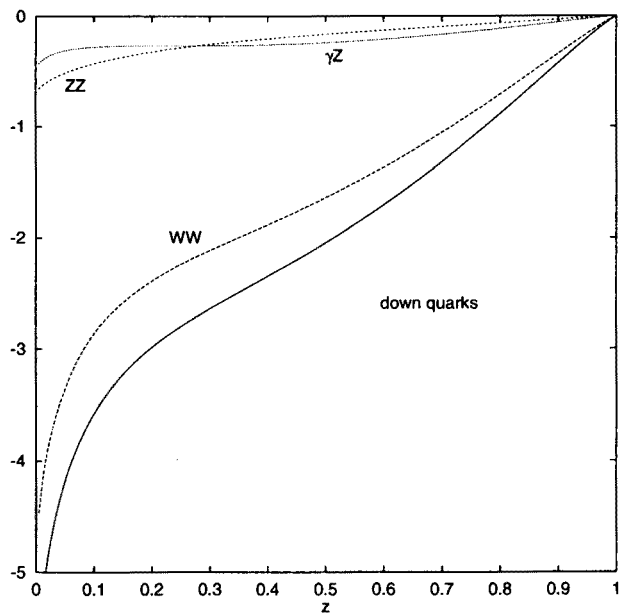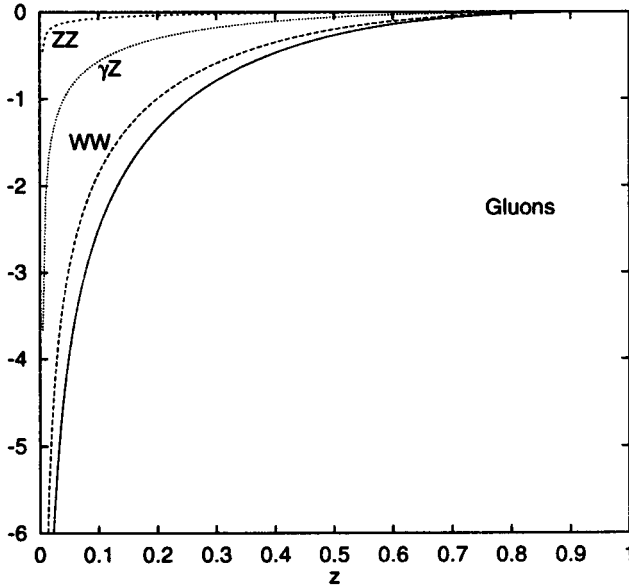Fig. 5. Unpolarized anti-quark distributions $z f_{\bar{u}/\bar{d}}^{as}(z)$ — solid line. The other lines show contributions from different electroweak bosons.

Fig. 6. Unpolarized gluon distribution $z f_{\mathrm{G}}^{\mathrm{as}}(z)$ — solid line. The other lines show contributions from different electroweak bosons.

Fig. 7. Polarized d-quark distribution $z \Delta f_{\mathrm{d}}^{\mathrm{as}}(z)$ — solid line. The other lines show contributions from different electroweak bosons.

Fig. 8. Polarized gluon distribution $z\Delta f_G^{as}(z)$ — solid line. The other lines show contributions from different electroweak bosons.

5. Conclusions

In the paper we investigated the hadronic content of the electron defining a new quantity — the electron structure function. We defined it and constructed its evolution equations. We also gave asymptotic solutions to these equations demonstrating rich flavour and spin dependence. In all cases the parton densities inside the electron grow as logarithm squared of the external momentum scale (in our calculation the gluon momentum P^2). This construction, which looks at first sight a formal manipulation only, brings in several new issues. The most important ones are the correct treatment of the intermediate electroweak bosons and precise formulation of the structure function evolution. In the first case one faces the necessity of introducing the electroweak boson density matrix with the off-diagonal (γ-Z) terms comparable to the diagonal ones. The probabilistic approach, one is used to in the partonic picture, can be restored only when looking at the whole process *i.e.* an electron emitting quarks and gluons. Also the QCD evolution can be treated correctly only when looking at the complete system. The virtuality of the intermediate boson depends in general on the external momentum and in this way the electroweak sector enters the QCD evolution. The derivation of the electron structure function presented above assumes a totally inclusive situation when the outgoing lepton is not tagged. In the opposite situation, which is possible only with neutral currents and
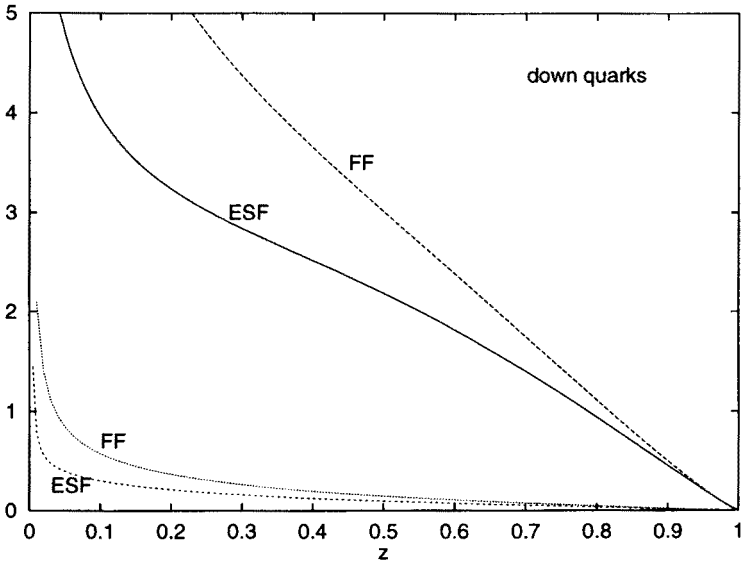
Fig. 9. Comparison of unpolarized d-quark distributions $zf_d^{as}(z)$ calculated by ESF and FF methods. The upper two lines result from contributions from all electroweak bosons while the lower two from $\gamma\gamma$ only.

away from the forward direction, the virtuality of the intermediate boson can be controlled by experiment and kept independent of the external momentum scale. In all cases the known boson structure function should be treated with great care in the lepton induced processes. The question becomes very important at momenta where all electroweak bosons come into play. But even at presently available momenta, where the 'resolved' photon dominates, one can see how the correct treatment of the scales changes the shape of parton distributions. In Fig. 9 we compare the asymptotic solutions of the evolution equations following from our procedure (ESF) with those following from naive application of the convolution Eq. (1.1) (FF). It is possible that the difference can be traced in the analysis of presently available data. The inclusion of effects connected with finite momenta and construction of the electron structure function which could be applied to present experiments is a natural next step along the proposed lines.

Appendix A

Here we present full results for the hadronic part of the QCD current matrix element squared as well as some kinematical relations.

The $h_{\rho\sigma}(x,\tau)$ functions of Eq. (3.35) read:

$$
\begin{aligned}
h_{--}(x,\tau) \quad = \quad & h_{++}(x,\tau) = (1-x)^2 \mathcal{L} \\
& +\frac{(1-x)^2}{16x^2}\left\{-\tau(1+x)^2 + \tilde{\tau}(1-14x+49x^2)\right. \\
& \qquad +3\tilde{\tau}^2(1-14x+25x^2) \\
& \qquad +3\tilde{\tau}^3(3-18x-19x^2) \\
& \qquad \left. +3\tilde{\tau}^4(1-x)(3-7x)+3\tilde{\tau}^5(1-x)^2\right\}\mathcal{L} \\
& +\frac{(1-x)^2}{8x^2}\left\{8\tilde{\tau}x(2-5x)-4\tilde{\tau}^2(1-8x+10x^2)\right. \\
& \qquad \left. -6\tilde{\tau}^3(1-x)(1-3x)-3\tilde{\tau}^4(1-x)^2\right\} \\
& -2(1-x)^2\,, \qquad\qquad\qquad\qquad\qquad\qquad\qquad (A.1)
\end{aligned}
$$

$$
\begin{aligned}
h_{-+}(x,\tau) \quad = \quad & h_{+-}(x,\tau) = x^2 \mathcal{L} \\
& +\frac{1}{16x^2}\left\{-\tau(1-x)^2(1+x)^2\right. \\
& \qquad +\tilde{\tau}(1-2x^2-32x^3+49x^4) \\
& \qquad +\tilde{\tau}^2(1-x)(3-13x+53x^2-75x^3) \\
& \qquad +\tilde{\tau}^3(1-x)^2(9-38x+57x^2) \\
& \qquad \left. +3\tilde{\tau}^4(1-x)^3(3-7x)+3\tilde{\tau}^5(1-x)^4\right\}\mathcal{L} \\
& +\frac{1}{8x^2}\left\{-8\tilde{\tau}(1-x)x^2(1-5x)\right. \\
& \qquad -4\tilde{\tau}^2(1-x)^2(1-4x+10x^2) \\
& \qquad \left. -6\tilde{\tau}^3(1-x)3(1-3x)-3\tilde{\tau}^4(1-x)^4-16x^4\right\},\ (A.2)
\end{aligned}
$$

$$
\begin{aligned}
h_{00}(x,\tau) \quad = \quad & \frac{(1-x)^2}{4x^2}\left\{-\tau(1+x)^2+\tilde{\tau}(1+2x+17x^2)\right. \\
& \qquad -\tilde{\tau}^2(1+18x-55x^2)+\tilde{\tau}^3(5-46x+53x^2) \\
& \qquad \left. +3\tilde{\tau}^4(1-x)(3-7x)+3\tilde{\tau}^5(1-x)^2\right\}\mathcal{L} \\
& +\frac{3(1-x)^2}{2x^2}\left\{-8\tilde{\tau}x^2+4\tilde{\tau}^2x(2-3x)\right. \\
& \qquad \left. -2\tilde{\tau}^3(1-x)(1-3x)-\tilde{\tau}^4(1-x)^2)\right\}, \qquad (A.3)
\end{aligned}
$$

$$
\begin{aligned}
h_{0-}(x,\tau) \quad &= \quad h_{0+}(x,\tau) = h_{-0}(x,\tau) = h_{+0}(x,\tau) \\
&= \frac{1-x}{8x^2}\Big\{\tau(1-x)(1+x)^2 - \tilde{\tau}(1+x-17x^2+31x^3) \\
&\qquad + \tilde{\tau}^2(1-23x+79x^2-65x^3) \\
&\qquad + \tilde{\tau}^3(1-x)(7-46x+55x^2) \\
&\qquad + 3\tilde{\tau}^4(1-x)^2(3-7x) + 3\tilde{\tau}^5(1-x)^3\Big\}\mathcal{L} \\
&\quad + \frac{1-x}{4x^2}\Big\{+4\tilde{\tau}x(1-7x+8x^2) \\
&\qquad -2\tilde{\tau}^2(1-x)(1-12x+19x^2) \\
&\qquad -6\tilde{\tau}^3(1-x)^2(1-3x) - 3\tilde{\tau}^4(1-x)^3 + 8x^3\Big\}, \quad \text{(A.4)}
\end{aligned}
$$

where

$$
\mathcal{L} = \log\frac{P^2}{x^2 Q^2}, \tag{A.5}
$$

$$
\tilde{\tau} = \frac{\tau}{1-\tau} \tag{A.6}
$$

and τ and x are defined by Eq. (3.37) and Eq. (3.36), respectively.
For fixed Q^2 $h_{\rho\sigma}(x,\tau)$ are functions of x only with

$$
\tau = \frac{x^2 Q^2}{P^2 + x^2 Q^2}. \tag{A.7}
$$

Introducing $\bar{s} \equiv s + P^2 - m^2 = 2lp$ we can define dimensionless variables

$$
\xi = \frac{Q^2}{\bar{s}}, \tag{A.8}
$$

$$
z = \frac{P^2}{\bar{s}} \tag{A.9}
$$

in terms of which we can write the formulae needed to change (x,τ) variables to (y,ξ) used in the integral (3.51):

$$
\tau = \frac{x^2 \xi}{z + x^2 \xi}, \tag{A.10}
$$

$$
x = \frac{\sqrt{y^2 + 4(1-y)z\xi} - y}{2(1-y)\xi}. \tag{A.11}
$$

Appendix B

In this appendix we present the formulae for the boson density matrix elements and electron to quark splitting functions for the fully polarized case. As can be seen from Eq. (3.51) and Eq. (3.55) the boson density matrix elements, $F^{e^{\lambda}}_{A_{\rho}B_{\rho}}$ are equal to $P^{e^{\lambda}}_{A_{\rho}B_{\rho}}$ times appropriate logarithms. The transverse components of $P^{e^{\lambda}}_{A_{\rho}B_{\rho}}$ can be read off Eq. (3.39) and Eq. (3.30) and are equal to

$$P^{e\pm}_{A_{\rho}B_{\rho}} = \hat{g}^{Ae}_{\pm}\hat{g}^{Be}_{\pm}\, yY_{\pm\rho}(y)\,. \tag{B.1}$$

Inserting electroweak couplings we derive following formulae for the transverse components of $F^{e^{\lambda}}_{A_{\rho}B_{\rho}}$ in the case of polarized electron:

$$F^{e\pm}_{\gamma_{\rho}\gamma_{\rho}}(y,Q^2) = \frac{\alpha}{2\pi}\, yY_{\pm\rho}(y)\, \log\frac{Q^2}{m_e^2}\,, \tag{B.2a}$$

$$F^{e-}_{Z_{\rho}Z_{\rho}}(y,Q^2) = \frac{\alpha}{2\pi}\rho^2_{\rm W}\tan^2\theta_{\rm W}\, yY_{-\rho}(y)\, \log\frac{Q^2+M_Z^2}{M_Z^2}\,, \tag{B.2b}$$

$$F^{e+}_{Z_{\rho}Z_{\rho}}(y,Q^2) = \frac{\alpha}{2\pi}\tan^2\theta_{\rm W}\, yY_{\rho}(y)\, \log\frac{Q^2+M_Z^2}{M_Z^2}\,, \tag{B.2c}$$

$$F^{e-}_{\gamma_{\rho}Z_{\rho}}(y,Q^2) = \frac{\alpha}{2\pi}\rho_{\rm W}\tan\theta_{\rm W}\, yY_{-\rho}(y)\, \log\frac{Q^2+M_Z^2}{M_Z^2}\,, \tag{B.2d}$$

$$F^{e+}_{\gamma_{\rho}Z_{\rho}}(y,Q^2) = -\frac{\alpha}{2\pi}\tan\theta_{\rm W}\, yY_{\rho}(y)\, \log\frac{Q^2+M_Z^2}{M_Z^2}\,, \tag{B.2e}$$

$$F^{e-}_{W_{\rho}W_{\rho}}(y,Q^2) = \frac{\alpha}{2\pi}\frac{1}{2\sin^2\theta_{\rm W}}\, yY_{-\rho}(y)\, \log\frac{Q^2+M_W^2}{M_W^2}\,, \tag{B.2f}$$

$$F^{e+}_{W_{\rho}W_{\rho}}(y,Q^2) = 0\,, \tag{B.2g}$$

where

$$yY_{-}(y) = \frac{(1-y)^2}{y}\,, \qquad yY_{+}(y) = \frac{1}{y}\,. \tag{B.3}$$

The splitting functions of a polarized electron into a polarized quark are defined by Eq. (3.58) with $F^{e^{\lambda}}_{A_{\rho}B_{\rho}}$ taken from Eq. (B.2) and a straightforward calculation leads to the following result:

$$P_{q+}^{e^-}(z, Q^2) = \frac{3\alpha}{2\pi} \Phi_-(z) \left\{ e_q^2 \log \frac{Q^2}{m_e^2} \right.$$

$$+ e_q^2 \rho_W^2 \tan^4 \theta_W \log \frac{Q^2 + M_Z^2}{M_Z^2}$$

$$\left. - 2e_q^2 \rho_W \tan^2 \theta_W \log \frac{Q^2 + M_Z^2}{M_Z^2} \right\} , \qquad \text{(B.4a)}$$

$$P_{q+}^{e^+}(z, Q^2) = \frac{3\alpha}{2\pi} \Phi_+(z) \left\{ e_q^2 \log \frac{Q^2}{m_e^2} \right.$$

$$+ e_q^2 \tan^4 \theta_W \log \frac{Q^2 + M_Z^2}{M_Z^2}$$

$$\left. + 2e_q^2 \tan^2 \theta_W \log \frac{Q^2 + M_Z^2}{M_Z^2} \right\} , \qquad \text{(B.4b)}$$

$$P_{q-}^{e^-}(z, Q^2) = \frac{3\alpha}{2\pi} \Phi_+(z) \left\{ e_q^2 \log \frac{Q^2}{m_e^2} \right.$$

$$+ z_q^2 \rho_W^2 \tan^4 \theta_W \log \frac{Q^2 + M_Z^2}{M_Z^2}$$

$$+ 2e_q z_q \rho_W \tan^2 \theta_W \log \frac{Q^2 + M_Z^2}{M_Z^2}$$

$$\left. + (1 + \rho_W)^2 \delta_{qd} \log \frac{Q^2 + M_W^2}{M_W^2} \right\} , \qquad \text{(B.4c)}$$

$$P_{q-}^{e^+}(z, Q^2) = \frac{3\alpha}{2\pi} \Phi_-(z) \left\{ e_q^2 \log \frac{Q^2}{m_e^2} \right.$$

$$+ z_q^2 \tan^4 \theta_W \log \frac{Q^2 + M_Z^2}{M_Z^2}$$

$$\left. - 2e_q z_q \tan^2 \theta_W \log \frac{Q^2 + M_Z^2}{M_Z^2} \right\} , \qquad \text{(B.4d)}$$

where $\Phi_-(z)$ and ρ_W are defined by Eq. (3.61) and Eq. (3.62), respectively. The splitting functions for an antiquark of the opposite helicity can be obtained from Eq. (B.4) by interchanging Φ_+ with Φ_-.

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