DECOHERENCE RATE OF MASS SUPERPOSITIONS

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Dedicated to Andrzej Białas in honour of his 60th birthday

We estimate the damping parameter characterizing the rate of loss of coherence between the components of a state consisting of a coherent superposition of masses. In a medium at temperature T the result is $D \approx T^3(G\Delta M)^2$, where ΔM is the mass spread and G the gravitational constant. Since each mass produces a different metric this may be viewed as a simple calculation of the decoherence rate between different metrics. In another application, we consider the loss of coherence of mixing neutrinos arriving from the early universe.

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1. Introduction

Coherent superpositions of states of different energy or mass are an essential part of quantum mechanics. Without them wavepackets would not move, spins would never rotate, and the world would be frozen in a strange state of stationarity. For an isolated system like an atom or an elementary particle in its rest frame, if we do not have a stationary state, we have an interfering superposition of masses. A particularly striking example of this are the famous oscillations of the K^0 system, where coherence between different mass states leads to striking effects, but many, more pedestrian, examples exist; a decaying atom in an excited state has a width and so is in a superposition of different masses.

Although this coherence between different energies or masses is an essential aspect of the world as we know it, it is not entirely obvious that it must necessarily be produced or retained under all conditions. The study of "quantum damping", or "decoherence" as it now is often called, has taught us how to calculate the loss or maintenance of this coherence, a least in simple cases. By the same token, it makes it possible to understand why certain "directions" in the apparently orientation-free Hilbert space of quantum mechanics may come to be preferred. With this understanding comes the realization that what is "natural" under some conditions may not be natural under others. To take the simple case of a single molecule, under certain conditions of the environment it may be thought of as having a more or less definite shape, under others as having a more or less definite value of a conjugate quantity like the parity or angular momentum. By varying the conditions of the environment, one can cause one or the other to appear [1].

2. Energy and gravity

In the same way, concerning coherent superpositions of energy or mass we can legitimately ask as to why or when coherence in mass or energy is natural and whether it is always to be expected and maintained.

Here we would like to estimate the loss of mass coherence using the methods we have previously applied to such things as optical isomers and neutrino oscillations. It is perhaps interesting to see what the answer looks like since these concepts come up in connection with the fundamentals of gravity. Since mass or energy is the source of the metric, by calculating the decoherence of different masses, we will be calculating something related to the loss of coherence between metrics.

We shall approach the question from the angle of asking why this coherence appears to be so stable and hard to break. Certainly it is not part of our everyday experience that the coherence necessary for the ordinary march of events breaks down easily. To begin to answer this, we must recall one of the major lessons of our previous work [1], a point which is essential in understanding how the above "directions" get selected. That is that "damping" or "decoherence" occurs between those states of a system to which the environment responds differently. Mere interaction with the environment is not enough, the medium must "feel" ("measure", some might like to say) the different components of the system differently. For example, in the case of the molecule with two states of handedness it turns out [2] that despite frequent collisions with an ambient gas most of the collisions (at low temperature) will not break the coherence between the two handedness. This is because the two states in question respond to the environment very much equally at low collision energy.

Now, as far as our present problem of the coherence between different energies or masses is concerned, what interaction is there that will distinguish between two states which are identical except for their mass or energy?

Evidently, there is only gravity. Hence, since under most circumstances gravity is very weak on the quantum level, practically no energy decoherence [3] is to expected under ordinary conditions.

However, if we are willing to consider extraordinary conditions, like energies near the planck mass, or lengths on cosmological scales, there may be some effects to look at.

3. Model calculation

Let us therefore consider the following simple problem: A state representing a mass having two components with mass M and M' scatters an external probe gravitationally. (Think of a K^0or neutrino mixture as a target in a beam which the probe). The probe will go into different states according to the interaction with M or M'; this induces a degree of decoherence. We wish to estimate this.

We use the damping parameter D of earlier work, which has the interpretation of the damping or decoherence rate; equivalently $1/D$ is a "decoherence time". For a two state system , D is given by the "unitarity deficit" for the S matrices for the two states (here, M, M') in question. This deficit is [2] Im $i(1 - S_M^\dagger S_{M'})$. S being the scattering matrix, and the imaginary part arising because we are dealing with a kind of damping or absorptive effect. We shall assume here that the same applies for two mass states so that in an impact parameter b representation

$$D = (\text{flux}) \int 2\pi b db \ \text{Im} \ i(1 - S_M^\dagger(b) S_{M'}(b)) \,. \tag{1}$$

For orientation in understanding this formula, we recall our old result that when we have two components, like say two neutrino types, with one scattering in some medium and the other not, this formula gives $D = 1/2\times$(scattering rate), where "scattering rate" refers to the interacting component. Thus, as might be expected, the decoherence rate is something like the scattering rate.

For the present problem of gravitational scattering we can use standard methods, as for coulomb scattering, to find the scattering amplitudes needed. Their M dependance will then determine the damping rate. Supposing the probe to be represented by a beam of particles and hence a plane wave, it is convenient to carry out the calculation in an impact parameter b representation where S is given by a sum over b with $S(b) = e^{2i\delta(b)}$

We use the well-known formula for the coulomb phase shift

$$\delta(b) = 2 \int_0^{l_{\max}} \alpha \frac{dl}{\sqrt{l^2 + b^2}} \,, \tag{2}$$

where here $\alpha = GEM/v$, M being M or M' and E the energy of the probe, v the velocity. G is the gravitational constant. We also call

$$\Delta\alpha = GE(\Delta M)/v\,, \tag{3}$$

where $\Delta M = M - M'$. The path of integration through the potential is along the straight path l at impact parameter b.

In this, as in all Coulomb-like problems, we shall have long distance logarithmic divergences, and so we introduce a large distance cutoff $l_{\mathbf{max}}$ which must be interpreted according to the physical conditions. Carrying out the integral we find that for a given b

$$S^{\dagger}_{M}(b)S_{M'}(b) = e^{i2\Delta\alpha \ln(l_{\max}/b)}\,. \tag{4}$$

Performing the b integration introducing again a long distance cutoff, we find

$$\int 2\pi b db\; (1 - S^{\dagger}_{M}S_{M'}) = \pi l^{2}_{\mathbf{max}} \left(\frac{b^{2}_{\mathbf{max}}}{l^{2}_{\mathbf{max}}} - \frac{(l^{2}_{\mathbf{max}}/b^{2}_{\mathbf{max}})^{1-i\Delta\alpha}}{1 - i\Delta\alpha} \right)\,. \tag{5}$$

This result may now be evaluated in various contexts. Typically we anticipate $l_{\mathbf{max}} = b_{\mathbf{max}}$, so that

$$\int 2\pi b db (1 - S^{\dagger}_{M}S_{M'}) = \pi l^{2}_{\mathbf{max}} \left(\frac{-i\Delta\alpha}{1 - i\Delta\alpha} \right)\,. \tag{6}$$

Taking the absorptive part leads to a second order expression in G. (The first order, real part, corresponds to an energy shift, which is not of great interest to us here.) Expanding for the usual case of $\Delta\alpha$ very small, gives finally for D

$$D = (\text{flux}) l^{2}_{\mathbf{max}} (\Delta\alpha)^{2}\,. \tag{7}$$

4. Thermal medium

As anticipated, the damping rate involves the difference of the masses and is usually very small. If we assume the system is in a medium of relativistic particles (and setting all purely numerical factors to one) at temperature T so that $E \approx T$, (flux)$\approx T^{3}$ and $l_{\mathbf{max}}$ is the distance between particles, or $l^{-3}_{\mathbf{max}} \approx$ density$= T^{3}$, we obtain, finally for such an environment

$$D \approx T^{3}(G\Delta M)^{2} = T^{3} \left(\frac{\Delta m}{M^{2}_{\mathbf{Planck}}} \right)^{2}\,. \tag{8}$$

For any usual temperature and masses this rate is exceedingly small. However, for energies around the Planck scale and beyond, Planck mass particles will not evolve, or at least not in the usual way. Time, in some sense, stands still.

5. Mixing primordial neutrinos

An oscillating neutrino mixture, starting out in the early universe, will see a flux of mass perturbations coming at it, like galaxies and stars, and a decoherence will be induced in the density matrix describing the mixture. We can examine this question in the above spirit, with a "beam" of galaxies of the present epoch with a density of about one per cubic megaparsec (mpc).

Here, however, $\Delta\alpha$ is not small. With a mass for a galaxy of 10^{11} solar masses we have $\Delta\alpha \approx GM\gamma\Delta m \approx \gamma 10^{20}\Delta m/\text{eV}$, where we normalize the neutrino mass difference in eV and γ is E/m for the neutrinos . We cannot expect this to be small. Rather, Eq. (6) will give us simply $l^2_{\mathbf{max}}$ the cross section of the galaxy. In other words we have, as in the old result referred to above, that the damping rate is simply given by the number of galaxies encountered per unit time.

Taking the size of a galaxy $l_{\mathbf{max}} \approx 0.1$ mpc we have

$$D \approx 10^{-2}/(\text{mpc})\,. \tag{9}$$

So in 100 megaparsecs, every neutrino will have encountered a galaxy, and have lost any phase coherence between different mass states. Of course, to have any oscillations for primordial neutrinos in the first place they would have to have been released ("decoupled") in the early universe in a time short compared to their oscillation period. This doesn't seem entirely impossible, especially since they are rather energetic at the time of decoupling. However, it would seem that they have little chance of reaching us as a still–oscillating mixture.

6. Gravitational vs. Coulomb phase

At this point we should admit to a slight over-simplification which we have committed in order not to distract from the main point. We should really use not just the Coulomb phase but rather that arising from the Schwarzschild metric, since we are dealing with gravity. The gravitational phase factor [4] is $\frac{1}{2}\int h_{\mu\nu}p_\mu dx_\nu$, where $h_{\mu\nu}$ is the deviation of the metric tensor from its flat (Minkowski) value. The h_{00} contribution will be the same as in the Coulomb case. However, since we have been dealing with relativistic particles, where the momentum and energy are essentially equal, we should also take into account the h_{rr} part of the metric. This indeed gives a contribution which is the same as h_{00}, up to a numerical factor. Since it is just a numerical factor, however, the results for the purposes of our qualitative discussion will be the same.

On the other hand, the fact that masses interfere — which is our whole point here — but charges do not [4] is a very fundamental and significant difference between electromagnetism and gravity.

REFERENCES

[1] For a review of these concepts see L. Stodolsky, *Quantum Damping and Its Paradoxes* in: *Quantum Coherence*, ed. J.S. Anandan, World Scientific, Singapore 1990.

[2] R.A. Harris, L. Stodolsky, *J. Chem. Phys.* **74**, 2145 (1981). For a derivation of such expressions under general conditions see G. Raffelt, G. Sigl, L. Stodolsky, *Phys. Rev. Lett.* **70**, 2363 (1993). Here we use Eq. (12) of Ref. [1] in impact parameter form.

[3] Students of thermodynamics will recognize that there is a familiar case where we have complete decoherence of the energy and no time evolution: ideal thermal equilibrium. There the density matrix is diagonal in the energy, meaning no coherence between different energies. However, as is clear at least from the considerations of this note, such a state is a rather formal concept and could only be reached in a very long time.

[4] L. Stodolsky, *Gen. Relativ. Gravitation* **11**, 391 (1979).