EQUATION OF MOTION OF A POINT-LIKE TEST BODY AND POST-NEWTONIAN APPROXIMATION IN THE PROJECTIVE UNIFIED FIELD THEORY

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The equation of motion of a point-like test body is derived from the field equations of Projective Unified Field Theory. We studied this equation of motion with respect to a potential violation of the equivalence principle. The Newtonian and the post-Newtonian approximations of the field equations and of the equation of motion of a test body are studied in detail. In analyzing some experimental data we performed some numeric estimates of the ratio of the inertial mass to the scalaric mass of matter.

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1. Introduction

Recently considerable interest was raised by experiments aiming at the verification of the equivalence principle, in context of searching for new effects in the gravitational field. Particularly a possible experimental discovery of a deviation from the Newtonian gravity has been widely discussed with respect to the fifth force hypothesis of Fischbach *et al.* [1].

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Let us mention that Jordan [2] tried to generalize Einstein's theory of gravitation by taking into account a scalar field occurring in the 5-dimensional projective relativity theory which was developed after the well-known 5-dimensional concept of Kaluza and Klein.

In the present paper we take the last version of the Projective Unified Field Theory (PUFT) of Schmutzer [3] as a basis of our investigations. This concept introduced a hypothetical new attribute of matter — the so-called scalarism (with scalaric as the related adjective). Schmutzer's PUFT has been developed in three stages. The version **I** of PUFT led to a possible violation of the equivalence principle which has been tested experimentally to a precision of 10^{-12} [4]. Two decades later this situation was one of the reasons for him to elaborate a second version of PUFT with a slightly changed projectian formalism using a "conformal factor" (version **II**). Recently he proposed new 5-dimensional field equations and returned to the projection formalism of version **I** (version **III**).

In the paper presented here we do not discuss the differences of these two versions which mathematically differ only insignificantly from each other. We rather treat both versions mentioned simultaneously, drawing our attention to the fulfillment of the equivalence principle.

Further we study the equation of motion of a test body and the post-Newtonian approximation of the theory. Our results allow to compare the predictions of PUFT and Einstein's theory with the experimental data.

2. Equation of motion of a point-like test body

The 4-dimensional field equations of PUFT (Gauss system of units) have the following form (Latin indices run from 1 to 4, Greek indices from 1 to 3; signature of the metric $(+,+,+,-)$; comma and semicolon denote partial and covariant derivatives, respectively) [3]:

GENERALIZED GRAVITATIONAL FIELD EQUATIONS

$$R_{mn} - \tfrac{1}{2} g_{mn} R + \Lambda e^{-2\sigma} g_{mn} = \kappa_0 (E_{mn} + S_{mn} + \Theta_{mn}) , \qquad (1)$$

where $\kappa_0 = 8\pi\gamma_N / c^4$ is Einstein's gravitational constant,

$$E_{mn} = \frac{1}{4\pi} (B_{mk} H^k{}_n + \frac{1}{4} g_{mn} B_{jk} H^{jk}) \qquad (2)$$

is the energy tensor of the electromagnetic field,

$$S_{mn} = -\frac{\lambda}{\kappa_0} (\sigma_{,m} \sigma_{,n} - \frac{1}{2} g_{mn} \sigma_{,k} \sigma^{,k}) \qquad (3)$$

is the energy tensor of the scalaric field, Θ_{mn} is the energy tensor of the non-geometrized matter (substrate), λ is a free constant, which can be chosen [3] as $\lambda = -2$, Λ is a free constant, which is an analog to Einstein's cosmological constant ($\Lambda = \lambda_0/S_0^2$, $\lambda = K_0\kappa_0$, where λ_0, K_0 and S_0 were used in [3]).

GENERALIZED ELECTROMAGNETIC FIELD EQUATIONS

$$\text{a) } H^{mn}{}_{;n} = \frac{4\pi}{c} j^m, \qquad \text{b) } B_{\langle mn,k\rangle} = 0, \qquad \text{c) } H_{mn} = e^{2\sigma} B_{mn}\,, \tag{4}$$

where H_{mn} and B_{mn} are the field strength tensor and the induction tensor of the electromagnetic field, respectively, j^k is the electric four-current density ($\langle \quad \rangle$ means cycle).

SCALARIC FIELD EQUATION

$$\sigma^{,k}{}_{;k} = \frac{\kappa_0}{\lambda}\left(\vartheta + \frac{1}{8\pi} B_{kj} H^{kj}\right) + \frac{2\Lambda}{\lambda} e^{-2\sigma}\,, \tag{5}$$

where ϑ is the scalaric substrate density.

The above field equations lead directly to the following conservation law:

$$\Theta^{mn}{}_{;n} = -\frac{1}{c} B^m{}_k j^k + \vartheta \sigma^{,m}\,. \tag{6}$$

Let us mention that the field equations of version **II** of PUFT [3] can be obtained from the present field equations (1) to (5) by the choice: $\lambda = \frac{2}{3}$, $\Lambda = 0$; $\sigma = \frac{3}{2}\sigma_{\mathbf{II}}$, $\vartheta = \frac{2}{3}\vartheta_{\mathbf{II}}$, where the index "**II**" denotes quantities which occur in version **II**.

It is well-known that the equation of motion in PUFT (as in the Einstein theory) can be derived from the field equations. Of course its explicit form requires concrete assumption for the energy tensor Θ^{mn}, electric four-current density j^k and scalaric substrate density ϑ.

As it is well-known, in the Einstein theory for incoherent matter $\Theta^{mn} = -\mu u^m u^n$ and $j^m = \rho_0 u^m$ (μ mass density, ρ_0 electric charge density). This means for the case of a point-like test body (material point) with the inertial mass M, scalaric mass $\mathcal{M}$ and charge e that the following formulas are valid:

$$\Theta^{mn} = -c \int \frac{M\delta^{(4)}(x - \xi(\tau)) u^m u^n}{\sqrt{g(\xi(\tau))}}\, d\tau\,, \tag{7}$$

$$j^m = ec \int \frac{\delta^{(4)}(x - \xi(\tau)) u^m}{\sqrt{g(\xi(\tau))}}\, d\tau\,, \tag{8}$$

$$\vartheta = c^3 \int \frac{\mathcal{M}\delta^{(4)}(x - \xi(\tau))}{\sqrt{g(\xi(\tau))}}\, d\tau\,, \tag{9}$$

where the integrals in the equations (7) to (9) must be taken along the world line (determined by $x^i = \xi^i(\tau)$) of the test body; $u^m = \frac{\mathrm{d}\xi^m}{\mathrm{d}\tau}$ is the 4-velocity of the test body and $g = -\det(g_{ij})$. It should be taken into account that the integral structure of ϑ follows directly from the expressions (7) and (8) using the equations (6). The multiplier c^3 in the expression (9) guarantees that the scalaric mass $\mathcal{M}$ has the same physical dimension as the inertial mass M. Therefore the quantity $D = \mathcal{M}c^2$ was called "scalaric substrate energy" [3].

By substituting (7) to (9) into equations (6) we obtain

$$\frac{\mathrm{D}(Mu^i)}{\mathrm{D}\tau} = \frac{e}{c}B^i{}_k u^k - c^2\mathcal{M}\sigma^{,i}\,. \tag{10}$$

Multiplying (10) with u_i and keeping in mind that $u^i u_i = -c^2$ instead of (10) we find the following equations which describe the motion of a point-like test body in a gravitational, electromagnetic and scalaric field:

$$M\frac{\mathrm{D}u^i}{\mathrm{D}\tau} = \frac{e}{c}B^i{}_k u^k - c^2\mathcal{M}h^i{}_j\sigma^{,j}\,, \tag{11}$$

$$\frac{\mathrm{d}M}{\mathrm{d}\tau} = \mathcal{M}\sigma^{,k}u_k = \mathcal{M}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\,, \tag{12}$$

where $h^i{}_j = g^i{}_j + \frac{1}{c^2}u^i u_j$ is the projection tensor.

We mention that the equation (11) and (12) coincide with the corresponding ones found by Schmutzer by performing a transition from the equation of motion of a perfect fluid [3]. From the equation (12) follows that there exists a dependence of the inertial mass on special features of the test body in PUFT. Further one realizes that in the restricted case of a vanishing scalaric field (*e.g.* at infinity, far from matter) PUFT goes over into the Einstein theory [3], in which $M = M_0 = \text{const}$. Therefore as a consequence of (12) it is quite natural to suppose that the variability of the inertial mass is exclusively caused by the scalaric field, *i.e.* $M = M(\sigma)$, where the function $M(\sigma)$ satisfies the condition $M(0) = M_0$. Hence follows that the scalaric mass is determined, in correspondence with equation (12), by $\mathcal{M} = \frac{\mathrm{d}M(\sigma)}{\mathrm{d}\sigma}$.

In the case of an electrically neutral particle ($e = 0$) or in the absence of an external electromagnetic field ($B_{mn} = 0$) equation (11) reads:

$$\frac{\mathrm{D}u^i}{\mathrm{D}\tau} = -c^2\eta(\sigma)\left(\sigma^{,i} + \frac{1}{c^2}\sigma^{,k}u_k u^i\right)\,, \tag{13}$$

where $\eta(\sigma) = \frac{\mathcal{M}}{M}$. The equation (13) contains an arbitrary function $\eta(\sigma)$ which according to the concept of Schmutzer [3].

Let as mention that from (13) one immediately learns that the equivalence principle is fulfilled if $\eta(\sigma)$ is a universal function for all matter. In the following we treat for the sake of simplicity the case $\eta = \text{const}$. Then the dependence of the inertial mass of the σ-field, acquires the specific form:

$$M(\sigma) = M_0 e^{\eta\sigma}, \quad \mathcal{M}(\sigma) = (\eta M_0) e^{\eta\sigma} . \tag{14}$$

The equation (13) affirms the non-geodesic motion of electrically neutral test bodies in a gravitational and scalaric field. Some experimental consequences of such a non-geodesic motion will be studied in following sections in more detail.

3. Newtonian approximation

In the case of a vanishing electromagnetic field and for $\Lambda = 0$ the equations (1) and (5) lead to the Newtonian approximation [3]:

$$\text{a)} \quad \Delta\phi = 4\pi\gamma_N \mu, \qquad \text{b)} \quad \Delta\sigma = \frac{\kappa_0}{\lambda}\vartheta , \tag{15}$$

where ϕ is the Newtonian gravitational potential, which is connected with the metric: $g_{44} = -1 - \frac{2\phi}{c^2}$.

Although (15) were obtained for the case of absence of an external electromagnetic field, we should not ignore the fact that the internal region of electrically neutral matter might contain local electromagnetic fields of appreciable strength, which could give birth to local scalaric fields. According to (5) these local scalaric fields σ_{loc} could give a considerable contribution to the global σ-field appearing in (15b). Therefore this problem will be studied now.

Let l be the characteristic size of the region of internal nonzero microscopic electromagnetic field. Now we study an isolated system of matter with the characteristic size L. Then from (5) the estimate of the microscopic scalaric field created by this microscopic electromagnetic field results:

$$\frac{\sigma_{loc}}{l^2} \sim \kappa_0 (B_{ij}H^{ij})_{loc} \sim \kappa_0 E_{loc}^2 , \tag{16}$$

where E_{loc} is the strength of the microscopic electric field. In a similar way one can estimate the value of macroscopic scalaric field from (15b):

$$\frac{\sigma}{L^2} \sim \kappa_0 \vartheta \sim \kappa_0 \eta\mu . \tag{17}$$

Comparison of (16) with (17) leads to the relation

$$\frac{\sigma_{loc}}{\sigma} \sim \frac{1}{\eta}\left(\frac{l}{L}\right)^2 \frac{E_{loc}^2}{\mu} . \tag{18}$$

The contribution of microscopic fields has no significance if the inequalities

$$\frac{\sigma_{loc}}{\sigma} \ll 1, \quad \text{or} \quad \eta \gg \left(\frac{l}{L}\right)^2 \frac{E_{loc}^2}{\mu} \tag{19}$$

are fulfilled, which under normal physical conditions is valid. Therefore we can neglect the scalaric field created by microscopic electromagnetic fields.

The equation of motion of a test body (13) in the Newtonian approximation takes the following form:

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + \mathrm{grad}\phi = -\eta c^2 \mathrm{grad}\sigma\,. \tag{20}$$

As a consequence of the assumed constant ratio of the scalaric mass to the inertial mass and under the assumption $\eta\sigma \ll 1$ we get

$$\text{a)}\quad \mu = \mu_0 e^{\eta\sigma} \approx \mu_0(1+\eta\sigma), \qquad \text{b)}\quad \vartheta = \eta c^2 \mu \approx \eta c^2 \mu_0 (1+\eta\sigma)\,, \tag{21}$$

where $\mu_0 = \mu(\sigma = 0)$. By means of the relations (21) the two equations (15) can be rewritten in the form:

$$\triangle\phi = 4\pi\gamma_N \mu_0 (1+\eta\sigma)\,, \tag{22}$$

$$\triangle\sigma = \frac{8\pi\gamma_N\eta}{\lambda c^2}\mu_0(1+\eta\sigma)\,. \tag{23}$$

Comparing the equations (22) and (23) one finds

$$\sigma = \frac{2\eta}{\lambda c^2}\phi + f\,, \tag{24}$$

where f is an arbitrary function satisfying Laplace's equation $\triangle f = 0$. The case of $f \neq 0$ takes into account sources of gravitational and scalaric fields outside the isolated system. Excluding such situations we can without any restrictions choose $f = 0$. From (20) and (24) one finds the final form of the equation of motion of a point-like test body in the Newtonian approximation:

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -(1+\frac{2}{\lambda}\eta^2)\mathrm{grad}\phi\,. \tag{25}$$

As it is well-known, the measurement of the frequency red shift in a gravitational field (experiments of Pound and Rebka [5] and Pound and Snider [6]) shows that the free motion of bodies corresponds to geodesics up to the precision of 10^{-6}. Looking at (25) one immediately recognizes that in the first nonzero (Newtonian) approximation the deviation of free fall from geodesic motion has the order η^2. Consequently we have the restriction: $\eta \leq 10^{-3}$.

Finally we note that the effective Newtonian gravitational potential reads: $\phi_{\text{eff}} = \left(1 + \frac{2}{\lambda}\eta^2\right)\phi$.

4. Post-Newtonian approximation

4.1. Solution of the field equations

Let us consider a system of slowly moving bodies bounded by gravitational interaction (planetary system). In order to describe the gravitational field of this system at large distances from its center, in the Einstein theory one can use the so-called post-Newtonian approximation (see for instance [7, 8]). In this paper we will show that the field equations of PUFT also allow an analogous approximation. Similarly to the procedure in the Einstein theory it is convenient to take the ratio $\beta = v/c$ as a small expansion parameter of the exact field equations, where v is the characteristic velocity of the motion of the bodies, which is related to the gravitational potential ϕ as follows:

$$\frac{v^2}{c^2} \sim \frac{\phi}{c^2} \sim \beta^2 \quad \text{(in solar system } \beta \sim 10^{-4} \text{ to } 10^{-3}\text{)}. \tag{26}$$

As an intermediate step we rewrite the field equations of PUFT (1) and (5) in the form:

$$R_{mn} = \kappa_0 \left(\Theta_{mn} - \tfrac{1}{2} g_{mn} \Theta^k{}_k\right) - \lambda \sigma_{,m} \sigma_{,n}\,, \tag{27}$$

$$\sigma^{,k}{}_{;k} = \frac{1}{\lambda}\kappa_0 \vartheta\,. \tag{28}$$

Taking into account the experience in the Einstein theory and in our analysis of exact solutions of PUFT field equations ([3, 9]), we suppose that there exists a coordinate system in which in zeroth order approximation the metric tensor equals the Minkowski tensor $\eta_{ij} \equiv \operatorname{diag}(1,1,1,-1)$. Then the following power series approximation is possible:

$$g_{\alpha\beta} = \delta_{\alpha\beta} + \overset{2}{g}_{\alpha\beta} + O(\beta^4)\,, \tag{29}$$

$$g_{\alpha 4} = \overset{3}{g}_{\alpha 4} + O(\beta^5)\,, \tag{30}$$

$$g_{44} = -1 + \overset{2}{g}_{44} + \overset{4}{g}_{44} + O(\beta^6)\,, \tag{31}$$

where the symbols $\overset{N}{g}_{ij}$ mean the terms of the order β^N. Using the solution of the field equations in the Newtonian approximation we claim that the expansion of the σ-field starts with a term of the order $\eta\phi \sim \eta\beta^2$:

$$\sigma = \overset{2}{\sigma} + \overset{4}{\sigma} + O(\beta^6)\,. \tag{32}$$

Therefore in (27) the σ-field can only appear in the combinations

$$(\sigma_{,4})^2 \sim \eta^2\beta^6, \qquad \sigma_{,4}\sigma_{,\alpha} \sim \eta^2\beta^5, \qquad \sigma_{,\alpha}\sigma_{,\beta} \sim \eta^2\beta^4\,. \tag{33}$$

Since $\eta \leq 10^{-3}$, the terms containing the σ-field can be omitted in (27). Substituting (29) and (32) into the field equations (27) and (28) one finds that indeed in harmonic coordinates (*i.e.* coordinates in which $\Gamma^i{}_{jm}g^{jm} = 0$) the field equations are in agreement with the expansions being used. We obtain the following results:

$$\Delta\overset{2}{g}_{44} = \frac{8\pi\gamma_N}{c^4}\overset{0}{\Theta}{}^{44}\,, \tag{34}$$

$$\Delta\overset{2}{g}_{\alpha\beta} = \frac{8\pi\gamma_N}{c^4}\delta_{\alpha\beta}\overset{0}{\Theta}{}^{44}\,, \tag{35}$$

$$\Delta\overset{3}{g}_{\alpha 4} = -\frac{16\pi\gamma_N}{c^4}\overset{1}{\Theta}{}^{\alpha 4}\,, \tag{36}$$

$$\Delta\overset{4}{g}_{44} - \overset{2}{g}_{44,44} - \overset{2}{g}_{\alpha\beta}\overset{2}{g}_{44,\alpha\beta} + (\nabla\overset{2}{g}_{44})^2 = \frac{8\pi\gamma_N}{c^4}(\overset{2}{\Theta}{}^{44} - 2\overset{2}{g}_{44}\overset{0}{\Theta}{}^{44} + \overset{2}{\Theta}{}^{\alpha\alpha})\,, \tag{37}$$

$$\Delta\overset{2}{\sigma} = \frac{8\pi\gamma_N}{\lambda c^2}\overset{0}{\vartheta}\,, \tag{38}$$

$$\overset{4}{\sigma}_{,\alpha\alpha} - \overset{2}{\sigma}_{,44} - \left(\overset{2}{g}_{\alpha\beta}\overset{2}{\sigma}_{,\alpha}\right)_{,\beta} + \frac{1}{2}\overset{2}{\sigma}_{,\alpha}\left(\overset{2}{g}_{\kappa\kappa,\alpha} - \overset{2}{g}_{44,\alpha}\right) = \frac{8\pi\gamma_N}{\lambda c^4}\overset{2}{\vartheta}\,. \tag{39}$$

One should realize that the equations (34) to (37) do not exhibit the constant λ. Therefore the post-Newtonian solution for the metric does not depend on λ. Though the equations (34) to (37) formally do not differ from the corresponding equations in the Einstein theory, their solutions can be different because of the influence of scalaric σ-field on the equation of motion of matter.

Taking into account the additional condition (21a), for a perfect fluid ($\Theta^{ij} = -\mu u^i u^j - p(\frac{1}{c^2}u^i u^j + g^{ij})$) the set of equations (34) to (39) can be integrated. The solutions read:

$$\begin{aligned}
&\text{a)} \quad g_{\alpha\beta} = \delta_{\alpha\beta}\left(1 - \frac{2\phi}{c^2}\right) + O(\beta^4)\,,\\
&\text{b)} \quad g_{\alpha 4} = \frac{1}{c^3}\zeta_\alpha + O(\beta^5)\,,\\
&\text{c)} \quad g_{44} = -\left[1 + \frac{2\phi}{c^2} + \frac{2}{c^4}\left(\phi^2 + \psi\right)\right] + O(\beta^6);
\end{aligned} \tag{40}$$

$$\sigma = \frac{2\eta}{c^2\lambda}\left[\phi + \frac{1}{c^2}(\psi + 2\Phi_1 + 3\Phi_4)\right] + O(\beta^4)\,, \tag{41}$$

where

$$\phi(\boldsymbol{x}, t) = -\gamma_N \int \frac{\mu_0(\boldsymbol{x}', t)}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3\boldsymbol{x}'\,, \tag{42}$$

$$\zeta_\alpha(\boldsymbol{x}, t) = 4\gamma_N \int \frac{\mu_0(\boldsymbol{x}', t) v_\alpha(\boldsymbol{x}', t)}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3\boldsymbol{x}'\,, \tag{43}$$

$$\psi(\boldsymbol{x}, t) = -\frac{1}{4\pi}\int \frac{\partial^2 \phi(\boldsymbol{x}', t)}{\partial t^2}\frac{1}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3\boldsymbol{x}' - 2\Phi_1 - 2\left(1 - \frac{\eta^2}{\lambda}\right)\Phi_2 - 3\Phi_4\,. \tag{44}$$

$$\Phi_1(\boldsymbol{x}, t) = \gamma_N \int \frac{\mu_0(\boldsymbol{x}', t) v^2(\boldsymbol{x}', t)}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3\boldsymbol{x}'\,, \tag{45}$$

$$\Phi_2(\boldsymbol{x}, t) = -\gamma_N \int \frac{\mu_0(\boldsymbol{x}', t) \phi(\boldsymbol{x}', t)}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3\boldsymbol{x}'\,, \tag{46}$$

$$\Phi_4(\boldsymbol{x}, t) = \gamma_N \int \frac{p(\boldsymbol{x}', t)}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3\boldsymbol{x}'\,. \tag{47}$$

As we expected, the solution found differs from the corresponding solution in the Einstein theory only in the order β^4 (see (44)). Concluding this section we present the equation of motion for a point-like test body in post-Newtonian approximation:

$$\begin{aligned}
\frac{d\boldsymbol{v}}{dt} = &- (1 + \frac{2}{\lambda}\eta^2)\nabla(\phi + \frac{2}{c^2}\phi^2 + \frac{1}{c^2}\psi) - (1 - \frac{2}{\lambda}\eta^2)\frac{v^2}{c^2}\nabla\phi \\
&+ (3 - \frac{2}{\lambda}\eta^2)\frac{\boldsymbol{v}}{c^2}\frac{\partial\phi}{\partial t} - \frac{1}{c^2}\frac{\partial\boldsymbol{\zeta}}{\partial t} + \frac{\boldsymbol{v}}{c^2}\times(\nabla\times\boldsymbol{\zeta}) + \frac{4\boldsymbol{v}}{c^2}(\boldsymbol{v}\nabla)\phi \\
&- \frac{2}{\lambda c^2}\eta^2\nabla(2\Phi_1 + 3\Phi_4)\,.
\end{aligned} \tag{48}$$

4.2. Perihelion motion of Mercury

In this section we investigate the motion of a test body (*e.g.* Mercury) around a central body (*e.g.* sun) which for simplicity will be considered as non-rotating and spherically symmetric. As before we suggest that the condition $\eta \ll 1$ is fulfilled. Under these assumptions the integration of the field equations (34) to (37) leads to

$$
\begin{aligned}
\text{a)}\quad & g_{\alpha\beta} = \delta_{\alpha\beta}(1 + \frac{2\gamma_N M_c}{c^2 R}) + O(\beta^4)\,, \\
\text{b)}\quad & g_{\alpha 4} = 0\,, \\
\text{c)}\quad & g_{44} = -1 + \frac{2\gamma_N M_c}{c^2 R} - \frac{2\gamma_N^2 M_c^2}{c^4 R^2} + O(\beta^6)\,,
\end{aligned} \tag{49}
$$

where M_c is a constant coinciding with the inertial mass of the central body if the scalaric field vanishes. Therefore in this approximation the metric has the same form as the metric in the Einstein theory.

The equation of motion of the test body reads:

$$
\begin{aligned}
\frac{\mathrm{d}^2 \boldsymbol{R}}{\mathrm{d}t^2} \equiv \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = & -\frac{\gamma_N M_c}{R^3}\boldsymbol{R}\left[\left(1 + \frac{2}{\lambda}\eta^2\right)\left(1 - \frac{4\gamma_N M_c}{c^2 R}\right)\right. \\
& \left. + \left(1 - \frac{2}{\lambda}\eta^2\right)\frac{v^2}{c^2}\right] + \frac{4\gamma_N M_c}{c^2 R^3}\boldsymbol{v}(\boldsymbol{v}\boldsymbol{R}) + O(\beta^4)\,.
\end{aligned} \tag{50}
$$

In the Newtonian approximation the equation (50) goes over into the equation of motion

$$
\frac{\mathrm{d}^2 \boldsymbol{R}}{\mathrm{d}t^2} \equiv \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{\gamma_N M_c}{R^3}\boldsymbol{R}\left(1 + \frac{2\eta^2}{\lambda}\right)\,. \tag{51}
$$

Integration leads to the following set of equations:

$$
R^2 \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \left[\gamma_N M_c \left(1 + \frac{2}{\lambda}\eta^2\right) p\right]^{1/2}\,, \tag{52}
$$

$$
\boldsymbol{v} \equiv \frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t} = \left[\frac{\gamma_N M_c (1 + \frac{2}{\lambda}\eta^2)}{p}\right]^{1/2} [-\boldsymbol{e}_x \sin\varphi + \boldsymbol{e}_y(\varepsilon + \cos\varphi)]\,, \tag{53}
$$

where

$$
\text{a)}\quad \boldsymbol{R} = R(\boldsymbol{e}_x \cos\varphi + \boldsymbol{e}_y \sin\varphi), \qquad \text{b)}\quad R = p(1 + \varepsilon\cos\varphi)^{-1}\,. \tag{54}
$$

As it is well-known, this solution describes the motion of the test body in a plane (spanned by the basis vectors $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$) orthogonal to the angular momentum. We remind that R and φ are polar coordinates in the plane of motion, p and ε (eccentricity) are the parameters of the ellipse.

Applying the method of successive approximation we find the following post-Newtonian solution:

$$\boldsymbol{v} = \left[\frac{\gamma_N M_c(1+\frac{2}{\lambda}\eta^2)}{p}\right]^{1/2} [-\boldsymbol{e}_x \sin\varphi + \boldsymbol{e}_y(\varepsilon + \cos\varphi)] + \delta\boldsymbol{v}\,, \tag{55}$$

$$R^2\frac{\mathrm{d}\varphi}{\mathrm{d}t} \equiv (\boldsymbol{R}\times\boldsymbol{v})_z = \left[\gamma_N M_c(1+\frac{2}{\lambda}\eta^2)p\right]^{1/2}(1+\delta h)\,, \tag{56}$$

where

$$\delta h = -\frac{4\gamma_N M_c}{c^2 p}\varepsilon\cos\varphi \tag{57}$$

and

$$\begin{aligned}\delta\boldsymbol{v} =& \sqrt{\frac{\gamma_N M_c(1+\frac{2}{\lambda}\eta^2)}{p}}\left(\frac{\gamma_N M_c}{c^2 p}\right)\left\{\boldsymbol{e}_x\left[\sin\varphi\left((3-\varepsilon^2)+\frac{2}{\lambda}\eta^2(1+\varepsilon^2)\right)\right.\right.\\ &\left. -\varepsilon(3-\frac{2}{\lambda}\eta^2)\varphi + \frac{1}{2}\varepsilon(1+\frac{2}{\lambda}\eta^2)\sin 2\varphi\right]\\ &\left. -\boldsymbol{e}_y\left[\cos\varphi(1+\varepsilon^2)(3+\frac{2}{\lambda}\eta^2)+\frac{\varepsilon}{2}\left(1+\frac{2}{\lambda}\eta^2\right)\cos 2\varphi\right]\right\}. \end{aligned}\tag{58}$$

Using the identity

$$\frac{\mathrm{d}}{\mathrm{d}\varphi}\frac{1}{R} = -\left(R^2\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^{-1}\left(\frac{\boldsymbol{R}\boldsymbol{v}}{R}\right), \tag{59}$$

we get from (58) for the trajectory of the test body:

$$\frac{p}{R} = 1+\varepsilon\cos\varphi + \frac{\gamma_N M_c}{c^2 p}\left[\frac{1}{2}\varepsilon\cos\varphi(7-\frac{2}{\lambda}\eta^2)+\varepsilon(3-\frac{2}{\lambda}\eta^2)\varphi\sin\varphi\right]. \tag{60}$$

The calculation of the perihelion motion leads to the result

$$\delta\varphi \approx \frac{6\pi\gamma_N M_c}{c^2 p}\left(1-\frac{2}{3\lambda}\eta^2\right) \tag{61}$$

for one revolution. This formula (61) is identical with the relation found by Schmutzer [3] using the exact spherically symmetric solution. If $\eta = 0$ (61) coincides with the corresponding result in the Einstein theory.

Let us finally mention that the deviation of the astronomical observations of the perihelion motion of Mercury from the predicted values in Einstein's theory is $0.1''$ to $0.01''$ (see [8]) and agrees with the equation (61) in PUFT if $\eta \leq 10^{-3}$.

5. Conclusion

In the present paper we investigated the equation of motion of a point-like test body in PUFT and the possibility of functional dependence of the inertial mass of an external scalaric field. Although the idea of variability of inertial mass is not new itself (see *e.g.* [10]), in PUFT it appears quite natural.

Let us remind that in PUFT the gravitational central mass M_c appears in the exact spherically symmetric solution. At the same time the acceleration of test bodies in an external gravitational field (see (50)) is characterized by another quantity $M_c\left(1+\frac{2}{\lambda}\eta^2\right)$. Naturally the latter must be identified as the mass of the Sun in experiments in the solar system, *i.e.* $M_\odot = M_c\left(1+\frac{2}{\lambda}\eta^2\right)$ since the mass of the Sun has to be determined by its gravitational interaction. Hence follows that the perihelion motion of Mercury reads

$$\delta\varphi \approx \frac{6\pi\gamma_N M_\odot}{c^2 p}\left(1-\frac{8}{3\lambda}\eta^2\right) . \tag{62}$$

The Einstein effects, as light deflection and photon frequency shift, are determined exclusively by the space-time geometry. Therefore the results in PUFT coincide with the corresponding ones in the Einstein theory if M_c is used [3]. Expressing the mass M_c by the experimentally determined value of the solar mass $M_\odot$, we obtain the following formulas for the light deflection:

$$\Delta\chi = \frac{4\gamma_N}{R_\odot c^2}M_\odot\left(1-\frac{2}{\lambda}\eta^2\right) \tag{63}$$

and for the photon frequency shift

$$\frac{\omega}{\omega_\infty} = 1+\frac{\gamma_N}{rc^2}M_\odot\left(1-\frac{2}{\lambda}\eta^2\right) , \tag{64}$$

where $R_\odot$ is the radius of the Sun. Of course (63) and (64) can be applied for calculation of post-Newtonian effects only if corrections connected with η^2 are greater than the post-post-Newtonian ones.

REFERENCES

[1] E. Fischbach, D. Sudarsky, A. Szafar, C. Talmadage, S.H. Aronson, *Phys. Rev. Lett.* **56**, 3 (1986); E. Fishbach, C. Talmadge, D. Krause, *Phys.Rev.* **D43**, 460 (1991); F.D. Stacey, G.J. Tuck, G.I. Moore, S.C. Holding, B.D. Goodwin, R. Zhou, *Rev. Mod. Phys.* **59**, 157 (1987); *New and Exotic Phenomena*, Proceedings of the 25th Rencontre de Moriond (10th Moriond Workshop), Les Ares, France, 1990. Edited by O. Fackler and J. Tran Thanh Van, Editions Frontières, Gif-sur-Yvetle 1990; V. De Sabbata, V.N. Melnikov, P.I. Pronin, *Progr.Theor.Phys.* **88**, 623 (1992).

[2] P. Jordan, *Z. Phys* **157**, 112 (1959).

[3] E. Schmutzer, *Relativistische Physik, Teubner Verlagsgesellschaft*, Leipzig 1968; E. Schmutzer, Proceedings of the 9th International Conference on General Relativity and Gravitation, Deutscher Verlag der Wissenschaften, Berlin 1983; *Ann. Phys.* **45**, 578 (1988); *Astron. Nachr.* **311**, 329 (1990); *Fortschr. Phys.* **43**, 613 (1995).

[4] C.M. Will, *Theory and experiment in gravitational physics*, Cambridge University Press, 1981.

[5] R.V. Pound, G.A. Rebka, *Phys. Rev. Lett.* **4**, 337 (1960).

[6] R.V. Pound, J.L. Snider, *Phys. Rev.* **B140**, 788 (1965).

[7] C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation*, W.H. Freeman and Company, San Francisco 1973.

[8] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, John Wiley and Sons, Inc., New York–London–Sydney–Toronto 1972.

[9] A.K. Gorbatsievich, Ho Si Mau Thuc, E. Schmutzer, *Docl. Acad. Nauk* Belarusi **38**, No 6, 41, (1994); *Acta Phys. Pol.* **B26**, 1439 (1995).

[10] J.D. Bekenstein, *Phys. Rev.* **D15**, 1458 (1977).