VACUUM POLARIZATION IN SCHWARZSCHILD SPACETIME: BOULWARE VACUUM

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Approximate regularized expectation values of the field fluctuation $\langle\phi^2\rangle_{reg}$ and the stress–energy tensor $\langle T^\mu_\nu\rangle_{reg}$ of the massless, conformally invariant, scalar field in the Boulware vacuum state in Schwarzschild spacetime are constructed by means of Hadamard regularization. It is shown that reconstruction of the characteristics of the vacuum polarization from its asymptotic behaviour leads to formulas that satisfactorily reproduce existing approximate expressions and closely follow exact numerical calculations.

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Despite of its inherent limitations quantum field theory in curved background has often been used as an useful tool for investigating problems that arise in strong gravitational fields and which should be analysed in the framework of still unknown quantum gravity. In the absence of the well-defined particle concept construction of characteristics of vacuum polarization is of principal importance. The special interest that we have in the regularized mean value of the field fluctuation $\langle\phi^2\rangle_{\rm reg}$ and the stress-energy tensor $\langle T^\mu_\nu\rangle_{\rm reg}$ evaluated in a suitable vacuum state follows from the fact that both quantities reflect physical content of the theory formulated in a spacetime describing black hole. Moreover, the stress-energy tensor couples to the gravitational field, and may be treated as a source term, allowing in principle, when a state of the quantized field is chosen properly, to determine the back reaction of the quantum field upon the spacetime geometry by means of the semi-classical Einstein field equations, whereas the field fluctuation plays an essential role in the phase transitions in the vicinity of the black hole.

Mathematical difficulties encountered in the attempts to construct and study the vacuum polarization in a general spacetime are well known and, beside the fact that the mode functions corresponding to vacuum states cannot be expressed in terms of known special functions, are connected mainly with the divergent mode sums. These obstacles invalidate purely analytical treatment of the problem and suggest the way of its partial solution. Indeed, thanks to the numerical investigations carried out by a number of authors we have, at least in the static case, satisfactory knowledge and understanding of the vacuum polarization effects for both massive and massless fields [1–4]. However, having in mind further applications the information concerning functional dependence of the mean value of the stress-energy tensor on wide class of background spacetimes as well as quantum states is essential. Unfortunately $\langle\phi^2\rangle_{\rm reg}$ and $\langle T^{\mu}_{\nu}\rangle_{\rm reg}$ critically depend on the metric tensor. It is natural therefore that much effort have been concentrated on developing the approximate methods which allow qualitative treatment of the problem. Such a programme have been successfully undertaken by Page [5], Brown and Ottewill [6], Zannias [7], Page, Brown and Ottewill [8], Frolov and Zel'nikov [9], Hiscock [10], Vaz [11], Nugaev [12], Anderson [13], and Anderson, Hiscock and Samuel [14, 15].

Among presently available methods of regularization the Hadamard regularization scheme seems to be most fruitful [16–18]. In the Hadamard formalism one assumes that the covariant scalar field equation in four dimensions has the solution with the following structure of singularities:

$$G^{(1)}(x,x') = \frac{1}{8\pi^2}\Delta^{1/2}(x,x')\left[\frac{2}{\sigma} + v(x,x')\ln\sigma + \omega(x,x;)\right], \tag{1}$$

where $2\sigma(x,x')$ is the square of the geodetic length between x and x' [19], and Δ the Van Vleck-Morette determinant is given by

$$\Delta(x,x') = -[g(x)]^{-1/2} Det\left[-\frac{\partial^2\sigma(x,x')}{\partial x^\mu \partial x^\nu}\right][g(x')]^{-1/2}. \tag{2}$$

Biscalars $v(x,x')$ and $\omega(x,x')$ are both smooth symmetric functions which expanded in symmetric covariant Taylor series

$$v(x,x') = v^{(0)} + v^{(1)}{}_{\mu}\sigma^\mu + v^{(2)}{}_{\mu\nu}\sigma^\mu\sigma^\nu + ..., \tag{3}$$

and

$$\omega(x,x') = \omega^{(0)} + \omega^{(1)}{}_{\mu}\sigma^\mu + \omega^{(2)}{}_{\mu\nu}\sigma^\mu\sigma^\nu + ..., \tag{4}$$

in terms of $\sigma^\mu = \sigma^{;\mu}$, and inserted into the field equation are to be determined from the recursive relations and boundary conditions.

In terms of $\omega^{(0)}$ and $\omega^{(2)\mu}_{\ \ \nu}$ the stress-energy tensor and the vacuum fluctuation in the case the massless conformally invariant scalar field are generally given by

$$8\pi^2\langle T_{\mu\nu}\rangle_{\rm reg} = \frac{1}{6}(\omega^{(0)}{}_{;\mu\nu} - \frac{1}{4}g_{\mu\nu}\Box\omega^{(0)}) - \omega_{\mu\nu} + \frac{1}{8}g_{\mu\nu}a_2, \tag{5}$$

and

$$\langle\phi^2\rangle_{\rm reg} = \frac{1}{16\pi^2}\omega^{(0)}, \tag{6}$$

where

$$a_2 = \frac{1}{180}(\Box R - R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}), \tag{7}$$

$\omega_{\mu\nu}$ is a traceless part of $\omega^{(2)}_{\mu\nu}$, and all symbols in (7) have their usual meaning. It seems that especially appealing feature of the method, when adopted in the spherically symmetric and static background, is the possibility of construction of the three omega functions which are simultaneous solutions to the constraint equations

$$\frac{1}{8}\omega^{(0)}{}_{;\mu\alpha}{}^{\alpha} - \omega^{(2)}{}_{\mu\alpha}{}^{;\alpha} = -\frac{1}{24}R_{\mu}{}^{\alpha}\omega^{(0)}{}_{;\alpha} - \frac{1}{12}a_{2;\mu}, \tag{8}$$

with appropriate boundary conditions. Since the constraint equation involves three unknown functions it has infinitely many solutions. To select the interesting one, it is necessary, beside the knowledge of the boundary conditions, to adopt some plausible hypotheses.

The idea of reconstructing $\langle T^{\mu}_{\nu}\rangle_{\rm reg}$ from its asymptotic behaviour is not new, and may be traced back to the celebrated Christensen and Fulling paper [20]. It is based on a safe anticipation that in the static and spherically symmetric case it is relatively easy to find the asymptotic characteristics of the vacuum polarization effects. In this note, which is motivated by the works of Tadaki [17], Bernard [18], and Vaz [11] we shall follow this approach and evaluate the approximate expectation value of the stress-energy tensor and the field fluctuation of the conformally invariant massless scalar field in the Boulware vacuum in the Schwarzschild spacetime. We assume that $\omega^{(0)}$, ω^{r}_{r}, and ω^{θ}_{θ} may be represented by the functions of the type

$$\sum_{m=-k}^{k'} (1-x)^m\, W_m(x), \qquad k, k' \geq 0, \tag{9}$$

where $x = 2M/r$ and $W_m(x)$, for each value of m, is a polynomial in x, and show that it is possible to construct solutions, which lead to the field fluctuation and stress-energy tensor which reflect principal features of the exact $\langle\phi^2\rangle_{\rm reg}$ and $\langle T^{\mu}_{\nu}\rangle_{\rm reg}$ with a reasonable success. We remark here that assuming very general conditions which the expected thermal stress tensor should satisfy, Tadaki [17] by means of the Hadamard regularization was able to construct the approximate expressions describing vacuum fluctuation and the stress-energy tensor in what he called the perturbed Israel-Hartle-Hawking state. In Ref. [21, 22] Hadamard regularization has been used to generalize Page's formulas describing the stress energy tensor and the field fluctuation of the massless scalar field in the Schwarzschild spacetime in the Israel–Hartle–Hawking vacuum.

For the Schwarzschild line element the stress-energy tensor has the form

$$8\pi^2\langle T^\mu_\nu\rangle_{\rm reg} = -\frac{1}{24}\left(1-\frac{2M}{r}\right)\frac{d^2}{dr^2}\omega^{(0)}{\rm diag}[1,-3,1,1], + \frac{1}{24}\left(\frac{6M}{r^2}-\frac{2}{r}\right)\frac{d}{dr}\omega^{(0)}{\rm diag}[1,1,-1,-1] - \omega^\mu_\nu + \frac{M^2}{30r^6}\delta^\mu_\nu. \tag{10}$$

In the latter, as a working hypothesis, we shall adopt the following: all the omega functions, i.e. $\omega^{(0)}, \omega^r_r, {\rm and}\omega^\theta_\theta$, are responsible for the asymptotic behaviour of the traceless part of the regularized mean value of the stress-energy tensor in the Boulware vacuum state. Moreover, we shall assume that the polynomials W_m are chosen in such a way that their contributions to the constraint equation is at most of the same order as the contribution of the last term in the rhs in Eq. (8). The latter condition results in necessary truncations and together with the simplicity demand allows us to construct general expressions of $\langle\phi^2\rangle_{\rm reg}$ and $\langle T^\mu_\nu\rangle_{\rm reg}$.

From general arguments Christensen and Fulling [20] have shown that near the event horizon the radial and tangential component of the traceless stress-energy tensor should diverge as $-(1-\frac{2M}{r})^{-2}$, whereas at large radii this components should fall off as $-r^{-6}$. Since the Boulware vacuum is expected to properly describe physical situation outside spherically symmetric star such a behaviour should not be treated as too worrisome [10]. Indeed, even in the ideal case of perfect fluid sphere, if the adiabatic sound speed is real, the minimal radius is $9M/4$, whereas assumption that the adiabatic sound speed does not exceed the speed of light yields even larger result, $2.8M$ [23, 24].

For the traceless radial and tangential components of the stress–energy tensor (10) near the event horizon one has

$$3\left(1-\frac{2M}{r}\right)\frac{d^2}{dr^2}\omega^{(0)} + \left(\frac{6M}{r^2}-\frac{2}{r}\right)\frac{d}{dr}\omega^{(0)} - \omega^r_r \sim -\left(1-\frac{2M}{r}\right)^{-2}, \tag{11}$$

and

$$-\left(1-\frac{2M}{r}\right)\frac{d^2}{dr^2}\omega^{(0)} - \left(\frac{6M}{r^2}-\frac{2}{r}\right)\frac{d}{dr}\omega^{(0)} - \omega^\theta_\theta \sim -\left(1-\frac{2M}{r}\right)^{-2}. \tag{12}$$

Similarly as $r\to\infty$

$$3\left(1-\frac{2M}{r}\right)\frac{d^2}{dr^2}\omega^{(0)} + \left(\frac{6M}{r^2}-\frac{2}{r}\right)\frac{d}{dr}\omega^{(0)} - \omega^r_r \sim -\frac{1}{r^6}, \tag{13}$$

and

$$-\left(1-\frac{2M}{r}\right)\frac{d^2}{dr^2}\omega^{(0)}-\left(\frac{6M}{r^2}-\frac{2}{r}\right)\frac{d}{dr}\omega^{(0)}-\omega_\theta^\theta\sim-\frac{1}{r^6}\,. \tag{14}$$

It is useful to investigate the role played by each omega function separately. At first, let us consider the function $\omega^{(0)}$. Inspection of equations (11) and (12) shows that first group of the Christensen–Fulling conditions are satisfied up to a sign if we choose $\omega^{(0)}\sim(1-\frac{2M}{r})^{-1}$, whereas in order to satisfy the conditions (13) and (14) the appropriate guess is $\omega^{(0)}\sim r^{-4}$. Therefore we put

$$\omega^{(0)}=T\left(1-\frac{2M}{r}\right)^{-1}r^{-4}+T_1\,, \tag{15}$$

where T and T_1 are arbitrary constants. Note that because of the adopted hypotheses the above form is unique.

Now, the constraint equation reduces to the following form:

$$\begin{aligned}&\left(1-\frac{2M}{r}\right)^{-3}T\left(-\frac{9}{r^7}+\frac{47M}{r^8}-\frac{86M^2}{r^9}+\frac{54M^3}{r^{10}}\right)\\&-\left\{\frac{d}{dr}\omega_r^r+\left(\frac{2}{r}-\frac{2M}{r^2}\right)\left(1-\frac{2M}{r}\right)^{-1}\omega_r^r\right.\\&\left.+\left(\frac{6M}{r^2}-\frac{2}{r}\right)\left(1-\frac{2M}{r}\right)^{-1}\omega_\theta^\theta\right\}-\frac{M^2}{5r^7}=0\,.\end{aligned} \tag{16}$$

Studying the influence of the various choices of ω_r^r and ω_θ^θ on the asymptotic form of the stress-energy tensor, one concludes that correct asymptotics may be obtained assuming:

$$\begin{aligned}\omega_r^r=T\frac{a_0}{r^6}+T\left(1-\frac{2M}{r}\right)^{-1}\left(\frac{a_1}{r^6}+\frac{a_2}{r^7}\right)\\+T\left(1-\frac{2M}{r}\right)^{-2}\left(\frac{a_3}{r^6}+\frac{a_4}{r^7}+\frac{a_5}{r^8}\right)\,,\end{aligned} \tag{17}$$

and

$$\begin{aligned}\omega_\theta^\theta=T\frac{A_0}{r^6}+T\left(1-\frac{2M}{r}\right)^{-2}\left(\frac{A_1}{r^6}+\frac{A_2}{r^7}+\frac{A_3}{r^8}\right)\\+T\left(1-\frac{2M}{r}\right)^{-1}\left(\frac{A_4}{r^6}+\frac{A_5}{r^7}\right)\,,\end{aligned} \tag{18}$$

where the coefficients A_i, a_i, T, and T_1 are to be determined.

Since the Boulware vacuum is defined by the mode functions which are of positive frequences with respect to the timelike Killing vector $\frac{\partial}{\partial t}$ one expects that at large radii the Boulware Green function reduces to

$$G(t,r,\theta,\phi;0,r,\theta,\phi) \sim \frac{-i}{4\pi^2 t^2}\,. \tag{19}$$

Moreover, simple calculations indicate that along the geodesic which joins these points [25]

$$\frac{1}{2\sigma} = -\frac{1}{t^2} + \frac{M^2}{12r^4}\,, \tag{20}$$

and consequently one has $T = -1/3M^2$, and $T_1 = 0$. It should be noted that such choice of the parameters yields the result obtained earlier by Frolov and Zel'nikov (see *e.g.* [26]).

Substituting equations (15), (17), and (18) into Eq. (16), collecting the terms with the like powers of r, one obtains four equations which allow to express four constants, say, a_1, a_2, A_2, and A_3 in terms of remaining ones:

$$a1 = -a_0 - a_3 - \frac{A_0}{2} - \frac{A_1}{2} - \frac{A_4}{2} + \frac{21}{10}\,, \tag{21}$$

$$a_2 = M\left(2a_0 - \frac{42}{5}a_3 - \frac{21}{5}a_4 - \frac{8}{5M}a_5 - \frac{6}{5}A_0 + \frac{6}{5}A_1 + \frac{6}{5}A_4 - \frac{9}{25}\right), \tag{22}$$

$$A_2 = M\left(16a_3 + \frac{8}{M}a_4 + \frac{4}{M^2}a_5 \; -4A_1 - 2A_4\frac{1}{M}A_5 - 4\right), \tag{23}$$

$$A_3 = M^2\left(-28a_3 - \frac{14}{M}a_4 - \frac{7}{M^2}a_5 \; +4A_1 + 4A_4 + \frac{2}{M}A_5 + 7\right). \tag{24}$$

Our choice, which is to a great extent arbitrary and limited only by tractability of the system of equations, was motivated by the fact that in such a parametrization the asymptotics of the stress-energy tensor are exclusively described by coefficients of one type. Indeed, equations (21)–(24) lead to the following asymptotic behaviour as $r \to \infty$

$$r^6\langle T_t^t\rangle_{\rm reg} = -\frac{M^2}{16\pi^2}(1 + A_0 + A_1 + A_4)\,, \tag{25}$$

$$r^6\langle T_r^r\rangle_{\rm reg} = -\frac{M^2}{720\pi^2}(19 + 15A_0 + 15A_1 + 15A_4)\,, \tag{26}$$

$$r^6\langle T_\theta^\theta\rangle_{\rm reg} = \frac{M^2}{360\pi^2}(19 + 15A_0 + 15A_1 + 15A_4)\,, \tag{27}$$

whereas as $r \to 2M$ one has

$$\left(1 - \frac{2M}{r}\right)^2 \langle T^\mu_\nu \rangle_{\text{reg}} = \frac{1}{90(8M)^4\pi^2}\left(24a_3 + \frac{12}{M}a_4 + \frac{6}{M^2}a_5 - 5\right) \text{diag}[-3, 1, 1, 1]^\mu_\nu . \quad (28)$$

In the latter we shall assume that the limit (27) is nonzero. Inspection of (10) shows that the resulting stress-energy tensor is independent of the coefficients a_0, and A_5. To specify the $\langle T^\mu_\nu \rangle_{\text{reg}}$ further one should know at least approximate values of remaining parameters.

Integrating the conservation equation, for sufficiently large r one has

$$\langle T^r_r \rangle_{\text{reg}} = \frac{1}{r^2}(H(r) + G(r)) + \frac{Q^2}{Mr^2}, \quad (29)$$

where

$$H(r) = \frac{1}{2}\int_{2M}^{r} (r' - 2M)\langle T^\mu_\mu \rangle_{\text{reg}} dr', \quad (30)$$

$$G(r) = 2\int_{2M}^{r} (r' - 3M)(\langle T^\theta_\theta \rangle_{\text{reg}} - \frac{1}{4}\langle T^\mu_\mu \rangle_{\text{reg}}) dr', \quad (31)$$

and Q is a state-dependent constant. By the Christensen–Fulling conditions it is expected that in the Boulware vacuum the terms which are proportional to r^{-2} should cancel. Since at spatial infinity $\langle T^\theta_\theta \rangle_{\text{reg}} - \frac{1}{4}\langle T^\mu_\mu \rangle_{\text{reg}} \sim -\alpha/r^6$, $\alpha > 0$, equations (7), (29), and (30) give:

$$\lim_{r\to\infty} \frac{\langle T^r_r \rangle_{\text{reg}}}{\langle T^\theta_\theta \rangle_{\text{reg}}} = -\frac{1}{2}. \quad (32)$$

The above condition is satisfied by our tensors, and, therefore, yields no more information.

Since we expect that at most two coefficients may be determined from asymptotic analyses and since we are looking for a simplest solution of the problem in the latter we shall equate to zero coefficients a_4, a_5, A_0, and A_4. It should be emphasized that the final result is independent of the concrete choice of the pair (a_i, A_j) of the retained parameters.

The first condition is provided by the Christensen–Fulling formula:

$$\langle T^\mu_\nu \rangle^\nu_{\text{reg}} - \langle T^\mu_\nu \rangle_{\text{reg}} = \frac{1}{90\pi^2(8M)^4}\left(1 - \frac{2M}{r}\right)^{-2} \text{diag}[-3, 1, 1, 1], \quad (33)$$

where superscript ν refers to the Israel–Hartle–Hawking vacuum. Originally it was supposed to hold everywhere, however only recently it was showed by Jensen, Ottewill and MacLaughlin [4] that (33) is valid only asymptotically, being generally incorrect. What we need is the behaviour of the difference $\langle T^{\mu}_{\nu}\rangle^{\nu}_{\rm reg} - \langle T^{\mu}_{\nu}\rangle_{\rm reg}$, as $r \to 2M$, and hence our conclusions will not be affected. Since the stress enegy tensor in the Israel-Hartle-Hawking vacuum state is expected to be regular on the event horizon one has

$$\left(1 - \frac{2M}{r}\right)^{+2} \langle T^{\mu}_{\nu}\rangle_{\rm reg} = \frac{1}{90\pi^2 (8M)^4} {\rm diag}[-3, 1, 1, 1]\,, \tag{34}$$

and consequently $a_3 = 49/240$. By virtue of asymptotic conditions (12), (13) one has

$$-\frac{44}{30} < A_1 < -\frac{35}{30}\,. \tag{35}$$

The latter constraint may be made even more stringent if we assume that at large radii $\langle T^{\theta}_{\theta}\rangle_{\rm reg} \sim -r^{-6}$. It yields the double inequality

$$-\frac{22}{15} < A_1 < -\frac{19}{15}\,. \tag{36}$$

Defining a new parameter $\delta = A_1 + \frac{4}{3}$ the one-parameter family of approximate stress-energy tensor in the Boulware vacuum state may compactly be written as

$$\langle T^{\mu}_{\nu}\rangle_{\rm reg} = \frac{M^2}{1440\pi^2 r^6}\left\{ \frac{(2 - \frac{3M}{r})^2}{(1 - \frac{2M}{r})^2} {\rm diag}[3, -1, -1, -1]^{\mu}_{\nu} + 6{\rm diag}[3, 1, 0, 0]^{\mu}_{\nu} \right\} + \Delta^{\mu}_{\nu}\,, \tag{37}$$

where Δ^{μ}_{ν} is given by

$$\Delta^{t}_{t} = \frac{\delta}{1440\pi^2 r^8}\left(1 - \frac{2M}{r}\right)^{-2} \left(-90M^2 r^2 + 348M^3 r - 336M^4\right)\,, \tag{38}$$

$$\Delta^{\theta}_{\theta} = \frac{\delta}{1440\pi^2 r^8}\left(1 - \frac{2M}{r}\right)^{-2} \left(60M^2 r^2 - 240M^3 r + 240M^4\right)\,, \tag{39}$$

$$\Delta^{r}_{r} = \frac{\delta}{1440\pi^2 r^8}\left(1 - \frac{2M}{r}\right)^{-2} \left(-30M^2 r^2 + 132M^3 r - 144M^4\right)\,, \tag{40}$$

and δ should satisfy

$$-\frac{2}{15} < \delta < \frac{1}{15}\,. \tag{41}$$

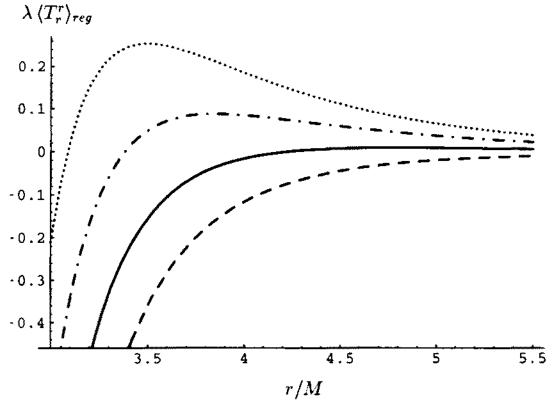

Fig. 1. $\lambda\langle T^r_r\rangle_{\rm reg}$ ($\lambda = 90\pi^2(8M)^4$) as a function of r, displayed for $\delta = -2/15$ (dotted line), $\delta = -1/15$ (dashed–dotted line), $\delta = 0$ (solid line), and $\delta = 1/15$ (dashed line).

Note that with $\delta = 0$, Eq. (37) reduces to the approximate stress-energy tensor obtained by Brown and Ottewil [6] who analysed conformal transformations of the one–loop effective action and by Frolov and Zel'nikov (see *e.g.* Ref. [26]).

Although we are unable to determine the parameter δ further, definite conclusions regarding the overall character of the stress energy tensor may be obtained studying the effects of the various choices of δ from the interval (41) on $\langle T^\mu_\nu\rangle_{\rm reg}$. From (37) one concludes that the maximum of $\langle T^r_r\rangle_{\rm reg}$ increases and is shifted towards smaller values of r as δ decreases. (Fig.1). A sample of these results is displayed in Fig. 1, where we show the radial component as a function of r for δ equal $-2/15, -1/15, 0$, and $1/15$. It should be noted that such a tendency, although desirable, is not sufficient to reconstruct acurately the radial component of the stress-energy tensor near $r = 3M$.

The finer structure of the rescaled components of the stres–energy tensor is displayed in Figs.2–4. Inspection of Fig. 2 shows that the energy density increases with δ. Comparison of the numerical results of Jensen, MacLaughlin, and Ottewill [4] and the results presented in this note shows that the negative values of δ are more favourable. In general it is possible to improve the accuracy of the Brown–Ottewill approximation suitably adjusting δ. For example, the choice $\delta = -1/15$ gives a good agreement of the energy density and tangential pressure of thus obtained $\langle T^\mu_\nu\rangle_{\rm reg}$ with the exact numerical results. However, because of vanishing of Δ^r_r for $r = 2.4M$, we have some unwanted features of the radial component of the stress energy tensor in the interval $2M < r < 2.4M$, for $-2/15 < \delta < 0$. Analyses carried out in Ref. [4] indicate that the exact radial component of the stress–

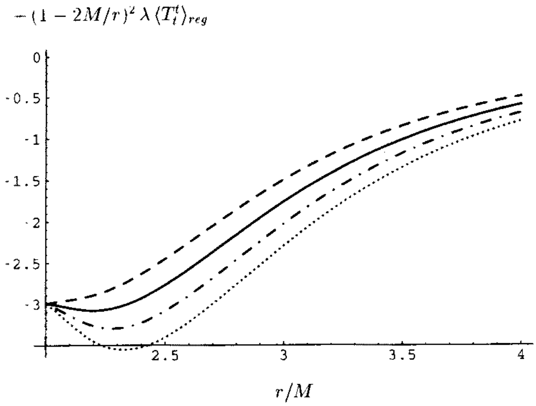

Fig. 2. $-\lambda(1-2M/r)^2\langle T_t^t\rangle_{\rm reg}$ as a function of r, displayed for $\delta = -2/15$ (dotted line), $\delta = -1/15$ (dashed–dotted line), $\delta = 0$ (solid line), and $\delta = 1/15$ (dashed line).

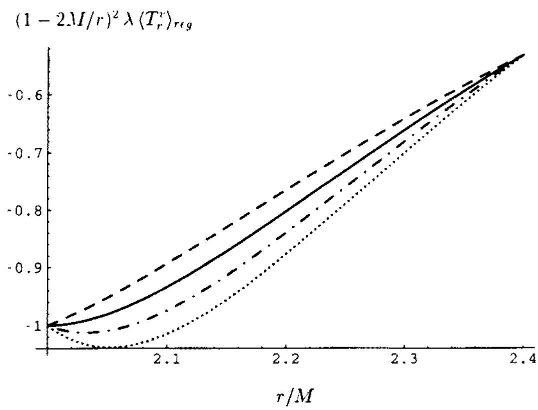

Fig. 3. $\lambda(1-2M/r)^2\langle T_r^r\rangle_{\rm reg}$ as a function of r, displayed for $\delta = -2/15$ (dotted line), $\delta = -1/15$ (dashed–dotted line), $\delta = 0$ (solid line), and $\delta = 1/15$ (dashed line).

energy tensor everywhere exceeds the radial component evaluated within the Brown–Ottewill approximation. On the other hand Fig. 3 shows that regardless of the value of the parameter the for $r < 2.4M$, the approximation is slightly spoiled. It should be emphasized however, that since it is expected that the Boulware vacuum in the vicinity of the event horizon does not describe real physical situation and that genuine spherical objects do have radii considerably exceeding $2M$ such unwanted behaviour is of lesser importance.

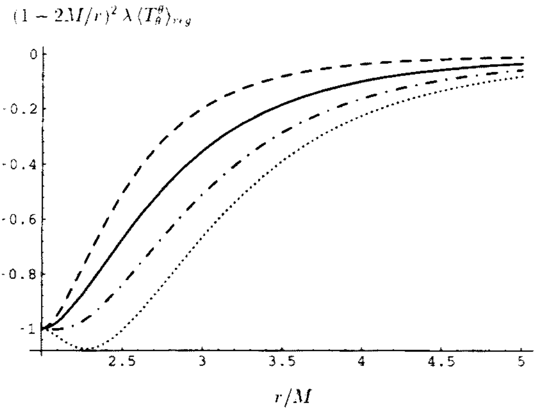

Fig. 4. $\lambda(1-2M/r)^2\langle T_\theta^\theta\rangle_{\rm reg}$ as a function of r, displayed for $\delta = -2/15$ (dotted line), $\delta = -1/15$ (dashed–dotted line), $\delta = 0$ (solid line), and $\delta = 1/15$ (dashed line).

Construction of the vacuum polarization near the spherical and static star model yields another problem: the stress–energy tensor for $r > r_{\rm star}$ may be written as the sum of its local and nonlocal parts. It is expected that the local part should be the same in the black hole case as in the stellar case, whereas the nonlocal contribution depends on the specific boundary conditions. The second component, usually not included, may lead to the differences of the vacuum polarization characteristics. On the other hand absence of such a term may invalidate analyses near the surface of highly compact objects.

Although to reproduce the exact form of the field fluctuation and the stress–energy tensor better the more sophisticated models are to be constructed that exploit more detailed informations it should be noted that such a simple model satisfies all consistency conditions and satisfactorily reproduces the overall character of the stress–energy tensor.

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