TEXTURE DYNAMICS*

W. KRÓLIKOWSKI

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00–681 Warszawa, Poland

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We show how quantum dynamics can be introduced into the "texture" of fundamental–fermion mass matrices by means of annihilation and creation operators acting in the space of three families. Then, at least one texture zero appears in a natural way. A model of such a texture dynamics is described for charged leptons, predicting $m_\tau = 1776.80$ MeV from experimental values of m_e and m_μ.The model is reasonably extended to quarks in a straightforward way.

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As is well known in today's physical theory the mass of a particle is still a free parameter, subject to experimental determination, as it was from the very beginning in Newtonian dynamics. Of course, differences between particle masses may arise from a composite model introducing a few constituent masses and their binding energies, and/or from radiation vacuum contributions to particle energies. So, if we are lucky, the mass spectrum of a particle multiplet may be parametrized by a few massdimensional constants to be determined from the experiment. Such a fortunate situation takes place for the periodic system of elements, where the mass spectrum is parametrized *grosso modo* by proton and neutron masses. To some extent, an analogical situation occurs also in the quark model of hadron multiplets. For the fundamental fermions, leptons and quarks, such a program in its composite (preonic) version or, favourably, noncomposite (elementary Higgs) version is nowadays a main challenge in particle physics. Meanwhile, in absence of a successful realization of such a program, some phenomenological ansatzes are tried for the "texture" of lepton and quark mass matrices [1,2] in hope that they will guide us to find out a solution for the fundamental–fermion mass problem.

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In the present note we try to introduce formally *quantum dynamics* into the texture of fundamental–fermion mass matrices, using — as its possibly simplest agents — annihilation and creation operators acting in the space of three fundamental–fermion families. Then, as will be seen, mass matrices containing at least *one texture zero* can be constructed in a natural way from these operators.

To this end, consider a horizontal or family triplet of three bispinors

$$(\psi_0(x)\,,\;\psi_1(x)\,,\;\psi_2(x)) \tag{1}$$

numerated by the family number $n = 0, 1, 2$ ascribed to three consecutive fundamental–fermion families (ν_e, e^-, u, d), (ν_μ, μ^-, c, s), (ν_τ, τ^-, t, b). Thus, there are four family triplets (1),

$$(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)\,,\;(e^-\,,\,\mu^-\,,\,\tau^-)\,,\;(u\,,\,c\,,\,t)\,,\;(d\,,\,s\,,\,b)\,, \tag{2}$$

which may be labelled by $f = \nu\,,\,e\,,\,u\,,\,d$. Then, introduce in the space of three families the matrices

$$\widehat{a} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}\,,\; \widehat{a}^\dagger \equiv \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}. \tag{3}$$

Of their mixed products

$$\widehat{n} \equiv \widehat{a}^\dagger \widehat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\,,\; \widehat{a}\widehat{a}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

the former defines the matrix for family number: $\widehat{n}|n\rangle = n|n\rangle\,,\ n = 0, 1, 2$ ($\psi_n(x) \equiv \langle n|\psi(x)\rangle$). The matrices (3) satisfy familiar commutation relations,

$$[\widehat{a}\,,\,\widehat{n}] = \widehat{a}\,,\; \left[\widehat{a}^\dagger\,,\,\widehat{n}\right] = -\widehat{a}^\dagger\,, \tag{5}$$

characteristic for annihilation and creation operators. They imply that $\widehat{a}|n\rangle = \sqrt{n}|n-1\rangle$ and $\widehat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$. However, $\widehat{a}^\dagger|n\rangle = 0$ *i.e.*, $|n+1\rangle = 0$ for $n = 2$ (in addition to $\widehat{a}|n\rangle = 0$ for $n = 0$), because

$$\widehat{a}^3 = 0\,,\; \widehat{a}^{\dagger\,3} = 0\,. \tag{6}$$

We will call the matrices $\widehat{a}$ and $\widehat{a}^\dagger$ *truncated* annihilation and creation operators. Notice that

$$\left[\widehat{a}\,,\,\widehat{a}^\dagger\right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \widehat{1} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}. \tag{7}$$

Below we list other nonvanishing quadratic, triple and quartic products of $\widehat{a}$ and $\widehat{a}^\dagger$:

$$\widehat{a}^2 = \begin{pmatrix} 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \widehat{a}\widehat{n}\widehat{a}\ , \ \widehat{a}^{\dagger\,2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \end{pmatrix} = \widehat{a}^\dagger\widehat{n}\widehat{a}^\dagger\,,$$

$$\widehat{a}^2\widehat{a}^\dagger = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \ \widehat{a}\widehat{a}^{\dagger\,2} = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\widehat{n}\widehat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \quad , \ \widehat{a}^\dagger\widehat{n} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix},$$

$$\widehat{a}\widehat{n} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \quad , \ \widehat{n}\widehat{a}^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2\sqrt{2} & 0 \end{pmatrix} \tag{8}$$

and

$$\widehat{a}\widehat{n}\widehat{a}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \widehat{n}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix},$$

$$\widehat{a}^2\widehat{a}^{\dagger\,2} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \widehat{a}\widehat{a}^\dagger\widehat{n} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \widehat{n}\widehat{a}\widehat{a}^\dagger\,, \ \widehat{a}^\dagger\widehat{n}\widehat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \tag{9}$$

Note also that : $\widehat{n}^2 := \widehat{a}^{\dagger\,2}\widehat{a}^2 = \widehat{a}^\dagger\widehat{n}\widehat{a} = \widehat{n}(\widehat{n} - \widehat{1})$, where :( ): denotes the normal ordering.

The overall wave function of family triplet (1) may include in general some nontrivial weight factors ρ_0 , ρ_1 , ρ_2,

$$\Psi(x) = \begin{pmatrix} \rho_0\psi_0(x) \\ \rho_1\psi_1(x) \\ \rho_2\psi_2(x) \end{pmatrix} = \widehat{\rho} \begin{pmatrix} \psi_0(x) \\ \psi_1(x) \\ \psi_2(x) \end{pmatrix}, \ \widehat{\rho} \equiv \begin{pmatrix} \rho_0 & 0 & 0 \\ 0 & \rho_1 & 0 \\ 0 & 0 & \rho_2 \end{pmatrix}, \tag{10}$$

where $\mathrm{Tr}\,\widehat{\rho}^2 = 1$ (in the trivial case $\widehat{\rho} = \widehat{1}/\sqrt{3}$). Thus, the mass matrix for this triplet gets the form

$$\widehat{M} = \widehat{\rho}\widehat{h}\widehat{\rho}\,. \tag{11}$$

Here, a strength matrix $\widehat{h}$ appears that has to be built up of our annihilation and creation operators $\widehat{a}$ and $\widehat{a}^\dagger$.

Consider the following simple operator form:

$$\widehat{h} \equiv \mu(\widehat{n}) + (\alpha\widehat{1} + \beta\widehat{n})\widehat{a}e^{i\varphi} + \widehat{a}^\dagger(\alpha\widehat{1} + \beta\widehat{n})e^{-i\varphi}\,, \tag{12}$$

where $\mu(n)$ is a massdimensional real function, $\alpha > 0$ and $\beta > 0$ play the role of massdimensional coupling constants, while φ denotes a constant phase. If $\mu(n) \equiv \omega(n+\frac{1}{2})$ and $\beta = 0$, this form would remind us of the simplest one–mode field theory. The matrix $\widehat{h}$ and so $\mu(n)$, α, β and φ depend generally on the label $f = \nu$, e, u, d of family triplet (1). On the other hand, the weight matrix $\widehat{\rho}$ may be universal for all f (*cf.* Eq. (16) later on).

Then, making use of Eqs. (11) and (12) we obtain the mass matrix of the following texture:

$$\widehat{M} = \begin{pmatrix} \mu(0)\rho_0^2 & \alpha\rho_0\rho_1 e^{i\varphi} & 0 \\ \alpha\rho_0\rho_1 e^{-i\varphi} & \mu(1)\rho_1^2 & (\alpha+\beta)\sqrt{2}\rho_1\rho_2 e^{i\varphi} \\ 0 & (\alpha+\beta)\sqrt{2}\rho_1\rho_2 e^{-i\varphi} & \mu(2)\rho_2^2 \end{pmatrix} . \quad (13)$$

It contains one texture zero at least. *A priori*, in Eq. (13) there are five real constants $\mu(n)$, α, β and one phase φ to be determined experimentally [if the weights ρ_n are given; otherwise, five real constants to be determined are:$\mu(n)\rho_n^2$, $\alpha\rho_0\rho_1$, $(\alpha+\beta)\rho_1\rho_2$].

In the particular case, when

$$\begin{aligned} &(\mathrm{u})\ \mu^{(u)}(0) = 0 = \mu^{(u)}(1)\,,\ \varphi^{(u)} = 0\,, \\ &(\mathrm{d})\ \mu^{(d)}(0) = 0\,, \alpha^{(d)} + \beta^{(d)} = 0\,, \\ &(\mathrm{e})\ \mu^{(e)}(0) = 0\,,\ \mu^{(e)}(1) = -3\mu^{(d)}(1)\,,\ \mu^{(e)}(2) = \mu^{(d)}(2)\,, \\ &\qquad \alpha^{(e)} = \alpha^{(d)}\,,\ \beta^{(e)} = \beta^{(d)}\,,\ \rho_n^{(e)} = \rho_n^{(d)}\,,\ \varphi^{(e)} = 0\,, \end{aligned}$$

the texture (13) reduces to the Georgi–Jarlskog ansatz (at the GUT scale) [1]. In this case $\widehat{M}^{(d)}$ and $\widehat{M}^{(e)}$ contain two texture zeros.

If the coupling constants α and β are small enough to allow us to apply the perturbative calculation to the mass matrix (13), we get in the lowest order the following mass eigenvalues for our family triplet:

$$\begin{aligned} M_0 &= \mu(0)\rho_0^2 - \frac{\alpha^2\rho_0^2\rho_1^2}{\mu(1)\rho_1^2 - \mu(0)\rho_0^2}\,, \\ M_1 &= \mu(1)\rho_1^2 + \frac{\alpha^2\rho_0^2\rho_1^2}{\mu(1)\rho_1^2 - \mu(0)\rho_0^2} - \frac{2(\alpha+\beta)^2\rho_1^2\rho_2^2}{\mu(2)\rho_2^2 - \mu(1)\rho_1^2}\,, \\ M_2 &= \mu(2)\rho_2^2 + \frac{2(\alpha+\beta)^2\rho_1^2\rho_2^2}{\mu(2)\rho_2^2 - \mu(1)\rho_1^2}\,. \end{aligned} \quad (14)$$

Also, we can calculate perturbatively in a straightforward way the diagonalizing matrix $\widehat{U}^{-1}$ for $\widehat{M}$,

$$\widehat{U}^{-1}\widehat{M}\widehat{U} = \mathrm{diag}(M_0\,,\ M_1\,,\ M_2)\,. \quad (15)$$

Then, $\Psi(x) = \widehat{U}^{-1}\Psi^{(0)}(x)$, where the unperturbed $\Psi^{(0)}(x)$ corresponds formally to $\alpha = 0 = \beta$. The Cabibbo–Kobayashi–Maskawa quark mixing matrix is given as $\widehat{V} = \widehat{U}^{(u)\,-1}\widehat{U}^{(d)}$. But the simple perturbative calculation of $\widehat{U}^{(u)}$ and $\widehat{U}^{(d)}$ is not expected to work in this case since quark mixing is considerable.

Some time ago we argued [3] that for fundamental fermions the weight matrix has the form

$$\widehat{\rho} = \frac{1}{\sqrt{29}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{4} & 0 \\ 0 & 0 & \sqrt{24} \end{pmatrix} , \tag{16}$$

if *all of them can be deduced* from Dirac's square–root procedure $\sqrt{p^2} \rightarrow \Gamma \cdot p$ by means of its (generally reducible) representations. In this approach, the weights ρ_n as given in Eq. (16) correspond to the multiplicities with which the wave–function components $\psi_n(x)$, $n = 0, 1, 2,$ appear (up to the phase factor) in the reduction procedure, while $N = 1 + 2n = 1, 3, 5$ is the number of Dirac bispinor indices appearing in *three* generally reducible representations of Dirac's square–root procedure, *allowed* for leptons and quarks by the intrinsic Pauli principle [3].

Then, in the case of charged leptons e^- , μ^- , τ^- we tried successfully the specific ansatz for the unperturbed term in Eq. (12):

$$\mu(\widehat{n}) \equiv \mu \left[\left(\widehat{1} + 2\widehat{n}\right)^2 - \frac{1-\varepsilon^2}{\left(\widehat{1} + 2\widehat{n}\right)^2} \right] , \tag{17}$$

where μ and ε^2 were real constants to be determined experimentally. In fact, in this case the unperturbed mass matrix $\widehat{M}^{(0)} = \widehat{\rho}\mu(\widehat{n})\widehat{\rho}$ gets the eigenvalues

$$\begin{aligned} m_e^{(0)} &\equiv M_0^{(0)} = \frac{\mu}{29}\varepsilon^2 , \\ m_\mu^{(0)} &\equiv M_1^{(0)} = \frac{4}{9}\frac{\mu}{29}\left(80 + \varepsilon^2\right) , \\ m_\tau^{(0)} &\equiv M_2^{(0)} = \frac{24}{25}\frac{\mu}{29}\left(624 + \varepsilon^2\right) , \end{aligned} \tag{18}$$

leading to the predictions

$$m_\tau^{(0)} = 1776.80\,\text{MeV} , \tag{19}$$

and

$$\mu = 85.9924\,\text{MeV} , \; \varepsilon^2 = 0.172329 , \tag{20}$$

when experimental values of m_e and m_μ are used for $m_e^{(0)}$ and $m_\mu^{(0)}$. The agreement of the predicted $m_\tau^{(0)}$ with recent experimental value $m_\tau^{\rm exp} = 1777.1^{+0.4}_{-0.5}$ MeV [4] is remarkable.

The first term μN^2 in the function $\mu(n) = \mu N^2 + \mu(\varepsilon^2 - 1)/N^2$ following from Eq. (17) may be interpreted as an "interaction" of $N = 1 + 2n = 1, 3, 5$ elements ("algebraic partons" corresponding to Dirac bispinor indices) treated on the same footing, while the second term $\mu(\varepsilon^2 - 1)\, P_N^2$ with $P_N = (N-1)!/N! = 1/N$ may describe an additional "interaction" with itself of one element (the "algebraic parton of the centre of mass") arbitrarily chosen among N elements of which $N-1$ are then *always* undistinguishable. This distinguished "algebraic parton" within a pointlike lepton or quark is coupled to the external gauge fields of the Standard Model [3]. The operators $\hat{a}$ and $\hat{a}^\dagger$ perturbing $\mu(\hat{n})$ in the matrix (12) annihilate and create pairs of undistinguishable "algebraic partons" within leptons and quarks.

Now, calculate from Eqs. (14), (16) and (17) the lowest-order perburbative corrections to unperturbed masses (18). Then, the resulting corrected masses are

$$\begin{aligned} m_e &= \frac{\mu}{29}\varepsilon^2 - \frac{36}{29}\left(\frac{\alpha}{\mu}\right)^2 \frac{\mu}{320 - 5\varepsilon^2}\,, \\ m_\mu &= \frac{4}{9}\frac{\mu}{29}\left(80 + \varepsilon^2\right) + \frac{36}{29}\left(\frac{\alpha}{\mu}\right)^2 \frac{\mu}{320 - 5\varepsilon^2} \\ &\quad - \frac{10800}{29}\left(\frac{\alpha+\beta}{\mu}\right)^2 \frac{\mu}{31696 + 29\varepsilon^2}\,, \\ m_\tau &= \frac{24}{25}\frac{\mu}{29}\left(624 + \varepsilon^2\right) + \frac{10800}{29}\left(\frac{\alpha+\beta}{\mu}\right)^2 \frac{\mu}{31696 + 29\varepsilon^2}\,. \end{aligned} \tag{21}$$

Hence, we can derive the formula

$$\begin{aligned} m_\tau &= \frac{6}{125}\left(351 m_\mu - 136 m_e\right) \\ &\quad - \frac{105192}{3625}\left(\frac{\alpha}{\mu}\right)^2 \frac{\mu}{320 - 5\varepsilon^2} + \frac{24094800}{3625}\left(\frac{\alpha+\beta}{\mu}\right)^2 \frac{\mu}{31696 + 29\varepsilon^2}\,. \end{aligned} \tag{22}$$

Using in the first term of Eq. (22) experimental values of m_e and m_μ and in its terms $O\left[\left(\frac{\alpha}{\mu}\right)^2\right]$ and $O\left[\left(\frac{\alpha+\beta}{\mu}\right)^2\right]$ the values (20) for μ and ε^2, we predict that

$$m_\tau = \left[1776.80 - 7.819\left(\frac{\alpha}{\mu}\right)^2 + 18.03\left(\frac{\alpha+\beta}{\mu}\right)^2\right] \text{MeV}\,. \tag{23}$$

Thus, the difference

$$m_\tau - m_\tau^{\rm exp} = \left[-7.82 \left(\frac{\alpha}{\mu} \right)^2 + 18.0 \left(\frac{\alpha + \beta}{\mu} \right)^2 - 0.3^{+0.4}_{-0.5} \right] \;\text{MeV} \tag{24}$$

becomes zero for

$$18.0 \left(\frac{\alpha + \beta}{\mu} \right)^2 - 7.82 \left(\frac{\alpha}{\mu} \right)^2 = 0.3^{+0.4}_{-0.5} , \tag{25}$$

what cannot exclude the option of $\alpha = 0 = \beta$ for charged leptons.

It is interesting to note that the same specific ansatz (17) as for charged leptons, when applied to the down quarks d , s , b , leads *via* the unperturbed mass matrix $\widehat{M}^{(0)} = \widehat{\rho}\mu(\widehat{n})\widehat{\rho}$ to the reasonable eigenvalues:

$$\begin{aligned} m_d^{(0)} &\equiv M_0^{(0)} = \frac{\mu}{29}\varepsilon^2 , \\ m_s^{(0)} &\equiv M_1^{(0)} = \frac{4}{9}\frac{\mu}{29} \left(80 + \varepsilon^2 \right) , \\ m_b^{(0)} &\equiv M_2^{(0)} = \frac{24}{25}\frac{\mu}{29} \left(624 + \varepsilon^2 \right) . \end{aligned} \tag{26}$$

In fact, from the first and second Eq. (26)

$$\mu = \frac{29}{320}(9m_s^{(0)} - 4m_d^{(0)}) \; , \; \varepsilon^2 = \frac{320 m_d^{(0)}}{9m_s^{(0)} - 4m_d^{(0)}} , \tag{27}$$

and then from the third Eq. (26)

$$m_b^{(0)} = \frac{6}{125}(351 m_s^{(0)} - 136 m_d^{(0)}) , \tag{28}$$

what nicely correlates the experimentally suggested values for m_d , m_s , m_b. For instance, with $m_d^{(0)} \simeq 7\,\text{MeV}$ and $m_b^{(0)} \simeq (4.7 \text{ to } 4.73)\,\text{GeV}$ Eq. (28) implies

$$m_s^{(0)} \simeq (282 \text{ to } 283)\,\text{MeV} , \tag{29}$$

while Eqs. (27) give

$$\mu \simeq (227 \text{ to } 229)\,\text{MeV} \; , \; \varepsilon^2 \simeq 0.893 \text{ to } 0.888 . \tag{30}$$

This nice situation is no longer the case for up quarks u , c , t , where the top mass turns out much too large to be described together with the up and charm masses by the specific ansatz (17). However, one may try instead the modified ansatz

$$\mu(\widehat{n}) \equiv \mu\left[(\widehat{1}+2\widehat{n})^2 - \frac{1-\varepsilon^2}{(\widehat{1}+2\widehat{n})^2} + \widehat{C}\right], \tag{31}$$

where

$$\widehat{C} = 2\widehat{n}\,2\left(\widehat{n}-\widehat{1}\right) N_{\rm C}\,(N_{\rm C}-1)\,[N_{\rm C}(Q+B)] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 144 \end{pmatrix} \tag{32}$$

leads to reasonable results ($\widehat{C} = 0$ for leptons and down quarks). Here, $N_{\rm C}$, Q and B are the number of colors, the charge and the baryon number, respectively [note that $2n\,2(n-1) = (N-1)(N-3)$, while for leptons $N_{\rm C} - 1 = 0$ and for down quarks $Q + B = 0$]. Then, the unperturbed mass matrix $\widehat{M}^{(0)} = \widehat{\rho}\mu(\widehat{n})\widehat{\rho}$ gets the eigenvalues

$$m_u^{(0)} \equiv M_0^{(0)} = \frac{\mu}{29}\varepsilon^2 ,$$

$$m_c^{(0)} \equiv M_1^{(0)} = \frac{4}{9}\frac{\mu}{29}\left(80+\varepsilon^2\right) ,$$

$$m_t^{(0)} \equiv M_2^{(0)} = \frac{24}{25}\frac{\mu}{29}\left(4224+\varepsilon^2\right) . \tag{33}$$

Hence

$$\mu = \frac{29}{320}(9m_c^{(0)} - 4m_u^{(0)}) \, , \, \varepsilon^2 = \frac{320 m_u^{(0)}}{9m_c^{(0)} - 4m_u^{(0)}} \tag{34}$$

and

$$m_t^{(0)} = \frac{24}{125}(594 m_c^{(0)} - 259 m_u^{(0)}) . \tag{35}$$

For instance, with $m_u \simeq 4\,\mathrm{MeV}$ and $m_c \simeq (1.5 \;\text{ to }\; 1.55)\,\mathrm{GeV}$ Eq. (35) implies

$$m_t^{(0)} \simeq (171 \;\text{ to }\; 177)\,\mathrm{GeV} , \tag{36}$$

whereas Eqs. (34) give

$$\mu \simeq (1.22 \;\text{ to }\; 1.26)\,\mathrm{GeV} \, , \, \varepsilon^2 \simeq 0.0949 \;\text{ to }\; 0.0919 . \tag{37}$$

The predicted value (36) is to be compared with the experimental figure $m_t^{\rm exp} = 174 \pm 10\,^{+13}_{-12}$ GeV [4].

The result of perturbing $\mu(\widehat{n})$ in the way described in Eq. (12) depends much on the massdimensional coupling constants α and β (in the case of up and down quarks there are *a priori* four such constants $\alpha^{(u)}$, $\beta^{(u)}$ and $\alpha^{(d)}$, $\beta^{(d)}$). Because of considerable quark mixing the simple perturbative calculation does not apply in this case (especially to small masses m_u and m_d). However, a somewhat complicated numerical evaluation of six quark masses and the 3×3 Cabibbo–Kobayashi–Maskawa unitary matrix $\widehat{V} = \widehat{U}^{(u)-1}\widehat{U}^{(d)}$ in terms of $\alpha^{(u)}$, $\beta^{(u)}$ and $\alpha^{(d)}$, $\beta^{(d)}$ as well as $\mu^{(u)}$, $\varepsilon^{(u)2}$ and $\mu^{(d)}$, $\varepsilon^{(d)2}$, is always at hand. Of course, further ansatzes about these eight *a priori* free parameters are desirable to increase the predictive power of the scheme and to connect quarks with leptons [for a previous, partly different scheme for such a numerical calculation *cf.* Ref. [3], where $\alpha^{(u)} = 12\mu^{(u)}\varepsilon^{(e)}$, $\alpha^{(d)} = 3\mu^{(d)}\varepsilon^{(e)}$ and $\beta^{(u)} = \beta^{(d)} = 0$ with $\varepsilon^{(e)2}$ as given in Eq. (20) for charged leptons, but $\mu^{(u)}(\widehat{n})$ and $\mu^{(d)}(\widehat{n})$ are there modified compared with the forms (31) and (17)].

Concluding, the idea of texture dynamics may provide us with a new physical guide how to solve the fundamental–fermion mass problem. The model of texture dynamics, especially for charged leptons, briefly discussed above, is promising in this respect, although, its specific form does not seem to follow from our hitherto existing experience. Perhaps, there is something dynamically new in this form.

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