POSSIBLE DEFORMATION OF TIME RUN INDUCED BY CHANGE OF PARTICLE NUMBER: PART TWO*

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A discussion is continued on the author's recent conjecture that the overall particle number, when it changes in a localized physical process, induces in its proximity a tiny deformation of the time run. In some cases, the corresponding weak time-deformation field can be emitted and also detected by matter sources. Sometimes, it can propagate freely through the spacetime as ultraluminal waves. Though these hypothetic waves cannot transport energy between matter sources, they can do it with a new thermodynamic-type quantity called here the energy width. This causes the quantum time evolution of matter sources to deviate slightly from the conventional unitary evolution.

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1. Introduction

In the first part of this paper [1] we reported on some developments of the author's recent conjecture that a change of the overall particle number in a localized physical process induces in its proximity a tiny deformation of the time run. Such a hypothetic quantum effect is caused by a thermodynamic-type mechanism not present in the Einsteinian classical theory of gravitation. But this effect seems to be natural if the familiar analogy [2] between the thermal equilibrium described by $\exp(-H/kT)$ and the unitary quantum time evolution expressed by $\exp(-iHt/\hbar)$ (called here "temporal equilibrium") is accepted as a profound physical correspondence: $kT \leftrightarrow -i\hbar/t$.

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Then, in analogy with the small deviations from the thermal equilibrium: $T \to T + \delta T(\vec{r}, t)$, where in the case of a homogeneous medium the heat conductivity equation holds for $\delta T(\vec{r}, t)$:

$$\left(\triangle - \frac{1}{\lambda_Q c} \frac{\partial}{\partial t}\right) \delta T(\vec{r}, t) = 0\,, \tag{1}$$

in some circumstances there should appear small deviations from the temporal equilibrium: $t \to t + \delta t(\vec{r}, t)$ or

$$\frac{1}{t} \to \frac{1}{t + \delta t(\vec{r}, t)} \equiv \frac{1}{t} + \varphi(\vec{r}, t)\,, \tag{2}$$

where a new conductivity equation should be valid for $\varphi(\vec{r}, t) \simeq -\delta t(\vec{r}, t)/t^2$ in the vacuum:

$$\left(\triangle - \frac{1}{\lambda_\Gamma c} \frac{\partial}{\partial t}\right) \varphi(\vec{r}, t) = 0\,. \tag{3}$$

Hence, $\varphi(\vec{r}, t) = \varphi(\vec{r}) \exp(-\gamma t)$, where $[\triangle + (\gamma/\lambda_\Gamma c)]\varphi(\vec{r}) = 0$.

We call "energy width" the new thermodynamic-type quantity being the analogue of heat Q. Then, we extend consequently the first law of thermodynamics to the form

$$dU = \delta W + \delta Q - i\delta\Gamma\,, \tag{4}$$

where

$$\delta Q \propto \int d^3\vec{r} \rho(\vec{r}, t) k\, d[T + \delta T(\vec{r}, t)]_{t=\text{fixed}} \tag{5}$$

and

$$-i\delta\Gamma \propto \int d^3\vec{r} \rho(\vec{r}, t)(-i\hbar)\, d\left[\frac{1}{t + \delta t(\vec{r}, t)}\right]_{t=\text{fixed}} \tag{6}$$

with $\rho(\vec{r}, t)$ being the averaged particle-number density. In Eq. (4) $\delta\Gamma$ denotes the amount of energy width transferred to the matter system from the physical spacetime treated as its surroundings.

In the thermal equilibrium (of a matter system with a thermostat or heat reservoir) we have $\delta T(\vec{r}, t) \equiv 0$ and so, an equal distribution of temperature. Analogically, in the temporal equilibrium (of a matter system with the physical spacetime that in this case may be called "chronostat" or "energy-width reservoir") we get $\delta t(\vec{r}, t) \equiv 0$ or $\varphi(\vec{r}, t) \equiv 0$ and, consequently, $\delta\Gamma \equiv 0$ due to Eq. (6). Thus, in the temporal equilibrium, time t runs equally at all space points and hence, for a so called closed matter system the conventional unitary quantum state equation

$$i\hbar \frac{d\Psi(t)}{dt} = H\Psi(t)\,, \quad H^\dagger = H \tag{7}$$

holds (in the Schrödinger picture).

From the extended first law of thermodynamics (4) we infer that in the case of small deviations from temporal equilibrium the quantum state equation (7) should be generalized to the nonunitary form:

$$i\hbar \frac{d\Psi(t)}{dt} = [H - i\mathbf{1}\Gamma(t)]\Psi(t)\,, \quad H^\dagger = H\,. \tag{8}$$

Here, **1** is the unit operator, while $\Gamma(t)$ denotes the real-parameter-valued energy width that will be assumed to have the relativistic form (15) giving the form (16) in the nonrelativistic approximation for matter. Of course, in contrast to Eq. (7), the new quantum state equation (8) implies (in general) a nonunitary quantum time evolution, although its deviations from the conventional unitary time evolution have to be small.

The conductivity equation (3) for the real-parameter-valued time-deformation field $\varphi(\vec{r},t)$ is obviously nonrelativistic. Note, however, that with the use of the d'Alembertian $\Box \equiv \Delta - (1/c^2)(\partial^2/\partial t^2)$ one obtains

$$\begin{aligned}\left(\Box + \frac{1}{4\lambda_\Gamma^2}\right)\left[\varphi(\vec{r},t)\exp\frac{ct}{2\lambda_\Gamma}\right] &= \exp\frac{ct}{2\lambda_\Gamma}\left(\Box - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t}\right)\varphi(\vec{r},t)\\ &\simeq \exp\frac{ct}{2\lambda_\Gamma}\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t}\right)\varphi(\vec{r},t)\,,\end{aligned} \tag{9}$$

the last step being valid if $(1/c^2)(\partial^2/\partial t^2)\varphi$ can be neglected nonrelativistically in comparison with $(1/\lambda_\Gamma c)(\partial/\partial t)\varphi$. The rhs of Eq. (9) is zero if $\varphi(\vec{r},t)$ satisfies the conductivity equation (3). Thus, in place of the conductivity equation (3), the tachyonic-type (and so ultraluminal) Klein–Gordon equation

$$\left(\Box + \frac{1}{4\lambda_\Gamma^2}\right)\chi(x) = 0\,, \tag{10}$$

relativistic in the sense of special relativity, is suggested for the new (relativistic) time deformation field

$$\chi(x) \equiv \varphi(\vec{r},t)\exp\frac{ct}{2\lambda_\Gamma}\,. \tag{11}$$

Of course, $\chi(x) \equiv 0$ in the temporal equilibrium.

Since the overall particle number, when it changes, is conjectured to cause departures from temporal equilibrium, a natural thing is to assume that the homogeneous equation (10) transits in the presence of matter sources into the inhomogeneous equation of the following relativistic form:

$$\left(\Box + \frac{1}{4\lambda_\Gamma^2}\right) \chi(x) = -4\pi g\lambda_\Gamma\, \partial_\mu j^\mu(x)\,. \tag{12}$$

Here, $g > 0$ is an unknown dimensionless coupling constant (likely to be extremely small, *cf.* Ref. [1]), while $(j^\mu(x)) = (c\rho(x), \vec{j}(x))$ denotes the averaged matter (particle-number) four-current,

$$j^\mu(x) \equiv \langle \Psi(t)|J^\mu(\vec{r})|\Psi(t)\rangle_{\rm av}\,, \tag{13}$$

corresponding to the operator $J^\mu(\vec{r})$ of overall particle-number four-current (here, $\Psi(t)$ and $J^\mu(\vec{r})$ are taken, for instance, in the Schrödinger picture). We are aware of the infrared problem existing when photons are included in the matter sources (as they should be in this theory).

Note from Eqs. (12) and (9) that the field $\varphi(\vec{r},t) \equiv \chi(x)\exp(-ct/2\lambda_\Gamma)$ satisfies the noncovariant equation

$$\left(\Box - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t}\right)\varphi(\vec{r},t) = -4\pi g\lambda_\Gamma\, [\partial_\mu j^\mu(x)]\exp\left(-\frac{ct}{2\lambda_\Gamma}\right)\,, \tag{14}$$

valid in the relativistic theory. The time-run deformation can be expressed as $\delta t(\vec{r},t) \equiv -\varphi(\vec{r},t)t^2\,[1+\varphi(\vec{r},t)t]^{-1}$ due to Eq. (2).

Eventually, we decide to assume the following relativistic form for the energy width $\Gamma(t)$ appearing in the nonunitary state equation (8) (in the combination $H - i1\,\Gamma(t)$):

$$\Gamma(t) \equiv g\hbar \int d^3\vec{r}\,\frac{1}{c}\sqrt{j_\mu(x)j^\mu(x)}\,\chi(x)\,, \tag{15}$$

where $(1/c)\sqrt{j_\mu(x)j^\mu(x)} = \rho(x)\sqrt{1-[\vec{v}(x)/c]^2}$ with $\vec{j}(x) = \rho(x)\vec{v}(x)$, while $g > 0$ is the coupling constant introduced in Eq. (12) (this definition is an improvement to the form (16) which was used in Ref. [1]). Note that in the so called comoving frame of reference $(1/c)\sqrt{j_\mu(x)j^\mu(x)} = \rho(x)$ [3]. In the nonrelativistic approximation for matter, where $|\vec{j}(x)/j^0(x)| = |\vec{v}(x)/c| \ll 1$, the energy width (15) becomes

$$\Gamma(t) \simeq g\hbar \int d^3\vec{r}\,\rho(\vec{r},t)\,\chi(\vec{r},t) \tag{16}$$

with $\chi(\vec{r},t) \equiv \varphi(\vec{r},t)\,\exp(ct/2\lambda_\Gamma)$.

2. Chronodynamics

The mixed set of two equations: (8) for $\Psi(t)$ and (12) for $\chi(x)$, together with the definitions (15) of $\Gamma(t)$ and (13) of $j^\mu(x)$, determine (hopefully) both the state vector $\Psi(t)$ and the time-deformation field $\chi(x)$. We call such a thermodynamic-type quantum theory "chronodynamics". This may be considered as a thermodynamic-type approximation to a future fully dynamical quantum theory including gravitation, where the Hilbert subspace of the physical spacetime (described then in a quantal way) is projected out from the whole Hilbert space by projecting the latter onto the Hilbert subspace of the matter only. Such a procedure leads for matter (after some averaging) to the nonHermitian time-evolution operator $H - i\mathbf{1}\Gamma(t)$ (for the general formalism involved *cf.* Ref. [4]).

Strictly speaking, the set of Eqs. (8) and (12) is nonlinear and nonlocal with respect to the state vector $\Psi(t)$, thus it violates slightly the superposition principle, fundamental for the probabilistic interpretation of the conventional quantum theory (which is valid in the temporal equilibrium). However, this set becomes linear and local if the approximation is used, where in the definition (13) of $j^\mu(x)$ the state vector $\Psi(t)$ is replaced in the zero order by $\Psi^{(0)}(t)$ satisfying the temporal-equilibrium state equation (7). This gives $j^{(0)\mu}(x)$, and then, Eqs. (8) and (12) with the definition (15) lead in the first order to

$$i\hbar\frac{d\Psi^{(1)}(t)}{dt} = \left[H - i\mathbf{1}\Gamma^{(1)}(t)\right]\Psi^{(1)}(t) \tag{17}$$

and

$$\left(\Box + \frac{1}{4\lambda_\Gamma^2}\right)\chi^{(1)}(x) = -4\pi g\lambda_\Gamma\,\partial_\mu j^{(0)\mu}(x) \tag{18}$$

with

$$\Gamma^{(1)}(t) = g\hbar\int d^3\vec{r}\,\frac{1}{c}\sqrt{j^{(0)}_\mu(x)j^{(0)\mu}(x)}\,\chi^{(1)}(x) = O(g^2) \tag{19}$$

or, nonrelativistically (for matter),

$$\Gamma^{(1)}(t) \simeq g\hbar\int d^3\vec{r}\,\rho^{(0)}(\vec{r},t)\chi^{(1)}(\vec{r},t) = O(g^2)\,. \tag{20}$$

From Eq. (17) it follows that

$$\Psi^{(1)}(t) = \Psi^{(0)}(t)\exp\left[-\frac{1}{\hbar}\int_{t_0}^{t} dt'\,\Gamma^{(1)}(t')\right]\,, \tag{21}$$

where we have

$$\Psi^{(0)}(t) = \exp\left[-\frac{i}{\hbar}H(t-t_0)\right]\Psi_H \tag{22}$$

with $\Psi_H \equiv \Psi(t_0)$ being the exact state vector in the Heisenberg picture (if this picture and the Schrödinger picture coincide at $t = t_0$). In general, the norm of the state vector $\Psi^{(1)}(t)$, given by

$$\langle\Psi^{(1)}(t)|\Psi^{(1)}(t)\rangle = \langle\Psi_H|\Psi_H\rangle \exp\left[-\frac{2}{\hbar}\int\limits_{t_0}^{t} dt'\, \Gamma^{(1)}(t')\right], \tag{23}$$

changes slightly in time (in the interval, where $\chi^{(1)}(\vec{r},t) \neq 0$ and $\Gamma^{(1)}(t) \neq 0$).

3. Emission of time-deformation waves

First, consider the case of a static spherically-symmetric pointlike source:

$$\partial_\mu j^\mu(x) \equiv \mathrm{div}\vec{j}(\vec{r}) \equiv \frac{1}{\tau}\delta^3(\vec{r}-\vec{r}_S) \tag{24}$$

with $\tau > 0$ being a time-dimensional constant. Then, the time-deformation field equation (12) gets the following fundamental solution:

$$\chi(\vec{r}) = \frac{g\lambda}{\tau}\frac{\cos\left[(1/2\lambda)|\vec{r}-\vec{r}_S|\right]}{|\vec{r}-\vec{r}_S|} \tag{25}$$

(here, the label Γ at λ is omitted). In Eqs. (24) and (25) the constant $1/\tau$ may be interpreted as the overall number of particles produced per unit of time within the static matter source (that may be, for instance, a rough model of the Sun looked at from a large distance).

Now, assume that this number of particles oscillates harmonically in time:

$$\partial_\mu j^\mu(x) \equiv \frac{1}{\tau}\delta^3(\vec{r}-\vec{r}_S)\cos\Omega(t-t_S) \tag{26}$$

with $\Omega > 0$. In this case, the fundamental solution to the time-deformation field equation (12) becomes

$$\bar{\chi}(\vec{r},t) \equiv \mathrm{Re}\chi_F(\vec{r},t) = \frac{g\lambda}{\tau}\frac{\cos\left[\sqrt{(1/2\lambda)^2+(\Omega/c)^2}\,|\vec{r}-\vec{r}_S|\right]}{|\vec{r}-\vec{r}_S|}\cos\Omega(t-t_S)\,. \tag{27}$$

Here,

$$\chi_{\rm F}(\vec{r},t) \equiv g\lambda \int d^4x'\Delta_{\rm F}(x-x')\frac{1}{\tau}\delta^3(\vec{r}'-\vec{r}_{\rm S})\,\cos\Omega(t'-t_{\rm S})$$
$$= \frac{g\lambda}{\tau}\frac{\exp\left[\sqrt{(1/2\lambda)^2+(\Omega/c)^2}|\vec{r}-\vec{r}_{\rm S}|\right]}{|\vec{r}-\vec{r}_{\rm S}|}\cos\Omega(t-t_{\rm S}) \qquad (28)$$

is the solution to Eq. (12) with the source (26), corresponding to the Feynman-type propagator $\Delta_{\rm F}$. For the latter

$$\left(\Box+\frac{1}{4\lambda^2}\right)\Delta_{\rm F}(x-x') = -4\pi\,\delta(x-x')\,, \qquad (29)$$

and more specifically

$$\Delta_{\rm F}(x-x') \equiv -4\pi\int\frac{d^4k}{(2\pi)^4}\frac{\exp[-ik\cdot(x-x')]}{k^2+1/(2\lambda)^2+i\varepsilon}$$
$$= 4i\int_0^\infty\frac{|\vec{k}|\,d|\vec{k}|}{2\pi}\frac{\sin|\vec{k}||\vec{r}-\vec{r}_{\rm S}|}{|\vec{r}-\vec{r}_{\rm S}|}\theta(t-t')$$
$$\times\left\{\theta\left(\vec{k}^2-\frac{1}{4\lambda^2}\right)\frac{\exp[-ic(k_0-i\varepsilon)(t-t')]}{2k_0}\right.$$
$$\left.+\,\theta\left(\frac{1}{4\lambda^2}-\vec{k}^2\right)\frac{\exp[-c|k_0|(t-t')]}{-2i\,|k_0|}\right\}_{k_0=\sqrt{\vec{k}^2-1/(2\lambda)^2}}$$
$$+\,(t\leftrightarrow t') \qquad (30)$$

(where both ε's > 0 and $\to 0$ after the integration over t'). Note that

$$\chi^{(1)}(\vec{r},t) \equiv -{\rm Im}\chi_{\rm F}(\vec{r},t)$$
$$= -\frac{g\lambda}{\tau}\frac{\sin\left[\sqrt{(1/2\lambda)^2+(\Omega/c)^2}\,|\vec{r}-\vec{r}_{\rm S}|\right]}{|\vec{r}-\vec{r}_{\rm S}|}\cos\Omega(t-t_{\rm S}) \quad (31)$$

is a particular solution to the free equation (10). One can show that $\Delta_{\rm F} = \bar{\Delta} - i\Delta^{(1)}$, where $\bar{\Delta}$ and $\Delta^{(1)}$ are the Schwinger-type propagators. Then, $\bar{\chi}$ and $\chi^{(1)}$ correspond to $\bar{\Delta}$ and $\Delta^{(1)}$, respectively. Note also that, if in the source (26) the oscillating factor $\cos\Omega(t-t_{\rm S})$ were replaced by the damping factor $\exp[-\gamma(t-t_{\rm S})]$ with $\gamma > 0$, then the expressions $\exp[-\gamma(t-t_{\rm S})]$ and $\sqrt{(1/2\lambda)^2-(\gamma/c)^2}$ should be substituted for $\cos\Omega(t-t_{\rm S})$ and $\sqrt{(1/2\lambda)^2+(\Omega/c)^2}$ in the solution (28). Now, $\sqrt{\;} = i|\sqrt{\;}|$ if $\gamma > c/2\lambda$ (and still $\sqrt{\;} > 0$ if $\gamma < c/2\lambda$).

Another particular solution to the free equation (10) is given by the formula

$$\chi^{(0)}(\vec{r},t) \equiv g\lambda \int d^4x'\,\Delta(x-x')\frac{1}{\tau}\delta^3(\vec{r}'-\vec{r}_{\rm S})\,\cos\Omega(t'-t_{\rm S})$$
$$= i\frac{g\lambda}{\tau}\frac{\sin\left[\sqrt{(1/2\lambda)^2+(\Omega/c)^2}\,|\vec{r}-\vec{r}_{\rm S}|\right]}{|\vec{r}-\vec{r}_{\rm S}|}\,\sin\Omega(t-t_{\rm S}) \quad (32)$$

corresponding to the Schwinger-type propagator Δ:

$$\left(\Box+\frac{1}{4\lambda^2}\right)\Delta(x-x')=0 \quad (33)$$

and more specifically

$$\Delta(x-x') \equiv -4\pi\int\frac{d^4k}{(2\pi)^4}\frac{k_0}{|k_0|}\pi\delta\left(k^2+\frac{1}{4\lambda^2}\right)\exp[-ik\cdot(x-x')]\,. \quad (34)$$

Note that for $\Delta^{(1)}$ we have

$$\left(\Box+\frac{1}{4\lambda^2}\right)\Delta^{(1)}(x-x')=0 \quad (35)$$

and

$$\Delta^{(1)}(x-x') \equiv -4\pi\int\frac{d^4k}{(2\pi)^4}\pi\delta\left(k^2+\frac{1}{4\lambda^2}\right)\exp[-ik\cdot(x-x')]\,. \quad (36)$$

The particular solution (31) to the free equation (10) corresponds to $\Delta^{(1)}$ as the solution (32) corresponds to Δ.

From Eqs. (26) and (31) we can see that

$$\chi(\vec{r},t) \equiv \bar{\chi}(\vec{r},t) - i\chi^{(0)}(\vec{r},t)$$
$$= \frac{g\lambda}{\tau}\frac{\cos\left[\sqrt{(1/2\lambda)^2+(\Omega/c)^2}\,|\vec{r}-\vec{r}_{\rm S}|-\Omega(t-t_{\rm S})\right]}{|\vec{r}-\vec{r}_{\rm S}|} \quad (37)$$

is the solution to Eq. (12) with the source (26), corresponding to the propagator $\bar{\Delta}-i\Delta$. This solution describes the continuous emission of harmonic, spherically-symmetric time-deformation waves satisfying at $t=t_{\rm S}$ the initial condition:

$$\chi_{t=t_{\rm S}} = \frac{g\lambda}{\tau}\frac{\cos\left[\sqrt{(1/2\lambda)^2+(\Omega/c)^2}\,|\vec{r}-\vec{r}_{\rm S}|\right]}{|\vec{r}-\vec{r}_{\rm S}|} \quad (38)$$

and

$$\frac{\partial \chi}{\partial t}_{t=t_S} = \Omega \frac{g\lambda}{\tau} \frac{\sin\left[\sqrt{(1/2\lambda)^2 + (\Omega/c)^2}\,|\vec{r} - \vec{r}_S|\right]}{|\vec{r} - \vec{r}_S|} . \tag{39}$$

As follows from Eq. (37), their group velocity is larger than the speed of light:

$$v_{\rm gr} = c\sqrt{1 + \left(\frac{c}{2\lambda\Omega}\right)^2} > c \tag{40}$$

(so, they are ultraluminal), while their phase velocity is smaller:

$$v_{\rm ph} = \frac{c}{\sqrt{1 + \left(\frac{c}{2\lambda\Omega}\right)^2}} < c\,, \tag{41}$$

giving still $v_{\rm gr} v_{\rm ph} = c^2$. When $\Omega \to 0$ or ∞, then $v_{\rm gr} \to \infty$ or $c + 0$, while $v_{\rm ph} \to 0$ or $c - 0$.

Such time-deformation waves can be detected by another potential harmonic oscillator for the overall particle number. In principle, a detector of this kind could be a sample of hydrogen atoms (or other particles) whose number would oscillate in time with the interacting time-deformation waves (for the case of time-deformation field aroused in proximity of a big collider *cf.* the last Section of Ref. [1]). In fact, during the nonunitary quantum time-evolution of matter induced in such a detector by time-deformation field, the oscillating energy width would be transferred from these waves to the hydrogen atoms (or other particles), making the number of the latter to oscillate also (most likely, with an extremely small amplitude). More precisely, it follows from the nonunitary state equation (8) and the definition (15) of the parameter-valued energy width $\Gamma(t)$ that the parameter-valued time-deformation field $\chi(x)$ can contribute to the operator of energy width $\mathbf{1}\Gamma(t)$ of the matter system, but not to its energy operator H (in this context *cf.* Appendix).

In conclusion, the hypothetic ultraluminal time-deformation waves, though not exchanging energy with matter sources, do exchange the energy width and so, still, can transport in principle information between their emittors and receivers: much to our surprise faster than the light. Unfortunately, in practice, the possible signal from the time-deformation waves is likely to be extremely weak. Nevertheless, such a new exciting option seems to be offered theoretically.

Appendix

Consider for the state vector satisfying Eq. (8) the unitary transformation (which, in fact, is a phase transformation):

$$\Psi'(t) = \exp\left[-\frac{i}{\hbar}\int_{t_0}^{t} dt' \mathbf{1} E(t')\right]\Psi(t) = \Psi(t)\exp\left[-\frac{i}{\hbar}\int_{t_0}^{t} dt' E(t')\right], \quad \text{(A1)}$$

where $E(t)$ is the counterpart of classical energy for the parameter-valued field $\chi(x)$,

$$E(t) = \frac{\hbar}{8\pi c}\int d^3\vec{r}\left\{\left[\frac{1}{c}\dot{\chi}(x)\right]^2 + \left[\vec{\partial}\chi(x)\right]^2 - \frac{1}{4\lambda^2}\left[\chi(x)\right]^2\right\}$$
$$+ \frac{g\lambda\hbar}{c}\int d^3\vec{r}\,[\partial_\mu j^\mu(x)]\,\chi(x)\,, \quad \text{(A2)}$$

giving Eq. (12) as the classical field equation for $\chi(x)$. Of course, $j^\mu(x)$ and $\Gamma(t)$ defined in Eqs. (13) and (15) are not changed under the transformation (A1). Though the state equation (8) transits now into the form containing, beside the operator of energy width $\mathbf{1}\Gamma(t)$, the operator of time-deformation energy $\mathbf{1}E(t)$,

$$i\hbar\frac{d\Psi'(t)}{dt} = \{H + \mathbf{1}\,[E(t) - i\Gamma(t)]\}\,\Psi'(t)\,, \quad \text{(A3)}$$

Eq. (12) for our time-deformation field $\chi(x)$ does not change. So, $\chi(x)$ cannot be unitarily transformed — although a quantum field could — to the Schrödinger picture, where it were time-independent, contributing instead to the total energy of the extended system.

Thus, we can infer that, in contrast to the energy width, the notion of energy for the time-deformation field $\chi(x)$ is (in our mixed quantum theory) rather an artefact related to a physically irrelevant phase transformation (A1) for the state vector $\Psi(t)$ (although this transformation is time-dependent).

Even if the unitary (in fact, phase) transformation (A1) for $\Psi(t)$ is supplemented by a classical canonical transformation for $\chi(x)$ leading to a new, canonically equivalent time-independent $\chi'(\vec{r})$, it gives the time-deformation energy $E(t)$ (expressed now by $\chi'(\vec{r})$ and its canonical momentum) that still ought to be considered as an artefact, at any rate from the viewpoint of quantum time-evolution of matter (described by our mixed quantum theory).

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