MULTIPARTON DENSITY MATRIX FOR THE QCD–CASCADE IN DLA APPROXIMATION*

B. Ziaja

Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Cracow, Poland
e-mail: beataz@ztc386a.if.uj.edu.pl

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The multiparton density matrices of the QCD gluon cascade are investigated. The generating functional and master equation for momentum space multiparton density in the double logarithmic approximation (DLA) are proposed.

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1. Introduction

Recently, several groups have analyzed in great detail the multiparton distributions in the QCD gluon cascades [1]. The results of the investigations show that perturbative QCD [2] provides a powerful framework not only for the description of hard quark and gluon jets but also of much softer multiparticle phenomena. Although not understood, the hypothesis of parton-hadron duality [3] provides an apparently successful link between theoretical parton distributions and observed particle spectra. This prescription was extensively tested in single particle spectra (and total multiplicities) and found in a good agreement with available data (see *e.g.* [4]). Recently, there appeared indications that it may also work for multiparticle correlations [5]. These unquestionable successes invite one to study further consequences of the theory for processes of particle production. At this point we would like to notice that, if one wants to exploit fully the quantum-mechanical aspects of the QCD cascade, it is necessary to study the multiparton *density matrix*.

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The multiparticle distributions calculated so far give only diagonal terms of the density matrix and thus represent rather restricted (although very important) part of the information available from the theory.

It is perhaps important to stress that in contrast to what is usually believed, the interest in studying the multiparticle density matrix is not purely academic. As suggested in [7] density matrix allows to obtain the multiparticle Wigner functions and consequently gives information about the space-time structure of the system. This allows to make predictions on the shape and range of the HBT interference with clear experimental consequences.

We present in this paper a study of the multiparton density matrices of the QCD gluon cascade in the double logarithmic approximation (DLA) [6] of QCD. We propose a master equation for the functional which generates momentum space density matrix of arbitrary order. The paper is organized as follows. In Section 2 the definition of the density matrix is given. In Section 3 we recall briefly the main assumptions of the DLA approximation for the QCD cascade, and the generating functional method for multiparticle densities. In Section 4 we propose a scheme to calculate the density matrix in DLA formalism. We formulate the master equation for the functional, which can generate density matrices at first at the most general level, and later in the approximation of the "quasi - diagonal" limit. The approximation allows to derive explicit integral equations for density matrices of arbitrary order. To show explicitly, how the method works, we derive master equation for single and double inclusive particle density matrices in the last section.

2. Definition of the density matrix

We would like to investigate the properties of the momentum space density matrix. Let us consider any particle production process where all the produced particles are already "real" *i.e.* they are on mass shell. Then the exclusive m-particle density matrix takes the form :

$$d^{\rm ex}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = \sum_{D',D} S_{(D')}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m) S_{(D)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) \prod_{i=1}^{m} (4\omega_{k'_i}\omega_{k_i})^{-1/2}\,, \tag{1}$$

where $S_{(D)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)$ denotes the probability amplitude to obtain exactly m particles with the momenta $k_1,\ldots,k_m$ for the given Feynman diagram D, and the sum is taken over all possible realization of the process *i.e.* over all possible Feynman diagrams for each amplitude separately. We have taken into account also phase space factors.

The inclusive m-particle density matrix is then given by:

$$d^{\rm in}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)$$
$$=\sum_{n=m}^{\infty}\frac{1}{(n-m)!}\int d_{m+1}[k]_n d^{\rm ex}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m,\boldsymbol{k}_{m+1},\ldots,\boldsymbol{k}_n;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)\,, \tag{2}$$

where $d_i[k]_j \equiv d^3k_i \ldots d^3k_j$.

The "diagonal" elements of the exclusive and inclusive density matrices give the multiparticle exclusive and inclusive densities respectively :

$$\rho^{\rm ex}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = d^{\rm ex}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)$$
$$\rho^{\rm in}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = d^{\rm in}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)\,. \tag{3}$$

The on-mass-shell Fourier transforms of the density matrices define the exclusive and inclusive multiparticle densities in the configuration space.

3. Cascade in momentum space. DLA formalism

In this section we recall the main assumptions of the DLA approximation in momentum space [6]. This approximation accounts for the leading double logarithmic contributions to multiparton cross section, but it describes quite well the structure of the parton cascade at high energies. We will shortly review the formalism for the simplest QCD process: $e^+e^- \to q\bar{q} + g\ldots g$. To simplify the analysis one uses here the planar gauge. In this gauge q and $\bar{q}$ emit soft gluons independently since the interference between two emission amplitudes vanishes. Virtual corrections appear in the form of the QCD form factors.

3.1. Multigluon amplitudes

For any tree Feynman diagram D (without 4-gluon vertices) the amplitude to obtain exactly m gluons (see Fig. 1) emitted by the initial q ($\bar{q}$) of 4-momentum P with the account of the virtual corrections reads [2]:

$$S_{e_1\ldots e_m}(k_1,\ldots,k_m) = (-1)^n e^{-\frac{w(P)}{2}} \prod_{i=1}^{m} M_{P_i}(k_i)\, e^{-\frac{w(k_i)}{2}}\,, \tag{4}$$

and $M_{P_i}(k_i)$ equals:

$$M_{P_i}(k_i) = g_S \frac{(e_i\cdot P_i)}{(k_i\cdot P_i)}\Theta_{P_i}(k_i)\, G_{P_i}\,, \tag{5}$$

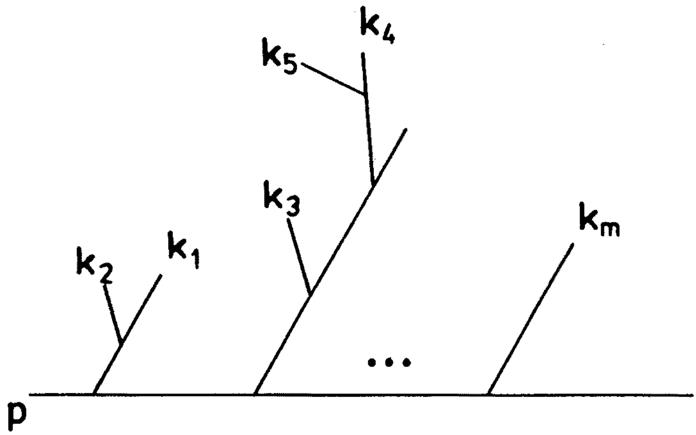

Fig. 1. Feynman diagram for the production of m gluons in DLA, where P is the 4-momentum of initial $\bar{q}(q)$; k_i denotes the 4-momentum of the i-th produced gluon.

where:
$g_S = \sqrt{4\pi\alpha_s}$,
n is the number of gluons emitted of quark (antiquark),
$k_i = (\omega_i, \boldsymbol{k}_i)$ denotes the 4-momentum of the i-th soft gluon,
$e_i \equiv e_i^{(j)}$, $j = 0, \ldots, 3$ describes its polarization 4-vector,
P_i is the 4-momentum of the parent of the ith gluon,
G_{P_i} denotes the color factor for the given vertex of the tree diagram D,
$\Theta_{P_i}(k_i)$ is a generalized step function which restricts the phase space for k_i:

$$\Theta_P(k) : \{k^0 \equiv \omega < P^0, \, \theta_{\boldsymbol{kP}} < \theta, \, \omega\theta_{\boldsymbol{kP}} > Q_0\}, \tag{6}$$

where P is the momentum of the parent of a given parton k, Θ denotes the emmision angle of the previous parton splitting on the P-line, and Q_0 is a cut-off parameter.

Radiation factor in (4) contains the function $w(P_i)$, defined as:

$$w(P) = \int d^3k |A_P(k)|^2_e \,, \tag{7}$$

where $A_P(k)$ introduces the phase space factor $(\omega_k)^{-1/2}$ in the form:

$$A_P(k) = \frac{M_P(k)}{\sqrt{2\omega_k}} \,. \tag{8}$$

The expression $|A_P(k)|^2_e$ in (7) has been already averaged over two physical transverse polarizations e^1, e^2. It should be also emphasized that produced gluons are "real" (on-mass-shell) particles, so the energy ω_k of the gluon of the momentum k can be approximated as:

$$\omega_k = |\boldsymbol{k}| \,. \tag{9}$$

Summing of the color factors G over the color indices gives the result :

$$G_{P_i} G^*_{P_i} = \begin{cases} C_F & \text{dla } P_i = P \\ C_V & \text{dla } P_i \neq P \end{cases} , \tag{10}$$

where P denotes the 4-momentum of the quark (antiquark) which initalizes the gluon cascade.

3.2. Multigluon densities

In the DLA approximation different tree diagrams come from different non-overlapping kinematic regions. Hence they do not interfere. Therefore, to calculate exclusive and inclusive multigluon densities it is enough to sum up the amplitudes (4) incoherently over all possible tree diagrams D. Hence one gets exclusive density $\rho^{\text{ex}}_P(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_m)$ in the form:

$$\rho^{\text{ex}}_P(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_m) = \sum_D \prod_{i=1}^{m} |A_{P_i}(k_i)|^2_{\epsilon_i} \, e^{-w(P_i)} \,, \tag{11}$$

parametrized by the momentum P of the quark (antiquark) which initializes the cascade. Multigluon inclusive density $\rho^{\text{in}}_P(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_m)$ follow from the formula (2), (3):

$$\rho^{\text{in}}_P(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_m) = \sum_{n=m}^{\infty} \frac{1}{(n-m)!} \int d_{m+1}[k]_n \; \rho^{\text{ex}}_P(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) \,. \tag{12}$$

3.3. Generating functional

Now the difficulty is how to perform the summation in (11) and (12) over all diagrams D in the convenient way. The problem has been solved by introducing the method of generating functional (GF) (see [2] and references therein). Generating functional for multigluon densities $Z_P[u]$, which fulfills the master equation:

$$Z_P[u] = e^{-w(P)} \, exp(\int d^3k |A_P(k)|^2_\epsilon \, u(k) Z_k[u]) \,, \tag{13}$$

reproduces contributions of all possible tree diagrams D and allows to express multigluon densities $\rho_P^{\rm in}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)$ and $\rho_P^{\rm ex}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)$ as:

$$\rho_P^{\rm ex}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)=\frac{\delta^m}{\delta u_1\ldots\delta u_m}Z_P|_{\{u=0\}} \tag{14}$$

and

$$\rho_P^{\rm in}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)=\frac{\delta^m}{\delta u_1\ldots\delta u_m}Z_P|_{\{u=1\}}\,, \tag{15}$$

where u_i denotes $u(k_i)$ and, respectively, $\frac{\delta}{\delta u_i}\equiv\frac{\delta}{\delta u(k_i)}$.

We would like to emphasize once more here the simplicity of description of the multigluon densities (14) and (15) in the language of the GF. The method allows to forget the complicated summation procedure especially in the case of inclusive densities, and express the multiplicities in a simple, compact form (for details see [8]).

4. Calculating of the density matrix in the DLA formalism

The DLA formalism in momentum space gives a good description of the structure of the gluon cascade. The GF scheme suggests a clear recipe how to construct multigluon densities, and allows to describe and investigate properties of the gluon distributions in a very convenient way.

In this chapter we would like to discuss calculating of the density matrices in the DLA approximation for the same process namely: $e^+e^-\to q\bar{q}+g\ldots g$. First we derive the general expression for the exclusive and inclusive density matrices

$$d_P^{\rm ex}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)\text{ and }d_P^{\rm in}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)\,.$$

The task looks more complicated because in this case the interference between different diagrams in (1) generally does not vanish. Let us define two GF, $Z_P[u]$ and $Z^*_{P'}[w]$, which generate the sum of all the tree amplitudes (4) and the sum of their complex conjugates respectively:

$$Z_P[u]=e^{-w(P)/2}\exp(\int d^3kA_P(k)u(k)Z_k[u])\,,$$
$$Z^*_{P'}[s]=e^{-w(P')/2}\exp(\int d^3kA^*_{P'}(k)s(k)Z^*_k[s])\,, \tag{16}$$

The multigluon density matrices can be then expressed as (see Appendix A):

$$d_P^{\rm ex}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)=\frac{\delta^m}{\delta s_{1'}\ldots\delta s_{m'}}\frac{\delta^m}{\delta u_1\ldots\delta u_m}Z_P[u]Z^*_{P'}[s]|_{\{u=s=0\},P=P'}\,,$$
$$d_P^{\rm in}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m)=\frac{\delta^m}{\delta s_{1'}\ldots\delta s_{m'}}\frac{\delta^m}{\delta u_1\ldots\delta u_m}Z_P[u]Z^*_{P'}[s]|_{\{u=\frac{\delta}{\delta s},s=0\},P=P'}\,. \tag{17}$$

Using the formulae (17) one can obtain complicated equations for the density matrices. The summation problem does not dissapear. It is only hidden in the compact form of (17). However, in the above formula we have taken into account *all* the possible interferences between different trees D and D'. Detailed analysis of the DLA gives the result that not all the diagrams mix up: one can distinguish classes of interfering diagrams. Nevertheless, at the general level we did not succeed in formulation of such a GF which would include only these interfering diagrams.

However, we do not need the most general formula for the density matrix. We are interested in its behaviour if the differences $|\boldsymbol{k}_1-\boldsymbol{k}'_1|, ..., |\boldsymbol{k}_m-\boldsymbol{k}'_m|$ are small (large momentum differences will not contribute to Fourier transforms). It can be checked that in this limit interferences between different diagrams vanish, and one can sum up only "squared" contributions from the same graphs. So we construct the new GF for density matrices which reproduces only these relevant contributions of the tree diagrams and their virtual corrections. The last step will be to derive explicit integral equations for single and double particle inclusive density matrix.

4.1. Density matrix in quasi-diagonal limit

We investigate the density matrix in the so-called quasi-diagonal limit *i.e.* if for the m-particle matrix the differences $|\boldsymbol{k}_1 - \boldsymbol{k}'_1|, \ldots, |\boldsymbol{k}_m - \boldsymbol{k}'_m|$ are small. This requirement simplifies the problem substantively. From the analysis of all diagrams contributing to single particle density matrix $d_P^{ex}(k'_1, k_1)$ it can be proved (see Appendix B) that interference between different diagrams appears only if $|\omega_1 - \omega_{1'}| \gg 0$ or $|\theta_{1P} - \theta_{1'P}| \gg 0$. This statement can be generalized for any m-particle density matrix. If we have m particles, and (m-1) from them are "close" to each other *e.g.* $k_1 \cong k'_1, \ldots, k_{m-1} \cong k'_{m-1}$ then the interference of the different diagrams will take place only if either energies or angles of k_m and k'_m are strongly ordered.

So, in our approximation we can exclude the interference between the different diagrams, and sum only the "square" contributions from the same ones. Hence, the exclusive and inclusive density matrices can be expressed as:

$$d_P^{\rm ex}(\boldsymbol{k}'_1, \ldots, \boldsymbol{k}'_m; \boldsymbol{k}_1, \ldots, \boldsymbol{k}_m) = \sum_D \prod_{i=1}^{m} (4\omega_{k'_i}\omega_{k_i})^{-1/2}$$

$$\langle S_{e_1 \ldots e_m}(k'_1, \ldots, k'_m) S_{e_1 \ldots e_m}(k_1, \ldots, k_m) \rangle_{(e_1 \ldots e_m)}, \qquad (18)$$

$$d_P^{\rm in}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = \sum_{n=m}^{\infty}\frac{1}{(n-m)!}\sum_D \int d_{m+1}[k]_n$$
$$\times \prod_{i=1}^{m}(4\omega_{k'_i}\omega_{k_i})^{-1/2}\prod_{j=m+1}^{n}(4\omega_{k_j}\omega_{k_j})^{-1/2}$$
$$\times \langle S_{e_1\ldots e_n}(k'_1,\ldots,k'_m,k_{m+1},\ldots,k_n)S_{e_1\ldots e_n}(k_1,\ldots,k_n)\rangle_{(e_1\ldots e_n)} \quad (19)$$

The summation in (18), (19) over D can be easily performed using the GF we define in the next subsection.

4.2. Generating functional for density matrices in the quasi-diagonal limit

We propose a master equation for the new generating functional $Z_{P'P}[u(k',k)]$ in the form:

$$Z_{P'P}[u] = e^{-W(P',P)}\sum_{n=0}^{\infty}\frac{1}{n!}\int d_1[k']_n\, d_1[k]_n u(k'_1,k_1)\ldots u(k'_n,k_n)$$
$$\times \langle A^*_{P'}(k'_1)A_P(k_1)\rangle_{e_1}\ldots\langle A^*_{P'}(k'_n)A_P(k_n)\rangle_{e_n}$$
$$\times Z_{k'_1k_1}[u]\ldots Z_{k'_nk_n}[u]P_{1',\ldots,n';1,\ldots,n}\,, \quad (20)$$

where the function $P_{1',\ldots,n';1,\ldots,n}$ introduces the requested parallel angular ordering for n particles in the form:

$$P_{1',\ldots,n';1,\ldots,n} = \sum_{(i_1,\ldots,i_n)\in{\rm Perm}(1,\ldots,n)} \Theta(\theta_{k'_{i_1}P'}>\ldots>\theta_{k'_{i_n}P'})\,\Theta(\theta_{k_{i_1}P}>\ldots>\theta_{k_{i_n}P})\,, \quad (21)$$

and the radiation factor fulfills the condition:

$$W(P',P) = \frac{w(P')+w(P)}{2}\,. \quad (22)$$

The multiparticle density matrices can be then expressed as (for proof see Appendix C):

$$d_P^{\rm ex}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = \frac{\delta^m}{\delta u_{m',m}\ldots\delta u_{1',1}}Z_{P'P}|_{\{u=0\},P=P'}\,, \quad (23)$$
$$d_P^{\rm in}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = \frac{\delta^m}{\delta u_{m',m}\ldots\delta u_{1',1}}Z_{P'P}|_{\{u=\delta^3(l'-l)\},P=P'}\,. \quad (24)$$

If the profile function $u(k', k)$ equals to 0 and $\delta^3(l'-l)$ respectively the functional $Z_{P'P}$ takes the form:

$$Z_{P'P}[u=0] = e^{-W(P',P)}$$
$$Z_{PP}[u=\delta^3(l'-l)] = 1\,. \tag{25}$$

The GF from (20) is the generalization of the GF defined in (13). Using GF (20) one can also easily obtain multigluon densities (11) and (12) in momentum space :

$$\rho_P^{\rm ex}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = \frac{\delta^m}{\delta u_{m,m}\ldots\delta u_{1,1}} Z_{P'P}|_{\{u=0\},P=P'}$$
$$\rho_P^{\rm in}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = \frac{\delta^m}{\delta u_{m,m}\ldots\delta u_{1,1}} Z_{P'P}|_{\{u=\delta^3(l'-l)\},P=P'}\,. \tag{26}$$

For further analysis we need the exact value of the product of the amplitudes $\langle A^*_{P'}(k')A_P(k)\rangle_\epsilon$ which occurs in (20). Averaging over gluon polarizations (see Appendix D) gives the result:

$$\langle A^*_{P'}(k')A_P(k)\rangle_\epsilon \equiv A_{P'P}(k',k)$$
$$= 4g_S^2\, G^*_{P'}G_P \frac{1}{\sqrt{4\omega^{3'}\omega^3}}\frac{1}{\theta_{Pk}\theta_{P'k'}}\Theta_{P'}(k')\Theta_P(k)\,. \tag{27}$$

4.3. Single and double inclusive density matrix

From the formula (24) one can derive iterative equations for inclusive density matrix of an arbitrary order. As an example we present equations for single and double particle density matrices:

$$d_P^{\rm in}(k_1';k_1) = \int d^3k A_{PP}(k,k) d_k^{\rm in}(k_1';k_1)$$
$$+ f_P(k_1',k_1)A_{PP}(k_1',k_1)\, Z_{1',1}[u=\delta^3(l'-l)] \tag{28}$$

and

$$d_P^{\rm in}(k_1',k_2';k_1,k_2) = \int d^3k A_{PP}(k,k) d_k^{in}(k_1',k_2';k_1,k_2)$$
$$+A_{PP}(k_1',k_1)\frac{\delta}{\delta u_{2',2}} Z_{1',1}[u=\delta^3(l'-l)] f_P(k_1',k_1)$$
$$+A_{PP}(k_2',k_2)\frac{\delta}{\delta u_{1',1}} Z_{2',2}[u=\delta^3(l'-l)] f_P(k_2',k_2)$$

$$+A_{PP}(k_1',k_1)Z_{1'1}[u=\delta^3(l'-l)]A_{PP}(k_2',k_2)Z_{2',2}[u=\delta^3(l'-l)]$$
$$\times f_P(k_1',k_2';k_1,k_2)$$
$$+A_{PP}(k_1',k_1)Z_{1'1}[u=\delta^3(l'-l)]\int d^3kA_{PP}(k,k)d_k^{\rm in}(k_2',k_2)f_P(k_1',k;k_1,k)$$
$$+A_{PP}(k_2',k_2)Z_{2'2}[u=\delta^3(l'-l)]\int d^3kA_{PP}(k,k)d_k^{in}(k_1',k_1)f_P(k_2',k;k_2,k)$$
$$+\left(\int d^3kA_{PP}(k,k)d_k^{\rm in}(k_1',k_1)\right)\left(\int d^3kA_{PP}(k,k)d_k^{\rm in}(k_2',k_2)\right). \qquad (29)$$

Factors $f_P(k_1',\ldots,k_m';k_1,\ldots,k_m)$ can be expressed as:

$$f_P(k_1',\ldots,k_m';k_1,\ldots,k_m)$$
$$=e^{-W(P,P)}\sum_{n=0}^{\infty}\frac{1}{n!}\int d_1[p]_nA_{PP}(p_1,p_1)\ldots A_{PP}(p_n,p_n)$$
$$\times P_{1,\ldots,n,k_1',\ldots,k_m';1,\ldots,n,k_1,\ldots,k_m}. \qquad (30)$$

For $m=1$ we have calculated $f_P(k',k)$ explicitly. It equals to:

$$f_P(k',k)=\Theta(\theta_{k'P}\geq\theta_{kP})e^{g_S^2C_F\left(\ln^2\frac{\theta_{kP}P}{Q_0}-\ln^2\frac{\theta_{k'P}P}{Q_0}\right)}$$
$$+\Theta(\theta_{k'P}<\theta_{kP})(k'\leftrightarrow k). \qquad (31)$$

Equations (28), (29) in the limit $k_1'=k_1, k_2'=k_2$ reduce to multiparticle density equations [8] as expected. However, also for any k,k' the structure of (28), (29) looks quite similar to the structure of multiparticle density equations. This suggests to get the solution using the technique similar to the technique described in [8].

5. Conclusions

Our conclusions can be listed as follows:

1. We have obtained master equation for the density matrices GF. From this relation one can derive equations for density matrices of arbitrary order. Integral equations for single and double inclusive densities look quite similar to the equations for multiparticle densities we already know. It allows to expect that they can be solved in a way similar to that used for multiparticle densities. Hence, as the next step in our approach we will try to derive multiparticle density explicitly and to investigate their behaviour in the coordinate space.

2. We know that the GF "works" in the quasi-diagonal region but we do not know what it really means quantitatively. The further quantitative analysis is needed. If the quasi-diagonal region is large enough, we could easily obtain the multiparticle densities in the configuration space taking the on-mass-shell Fourier transform of the density matrix (it needs also to be checked if the functions we want to transform are "smooth" enough to get the Fourier integral vanishing in large $|k_1 - k'_1|, \ldots, |k_m - k'_m|$ limit).

3. One can try to find the general GF for the density matrices which would work also far from the quasi-diagonal region. It would solve the problems mentioned above. Nevertheless, it seems to be quite complicated. Till now, we did not find the rule which could iteratively distinguish classes of the interfering diagrams.

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Appendix A

Functionals $Z_P[u]$ and $Z_{P'}[s]$ defined in (16) generate the sum of the amplitudes (4) and their complex conjugates over all possible tree diagrams respectively. It can be seen when one rewrites *i.e.* the master equation for Z_P in the form of the diagram series (see Fig. 2). From that construction follows the form of the exclusive density matrix (17):

$$d_P^{\mathrm{ex}}(\boldsymbol{k}'_1, \ldots, \boldsymbol{k}'_m; \boldsymbol{k}_1, \ldots, \boldsymbol{k}_m) = \frac{\delta^m}{\delta s_{1'} \ldots \delta s_{m'}} Z_P^*[s]|_{\{s=0\}} \frac{\delta^m}{\delta u_1 \ldots \delta u_m} Z_P[u]|_{\{u=0\}} . \tag{32}$$

Then the inclusive density matrix calculated from (2) and (32) looks like :

$$d_P^{\mathrm{in}}(\boldsymbol{k}'_1, \ldots, \boldsymbol{k}'_m; \boldsymbol{k}_1, \ldots, \boldsymbol{k}_m) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\int d^3k \frac{\delta}{\delta s} \frac{\delta}{\delta u} \right)^n \times \frac{\delta^m}{\delta s_{1'} \ldots \delta s_{m'}} Z_P^*[s]|_{\{s=0\}} \frac{\delta^m}{\delta u_1 \ldots \delta u_m} Z_P[u]|_{\{u=0\}} , \tag{33}$$

and from the identity for the product of any two functionals F, F':

$$F[u]F'[w]|_{\{u=\frac{\delta}{\delta w}, w=0\}} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\int d^3k \frac{\delta}{\delta w} \frac{\delta}{\delta u} \right)^n F[u]|_{\{u=0\}} F'[w]|_{\{w=0\}} \tag{34}$$

follows (17).

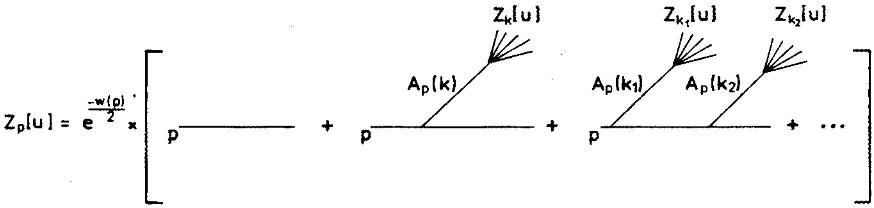


Fig. 2. GF(16) as a diagram series

$$Z_P[u] = \mathrm{e}^{-\frac{w(P)}{2}}\{1 + \int d^3k A_P(k) u(k) Z_k[u] + \int d^3k_1 d^3k_2 A_P(k_1) A_P(k_2) u(k_1) u(k_2) Z_1[u] Z_2[u] + \ldots\}.$$

Appendix B

In DLA one gets for the two gluon contributions four different tree graphs M_a, M_b, M_c, M_d defined on non-overlapping kinematic regions (see Fig. 3). Emmited gluons are either angular (AO) or energy ordered (EO).

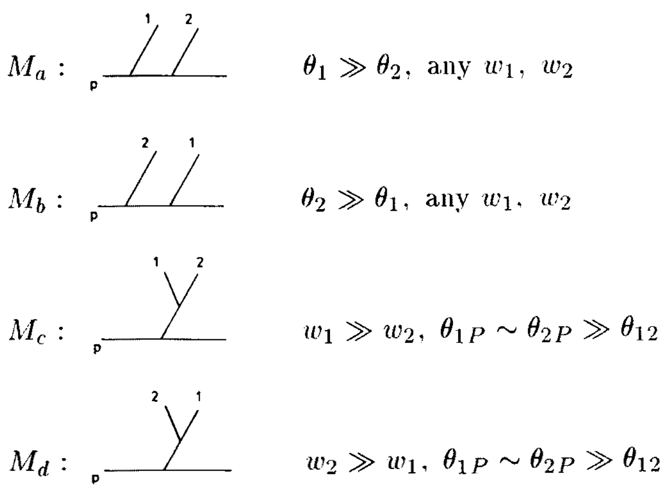
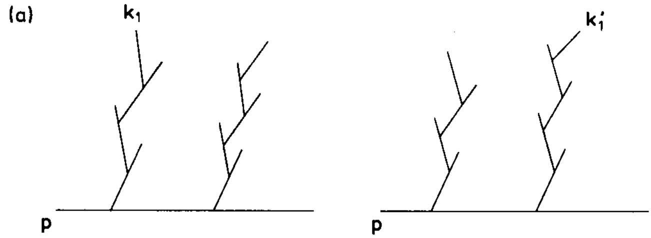
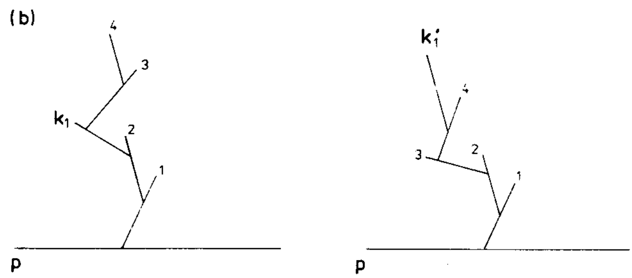


Fig. 3. Redefined phase space in DLA.

Let us consider all the diagrams contributing to the single particle density matrix $d_P^{ex}(k_1', k_1)$. From the (AO) and (EO) follows that the interference between any two different graphs will appear only, if either energies ω_1, ω_1' or emmision angles $\Theta_{1P}, \Theta_{1'P}$ of produced gluons are *strongly* ordered (Fig. 4).



(AO) Interference if $\theta_{1P} \gg \theta_{1'P}$



(EO) Interference if $w_1 \gg w_1'$

Fig. 4. Interference between different diagrams for $d_P^{ex}(k_1'; k_1)$. Remark: Diagrams (a), (b) are identical except of the position of $k_1(k_1')$ leg.

This statement can be generalized for any m-particle density matrix by induction. If we have m particles, and (m-1) from them are "close" to each other *e.g.* $k_1 \cong k_1', \ldots, k_{m-1} \cong k_{m-1}'$ then the intereference of the different diagrams will take place only if either energies or angles of k_m and k_m' are strongly ordered.

Appendix C

$Z_{P'P}[u(k', k)]$ defined in (20) produces correct exclusive density matrices (23). This statement follows from the master equation for $Z_{P'P}$ represented in the form of diagram series (see Fig. 5). The series reproduces all "squared" contributions of the same tree diagrams, and excludes interference of the different ones (function $P_{1',\ldots,n';1,\ldots,n}$).

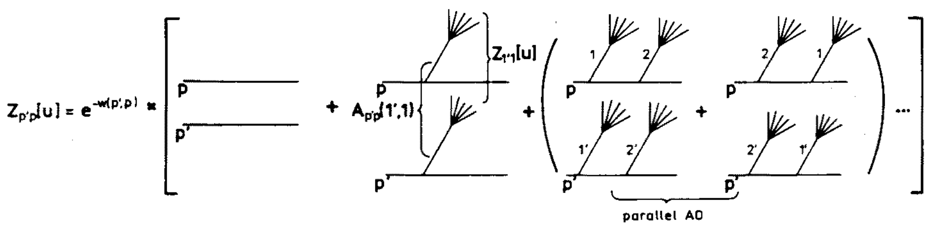


Fig. 5. GF(20) as a diagram series.

The form of the inclusive density (24) follows from the formulae (2) and (23). Substituting (23) into (2) we get as a result :

$$d_P^{\rm in}(\boldsymbol{k}'_1,\ldots,\boldsymbol{k}'_m;\boldsymbol{k}_1,\ldots,\boldsymbol{k}_m) = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\int d^3k d^3k'\delta^3(k'-k)\frac{\delta}{\delta u_{k',k}}\right)^n \times \frac{\delta^m}{\delta u_{m',m}\ldots\delta u_{1',1}} Z_{P'P}[u]|_{\{u=0\}} \qquad (35)$$

which represents the functional $Z_{P'P}[u]$ expanded around "null" in the "point" $u=\delta^3(l'-l)$.

Appendix D

We want to average relation (27) over the physical polarizations of the produced gluon. The exact expression to be summed over polarizations looks like:

$$\frac{(e\cdot P)(e'\cdot P')}{(k\cdot P)(k'\cdot P')}, \qquad (36)$$

where e, e' are polarization of the same gluon (one gets e' from e taking the limit $k'=k$). The gauge fixing we use in the approach allows to neglect contributions to (36) coming from the "nonphysical" polarizations e^0 and e^3. Furthermore, it requests time components of the physical polarizations e^1, e^2 to be equal to 0. Space components of e^1, e^2 can be then constructed in the form:

$$\boldsymbol{e}^2 = \frac{\boldsymbol{P}\times\boldsymbol{k}}{|\boldsymbol{P}\times\boldsymbol{k}|}, \qquad \boldsymbol{e}^1 = \frac{\boldsymbol{e}^2\times\boldsymbol{k}}{|\boldsymbol{e}^2\times\boldsymbol{k}|}, \qquad (37)$$

and respectively for e' :

$$e^{2'} = \frac{\boldsymbol{P'} \times \boldsymbol{k'}}{|\boldsymbol{P'} \times \boldsymbol{k'}|},$$
$$e^{1'} = \frac{\boldsymbol{e}^{2'} \times \boldsymbol{k'}}{|\boldsymbol{e}^{2'} \times \boldsymbol{k'}|}. \tag{38}$$

Summing over these 2 polarizations and expanding scalar products in (36) one obtains finally:

$$\sum_{j=1,2} \frac{(e^j \cdot P)(e^{j'} \cdot P')}{(k \cdot P)(k' \cdot P')} = \frac{4}{\omega\omega'} \frac{1}{\Theta_{Pk}\Theta_{P'k'}}. \tag{39}$$

The result can be easily confirmed for any two physical polarizations $\boldsymbol{\epsilon}^1, \boldsymbol{\epsilon}^2$ lying in the plane $\boldsymbol{e}^1\boldsymbol{e}^2$:

$$\boldsymbol{\epsilon}^1 = \cos\varphi\, \boldsymbol{e}^1 + \sin\varphi\, \boldsymbol{e}^2,$$
$$\boldsymbol{\epsilon}^2 = -sin\varphi\, \boldsymbol{e}^1 + cos\varphi\, \boldsymbol{e}^2, \tag{40}$$

and respectively for $\boldsymbol{\epsilon}^{1'}, \boldsymbol{\epsilon}^{2'}$ with the phases $\varphi = \varphi'$.

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