RELATIVISTIC CORRECTIONS IN THE AVERAGE FIELD OF NEUTRON AND PROTON SHELLS

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We have calculated the relativistic corrections for the mass and potential energy to one-nucleon levels. We have found new depths of Woods–Saxon potentials for the nuclei in the region $15 \leq A \leq 209$. The semi-relativistic equation has been reduced to the integral-differential equation with the kernel, which is proportional to the Green's function. It can be expressed by unperturbed wave functions and nonphysical solutions of the Schrödinger equation. It has been shown that for an average field of the nuclei this approach is sufficiently exact. The corrections for mass are comparable with the energies of excited states and they are increasing the binding energies. The corrections to potential are positive and small, except for some light nuclei, where they can compensate the negative corrections for mass.

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1. Introduction

Usually we consider the nuclei like the nonrelativistic systems. But we must take into consideration [1] that nuclear force has a repulsive core (-0.4 fm) and a great spin-orbit interaction. The repulsive core is generating the wave functions with high impuls [2] and we cannot solve this problem using the nonrelativistics Schrödinger or Hartree–Fock equations. In paper [2] the calculations for relativistic corrections for the mass or kinetic energy and potential energy were provided

$$\delta\hat{M} = \delta\hat{T} + \delta\hat{V}\,, \tag{1.1}$$

$$\delta\hat{T} = \sum_i \left[\sqrt{(\hat{p}_i)^2 + m^2} - m - \frac{(\hat{p}_i)^2}{2m}\right]. \tag{1.2}$$

There was obtained that the increase in the binding energy for the four nucleon system is about -0.6 MeV. For the deuteron I obtained -0.0727 MeV because the relativistic correction for mass is -0.2127 MeV and to potential energy $+0.1400$ MeV. The insignificant contribution of the relativistic corrections to the binding energies for the light nuclei is the result of their compensation. But for the large nuclei and excited states we have different situation [3, 4]. The corrections for the mass of the nucleons in the average potential of the nuclei $15 \leq A \leq 209$ are significant and cannot be compensated by small positive relativistic corrections to the potential energy of the main field. The relativistic corrections for the mass must be taken into account when interpreting of resonance [5], stripping [6] and knockout [7] reactions. The single-particle eigenfunctions and energies can be obtained by solving the Hartree-Fock equations with phenomenological potentials [8] or Skyrme's forces [9]. In that case the relativistic corrections for mass, depending on state[4], can achieve the significant value -1.12 MeV. These corrections must be calculated using the perturbation theory.

2. The general properties of the semi-relativistic equation

Semi-relativistic Hamiltonian [4] can be written in the form

$$\hat{H}_r = \frac{-\hat{p}^4}{8 \cdot m^3 \cdot c^2} + \frac{\hat{p}^2}{2m} - \frac{\hbar^2}{4 \cdot m^2 \cdot c^2} \left(\frac{d}{dr} V(r) \right) \frac{d}{dr} + V(r) + V_{sl}(r) \,. \quad (2.1)$$

The first term of the Hamiltonian and the third term include relativistic corrections for the mass and the potential. The following term

$$V_{sl}(r) = -\kappa \frac{1}{r} \left(\frac{d}{dr} V(r) \right) (\vec{\sigma} \cdot \vec{l}) \quad (2.2)$$

is the spin-orbital potential which has also the relativistic origin. The semi-relativistic equation for the eigenfunction $R_\alpha = U_\alpha / r$ can be obtained [4] from the Hamiltonian (2.1) for the central potential $V(r)$ in the form

$$C_1 \hat{D} U_\alpha + C_2 r \frac{d}{dr} V \frac{d}{dr} \frac{U_\alpha}{r} + \frac{d^2}{dr^2} U_\alpha - \frac{L(L+1)}{r^2} U_\alpha + (CE_\alpha - CV - CV_{sl}) U_\alpha = 0$$

$$C = \frac{2m}{\hbar^2} \,, \quad C_1 = \left(\frac{\hbar}{2mc} \right)^2 \,, \quad C_2 = \frac{1}{2mc^2} \,, \quad (2.3)$$

where $\hat{D}$ is a differential operator of fourth order

$$\hat{D} U_\alpha = \frac{d^4}{dr^4} U_\alpha - \frac{2L_0}{r^2} \frac{d^2}{dr^2} U_\alpha + \frac{4L_0}{r^3} \frac{d}{dr} U_\alpha + \left[\frac{(L_0)^2 - 6L_0}{r^4} \right] U_\alpha \,,$$

$$L_0 = L(L+1)\,. \tag{2.4}$$

Substituting asymptotic expression of the eigenfunction $U_\alpha \simeq r^\beta$ as $r \to 0$ in (2.3) we get four partial independent asymptotic solutions

$$U_{\alpha 0} \simeq r^{L+1}\,, \quad F_{\alpha 0} \simeq r^{-L}\,, \quad U_{\alpha 1} \simeq r^{L+3}\,, \quad F_{\alpha 1} \simeq r^{-L+2}\,. \tag{2.5}$$

Assuming that the potential energy vanishes at great distances, we can find the four asymptotic solutions of (2.3) in the exponential form $U_\alpha \simeq \mathrm{e}^{k_\alpha r}$. In this case we get

$$k_{\alpha 1} = -k_\alpha\,, \quad k_{\alpha 2} = k_\alpha\,, \quad k_\alpha = \frac{1}{\sqrt{2C_1}}\left(-1+\sqrt{1-4C_1CE_\alpha}\right)^{1/2}\,, \tag{2.6}$$

$$k_{\alpha 3} = ik_{\alpha m}\,, \quad k_{\alpha 4} = -ik_{\alpha m}\,, \quad k_{\alpha m} = \frac{1}{\sqrt{2C_1}}\left(1+\sqrt{1-4C_1CE_\alpha}\right)^{1/2}\,. \tag{2.7}$$

Usually $4C_1CE_\alpha < 1$ and we can express $k_\alpha = \sqrt{-CE_\alpha}$. From this, we can make the conclusion that for the vanishing at large distances potentials the semi-relativistic wave functions are decreasing at large distances more rapidly than the Schrödinger's wave functions for the same states because they represent the states with the larger binding energies [2, 3]. The solutions of (2.3) can be expressed in the form

$$U_\alpha(r) = r^\beta W(r)\mathrm{e}^{-k_\alpha r}\,. \tag{2.8}$$

When relativistic corrections are very small the solutions (2.8) are decreasing at infinity like the solutions of the Schrödinger equation. But at the origin we have essentially different situation. We have two different physical solutions (2.5) $U_{\alpha 0}$ and $U_{\alpha 1}$ with the essentially different behaviour at the origin. Wavefunctions $U_{\alpha 1}$ represent bound states spatially localized at larger distances with smaller binding energies.

For convenience we have introduced a dimensionless parameter $\rho = \frac{r}{F}$ and then the radial semi-relativistic equation (2.3) obtains the form

$$C_F\hat{D}(\rho)U_\alpha + C_FC\rho\left(\frac{d}{d\rho}V\right)\frac{d}{d\rho}\frac{U_\alpha}{\rho} + \frac{d^2}{d\rho^2}U_\alpha - \frac{L(L+1)}{\rho^2}U_\alpha + CF^2(E_\alpha - V)U_\alpha = 0$$

$$C_F = \frac{C_1}{F^2}\,, \quad C_2 = C_1C\,. \tag{2.9}$$

Now we can see that for large F the semi-relativistic equation reduces into the Schrödinger equation. For the nucleons localized around the center of

force about 1 fm $C_F = 0.011$. In the region of repulsive core (0.4 fm) $C_F = 0.07$. For the electrons in the first Bohr orbit $C_F = 1.3 \cdot 10^{-5}$. These results show that the theory of electronic spectra we can calculate the relativistic corrections with sufficient accuracy in the first approximation of the perturbation theory [10]. But for the calculations of the nuclear energy levels we must include the high order perturbations. In this paper the multiplicative perturbation theory [11, 12] is used. In the average potentials of a nucleus the nucleons are localized at sufficiently large distances from the center of force. The Green's functions of the semi-relativistic equation can be expressed by the linear independent solutions $U_{\alpha 0}$ and $F_{\alpha 0}$ of the Schrödinger equation. A very simple and exact iteration method can be used for finding eigenfunctions and eigenvalues of the integral-differential semi-relativistic equation.

3. The integral-differential semi-relativistic equation

If we consider relativistic corrections for mass and potential like perturbation, the semi-relativistic equation (2.3) can be written in the form

$$\frac{d^2}{dr^2}U_\alpha - \frac{L(L+1)}{r^2}U_\alpha + C[E_\alpha - \hat{V}_D - V_1(r)]U_\alpha = 0\,, \tag{3.1}$$

where we introduce the differential operator

$$\hat{V}_D = V(r) + V_{sl}(r) - V_1(r) + \frac{C_1}{C}\hat{D}(r) + C_1 r\frac{d}{dr}V(r)\frac{d}{dr}\frac{1}{r}\,. \tag{3.2}$$

There $V(r)$ is the average potential of the nucleus, whereas

$$V_1(r) = \frac{m\omega^2 r^2}{2} \tag{3.3}$$

is the model potential. The fourth and the fifth members in (3.2) represent the operators of the relativistic corrections for mass and for potential. The radial wave functions of the Schrödinger equation for the harmonic oscillator potential (3.3) are presented in [13]

$$U_{nL} = \mathrm{e}^{-\frac{1}{2}\rho}\rho^{\frac{1}{2}(L+1)}\sum_{k=0}^{n-1} a_k\rho^k\,,\quad \rho = \frac{m\omega r^2}{\hbar}\,,\quad n = 1,2,3\ldots \tag{3.4}$$

$$a_{k+1} = \frac{k-n+1}{(k+1)\,(k+L+\frac{3}{2})}a_k\,. \tag{3.5}$$

In the recurrence relation $a_0 = 1$ was chosen. Using the standard methods we obtained the linearly independent solution

$$F_{nL} = \mathrm{e}^{-\frac{1}{2}\rho}\rho^{-\frac{1}{2}L}w(\rho)\,, \quad w = \sum_{k=0}^{\infty} b_k\rho^k\,,$$

$$b_{k+1} = \frac{k - \frac{1}{2}\left(\varepsilon_{nL} + L - \frac{1}{2}\right)}{(k+1)\left(k - L + \frac{1}{2}\right)}b_k\,, \quad b_0 = 1\,. \tag{3.6}$$

The eigenvalues are also known as $E_{nL} = \varepsilon_{nL}\hbar\omega$, $\varepsilon_{nL} = 2n + L - \frac{1}{2}$. From the asymptotic behaviour of the wave function $U_\alpha(r)$ and nonphysical linearly independent solution $F_\alpha(r)$ at the origin we can obtain Wronskian of these linearly independent solutions

$$W_0 = (2L+1)\left(\frac{m\omega}{\hbar}\right)^{1/2}\,. \tag{3.7}$$

The eigenfunctions of Eq. (3.1) in the case of multiplicative perturbation theory [11, 12] must be expressed by multiplying the eigenfunctionU_{nL} for model potential $V_l(r)$ by the factor function [11] $\phi_{2,nLj}$ which depends on the potential operator $\hat{V}_D(r)$

$$U_\alpha = \phi_{2,nLj}U_{nL}\,. \tag{3.8}$$

Substituting (3.8) into (3.1) we obtain the equation (3.1) in the potential [11, 12] representation

$$U_{nL}\frac{d^2}{dr^2}\phi_2 + 2\left(\frac{d}{dr}U_{nL}\right) - C\hat{V}_\delta U_{nL}\phi_2 = 0\,,$$

$$\hat{V}_\delta(r) = \hat{V}_D(r) - \Delta E_{nLj}\,, \quad E_\alpha = E_{nL} + \Delta E_{nLj}\,. \tag{3.9}$$

Using the modified method of Lagrange in the paper [11] equation of the same type (3.9) has been reduced to integral equation

$$\phi_2 U_{nL} = U_{nL}\beta - \frac{F_{nL}}{W_0}\int_0^r U_{nL}C\hat{V}_\delta\phi_2 U_{nL}dx - \frac{U_{nL}}{W_0}\int_r^\infty F_{nL}C\hat{V}_\delta\phi_2 U_{nL}dx\,,$$

$$\beta = 1 + \frac{1}{W_0}\int_0^\infty F_{nL}C\hat{V}_\delta\phi_2 U_{nL}dx\,. \tag{3.10}$$

This equation can be written as

$$\phi_2(r)U_{nL}(r) = \beta U_{nL}(r) + \int_0^\infty G(r > r_1,\ r < r_1) C\hat{V}_\delta \phi_2 U_{nL} dr_1 , \qquad (3.11)$$

where according to [14] the kernel of this integral equation is the Green's function

$$G(r > r_1,\ r < r_1) = -\frac{1}{W_0} F_{nL}(r > r_1) U_{nL}(r < r_1) . \qquad (3.12)$$

The solution $\phi_2 U_{nL}$ regular at the origin for bound state must decrease to zero at infinity. Using this boundary condition from (3.10) we obtain

$$\Delta E_{nLj} = \frac{\int_0^\infty U_{nL} \hat{V}_D \phi_{2,nLj} U_{nL} dr}{\int_0^\infty U_{nL} \phi_{2,nLj} U_{nL} dr} . \qquad (3.13)$$

The equation (3.10) can be reduced to more handy form

$$\phi_2 U_{nL} = U_{nL} + \frac{U_{nL}}{W_0} \int_0^r F_{nL} C\hat{V}_\delta \phi_2 U_{nL} dr_1 - \frac{F_{nL}}{W_0} \int_0^r U_{nL} C\hat{V}_\delta \phi_2 U_{nL} dr_1 . \qquad (3.14)$$

The integral equations (3.14), (3.13) can be solved by the iteration method. For the zero approximation at the right-hand side of the integral equations we must take $\phi_2 = 1$ and then find ΔE_{nLj} from (3.13) and $\phi_2 U_{nL}$ from (3.14). We must choose freely the model potential, but it is better when unperturbed wave functions are close to perturbed wave functions $\phi_2 U_{nL}$. Then a small number of the iterations provide the results of high accuracy. In our method the frequency $\omega = d\omega_0$ for the model harmonic oscillator potential (3.3) can be determined by the r.m.s. radius of the nuclei [1]

$$\omega_0 = 41 \cdot A^{-1/3} \frac{\text{MeV}}{\hbar} . \qquad (3.15)$$

The constant d was found by demanding the minimum of the energy. The control calculations are in good agreement with the results obtained in [4, 15], where discretization method has been used.

4. Results of calculations

The control calculations of the neutron energy levels of ^{208}Pb in the states $1s_{1/2}$, $2s_{1/2}$, $3s_{1/2}$ give second terms -39.85 MeV, -29.43 MeV, -15.27 MeV while in paper [4] we get -39.9 MeV, -29.5 MeV, -15.3 MeV. The coincidence is sufficiently good because the accuracy of the calculations in [4] is not very high. The solutions of integral equations are always more exact.

The energies of the one-nucleon levels E_{nLj}, the relativistic corrections for the mass E_m and the potential E_v in the region $15 \leq A \leq 209$ have been calculated with the spherically symmetric Woods–Saxon potential

$$V(r) = -V^{n,p}\Big[1 + exp\Big[\alpha^{n,p}(r - R)\Big]\Big]^{-1}, \tag{4.1}$$

and the spin-orbit potential (2.4) with parameters [15] $\alpha^{n,p} = 1.5873\ \mathrm{fm}^{-1}$, $R = 1.24A^{1/3}$ fm and

$$V^{n,p} = V_m\left(1 \mp \gamma\frac{N-Z}{A}\right), \quad \gamma = 0.63, \quad V_m = 53.3\ \mathrm{MeV}, \tag{4.2}$$

$$\kappa = 0.263\left(1 + 2\frac{N-Z}{A}\right)\ \mathrm{fm}^2. \tag{4.3}$$

The Coulomb potential has been introduced in the usual form [16]

$$V_c(r) = \frac{(Z-1)e^2}{4\pi\varepsilon_0 r}P, \quad P = \frac{3}{2}\frac{r}{R} - \frac{1}{2}\Big(\frac{r}{R}\Big)^3, \quad r \leq R,$$
$$P = 1 \quad , \quad r > R. \tag{4.4}$$

At first the integral equation (3.14) was solved for the presented potentials without the differential operator $\hat{D}(r)$ and the relativistic corrections for the mass E_m and the relativistic corrections for the potential E_V were obtained in the first approximation of the perturbation theory. The one-nucleon levels E_{nLj} very well coincide with the appropriate energy levels obtained in [15]. The probability density of obtained wave functions at infinity is about 10^{-6} of their maximum values and without oscillations. This fact is very important in the calculations of E_m which are presented in Tables I and II. The relativistic corrections for the mass depend on the main and the orbital quantum numbers and do not have any influence on the definition of the constant of the spin-orbit interaction. For the neutrons the negative corrections for the mass vary from -0.2127 MeV in the state $1s_{1/2}$ of $^2_1$H to -0.9515 MeV in the state $2p_{3/2}$ of ^{53}Cr. For the protons the relativistic corrections for the mass vary from -0.2268 MeV in the state $2s_{1/2}$ of ^{17}F to -1.118 MeV in the state $2p_{3/2}$ of ^{59}Cu.

TABLE I

The neutrons levels and relativistic corrections for the considered potentials

Nucleus	nLj	E_m MeV	$(E_m)^D$ MeV	$(E_V)^D$ MeV	$(E_{nLj})^D$ MeV	V^n MeV	d
^{17}O	$1d_{5/2}$	−0.438	−0.427	0.061	−4.148	50.10	0.85
^{17}O	$2s_{1/2}$	−0.535	−0.414	0.012	−3.254	50.10	0.613
^{15}O	$1p_{1/2}$	−0.534	−0.346	0.110	−15.654	57.10	0.90
^{41}Ca	$1f_{7/2}$	−0.524	−0.494	0.062	−8.388	52.00	1.00
^{41}Ca	$2p_{3/2}$	−0.488	−1.124	0.017	−6.135	52.00	0.80
^{39}Ca	$1d_{3/2}$	−0.334	−0.337	0.075	−15.72	54.30	0.90
^{39}Ar	$1f_{7/2}$	−0.502	−0.502	0.058	−6.600	50.25	1.00
^{37}Ar	$1d_{3/2}$	−0.354	−0.344	0.070	−11.88	50.35	1.00
^{37}S	$1f_{7/2}$	−0.490	−0.506	0.054	−4.500	48.00	1.00
^{35}S	$1d_{3/2}$	−0.353	−0.348	0.069	−9.882	49.35	1.00
^{49}Ca	$2p_{3/2}$	−0.418	−0.433	0.015	−5.152	46.70	0.80
^{49}Ca	$2p_{1/2}$	−0.395	−0.356	0.012	−3.120	46.70	0.80
^{47}Ca	$1f_{7/2}$	−0.438	−0.445	0.059	−9.937	49.80	1.00
^{53}Cr	$2p_{3/2}$	−0.951	−0.664	0.023	−7.978	49.10	0.80
^{51}Cr	$1f_{7/2}$	−0.432	−0.429	0.061	−12.00	51.32	1.00
^{55}Fe	$2p_{3/2}$	−0.550	−0.752	0.027	−9.284	50.27	0.80
^{53}Fe	$1f_{7/2}$	−0.445	−0.447	0.065	−13.63	52.80	1.05
^{57}Ni	$2p_{3/2}$	−0.669	−0.611	0.031	−10.27	51.10	0.80
^{89}Sr	$2d_{5/2}$	−0.528	−0.529	0.017	−6.390	48.10	0.75
^{87}Sr	$1g_{9/2}$	−0.458	−0.461	0.051	−11.11	49.60	1.05
^{91}Zr	$2d_{5/2}$	−0.546	−0.540	0.019	−7.184	48.76	0.75
^{89}Zr	$1g_{9/2}$	−0.454	−0.456	0.052	−12.00	50.40	1.05
^{93}Mo	$2d_{5/2}$	−0.520	−0.515	0.018	−8.046	50.02	0.90
^{91}Mo	$1g_{9/2}$	−0.479	−0.450	0.052	−12.58	50.77	1.05
^{139}Ba	$2f_{7/2}$	−0.539	−0.541	0.012	−4.725	46.65	1.05
^{137}Ba	$2d_{3/2}$	−0.430	−0.424	0.025	−9.923	47.00	0.90
^{137}Ba	$1h_{11/2}$	−0.495	−0.502	0.043	−9.157	47.00	1.10
^{209}Pb	$2g_{9/2}$	−0.693	−0.639	0.014	−4.158	45.50	1.15
^{209}Pb	$1i_{11/2}$	−0.585	−0.629	0.039	−2.946	45.50	1.20
^{209}Pb	$3d_{5/2}$	−0.486	−0.480	0.001	−2.215	45.50	0.90
^{207}Pb	$3p_{1/2}$	−0.494	−0.477	0.010	−7.379	45.38	1.00
^{207}Pb	$2f_{5/2}$	−0.497	−0.489	0.024	−8.023	45.38	1.20
^{207}Pb	$3p_{3/2}$	−0.654	−0.688	0.011	−8.696	45.38	0.95
^{211}Po	$2g_{9/2}$	−0.663	−0.871	0.013	−4.538	45.50	1.15
^{211}Po	$3p_{1/2}$	−0.555	−0.587	0.012	−7.654	45.20	0.93
^{2}H	$1s_{1/2}$		−0.213	0.140	−2.218	26.60	0.34

TABLE II

The protons levels and relativistic corrections for the considered potentials.

Nucleus	nLj	E_V MeV	$(E_m)^D$ MeV	$(E_V)^D$ MeV	$(E_{nLj})^D$ MeV	V^p MeV	d
^{17}F	$1d_{5/2}$	−0.664	−0.751	0.053	−0.606	50.65	0.80
^{17}O	$2s_{1/2}$	−0.227	−0.216	0.007	−0.122	50.65	0.551
^{15}N	$1p_{1/2}$	−0.341	−0.361	0.105	−12.00	57.60	0.75
^{41}Sc	$1f_{7/2}$	−0.480	−0.482	0.056	−1.096	52.44	1.00
^{39}K	$1d_{3/2}$	−0.366	−0.350	0.070	−8.301	54.24	1.05
^{49}Sc	$1f_{7/2}$	−0.473	−0.474	0.068	−9.643	58.10	1.05
^{49}Sc	$2p_{3/2}$	−0.737	−0.705	0.022	−5.886	58.10	0.77
^{55}Co	$1f_{7/2}$	−0.413	−0.419	0.062	−7.207	55.40	1.03
^{59}Cu	$2p_{3/2}$	−1.118	−1.094	0.031	−3.387	53.70	0.77
^{57}Co	$1f_{7/2}$	−0.441	−0.441	0.061	−8.105	55.37	1.03
^{121}Sb	$2d_{5/2}$	−0.516	−0.499	0.026	−5.753	59.05	0.80
^{121}Sb	$1g_{7/2}$	−0.566	−0.470	0.052	−5.109	59.05	1.20
^{119}In	$1g_{9/2}$	−0.373	−0.420	0.053	−10.86	58.40	1.05
^{209}Bi	$1h_{9/2}$	−0.417	−0.418	0.043	−3.792	60.23	1.15
^{209}Bi	$2f_{7/2}$	−0.474	−0.474	0.026	−3.601	60.23	1.15
^{207}Tl	$3s_{1/2}$	−0.384	−0.370	0.023	−8.054	60.00	0.90

The relativistic corrections for the potential are positive and vary for neutrons from −0.0105 MeV in the state $3p_{1/2}$ of ^{207}Pb to 0.1400 MeV in the state $1s_{1/2}$ of ^{2}H with parameters $\alpha = 3.75$ fm^{-1}, $R = 2.4$ fm, $V^n = 26.6$ MeV. $(E_{nLj})^D$, $(E_m)^D$, $(E_V)^D$ were obtained solving the integral-differential equation (3.14) for the operator potential (3.2). The depths of Saxon–Woods potentials V^n and V^p were obtained from requiring the coincidences with experimental one-nucleons levels [15]. The differences between E_V and $(E_V)^D$ are insignificant. That is why E_V is not presented in the Tables.

5. Conclusions

The relativistic corrections to potential are small and they are about 0.05 MeV. We can argue on the possibility of calculating the energies of the single-particle states without taking into account the relativistic corrections for mass. In order to get the exact values of energy levels we must solve the integral-differential equation (3.14) with the operator potential (3.2). For the nuclei where two or three energy levels are presented we have obtained good coincidence of the calculated levels with experiments [15]. We

can confirm that taking into account the relativistic corrections for mass improves the results. These corrections must be taken into consideration in the calculations of the elastic scattering phase shifts, stripping pick-up, knockout reactions.

They are important for solving the problems of the dependence of the potentials on energy in the shell model [17] and the range of the nuclear force [18]. The relativistic corrections for mass significantly increase the binding energies of the nucleons in the external shells of the heavy nuclei like ^{209}Pb, ^{209}Bi, ^{209}Po and increase the probability of existence of superheavy unstable nuclei [15]. The obtained exact $(E_m)^D$, $(E_V)^D$ values in many cases coincide with E_m, E_V obtained in the first approximation of the perturbation theory. It is well known [19] that bound states and elastic scattering phase shifts can be described by the same real part of the optical potential. Consequently we can draw a conclusion that the relativistic corrections to mass must be included into the optical model calculations.

A new procedure for the solution of semi-relativistic equation can significantly improve the shell model calculations and can be useful for the explanation of the properties of the recently established neutron-rich nuclei with very large r.m.s. radii. The relativistic corrections to mass for the neutrons hole and particle states $1f_{7/2}$, $2d_{5/2}$, $2f_{7/2}$, $2g_{9/2}$, $1i_{11/2}$ significantly increase the separation energies of the neutrons in the external shells and the probability existence of the nuclei with abnormally high number neutrons in these shells.

In the Nillson Hamiltonian applicable for consideration of the single-particle energy levels of the nucleons moving in an axially deformed harmonic oscillator potential we have angular momentum dependent term $-D \cdot L^2$ which diminishes the energy for higher L. A similar dependence of the relativistic corrections for the mass was obtained [3] and [4]. We can confirm that relativistic corrections for the mass are important in the case of Nilsson's model.

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