ENERGY DEPENDENCE OF DCX CROSS SECTION IN GROUND STATE TRANSITION ON ^{56}Fe*

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Differential cross section of the double charge exchange (DCX) reaction ^{56}Fe$(\pi^+,\pi^-)^{56}$Ni at the pion energies 10 and 60 MeV is calculated in the proton-neutron quasiparticle random phase approximation using a realistic nucleon-nucleon interaction. A detailed structure of the transition amplitude through intermediate states is discussed in some extent. It is shown that the observed resonance-like behaviour can be explained at least semi-quantitatively in terms of an ordinary NN process due to all over increase of the transition amplitudes with pion energy for each J^π-multipolarity. The pn-QRPA seems to be a good framework for a description of structure of nuclei involved in double charge exchange processes.

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1. Introduction

The double charge exchange (DCX) with low energy pions has attracted constant interest since mid-eighties. Past reviews on this reaction include one by Becker and Batusov [1] and one by Dzhibuti and Kezerashvili [2]. Most recent activity in the field is summarized in reviews by Clement [3] and Johnson and Morris [4].

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In spite of such a long interest the data for DCX transitions are still very limited. For example in the case of the DCX reaction on ^{12}C the energy dependence of the cross section has been measured at one value of the scattering angle ($\theta = 35^o$) and at the pion energy $T_\pi = 49, 59, 79, 89$ MeV [5]. More complete data are collected for the ground state transitions on calcium isotopes $^{44,48}Ca$ [6]. Recently three-point angular distribution at $T_\pi = 50$ MeV has been obtained at the Paul Scherrer Institute for the ground state transition on ^{56}Fe [7]. In this case also data from Los Alamos National Laboratory are available [8]. All these experimental observations show the strong energy dependence of the DCX forward-angle cross section, which rises by some factors of 3 or more in the energy domain 20–50 MeV as well as between 100–70 MeV [3, 9–13].

In contrast to such an observed behaviour almost all of the theoretical models with a plane wave as well as with distorted wave approximation are not able to predict even roughly this strong energy dependence of the DCX cross section. The microscopic calculations give a rather smooth energy behaviour around $T_\pi = 50$ MeV except the predictions by Martemyanow and Schepkin [14] who have introduced the dibaryon resonance "by hand" to explain this dependence. This explanation is still not widely accepted and arguments against the approach are still discussed [15–17]. In this paper we are going to argue that also in the conventional two-nucleon mechanism one is able to describe gross features of the energy dependence of the DCX cross section within the proton–neutron Quasiparticle Random Phase Approximation (QRPA).

2. The model

In a series of papers [18–21] a new approach to the DCX process was formulated in terms of the QRPA framework. It was shown that the approach was successful in quantitative explanation of the DCX angular distribution and cross section for the defined pion energy. Here we shall discuss the dependence of the DCX cross section on incoming pion energy in the same approximation.

A detailed description of our formalism can be found elsewhere [20, 21]. In this paper we recapitulate main features of the theory to make the text readable.

The differential cross section for the DCX reaction is defined by the total amplitude such that

$$\frac{d\sigma}{d\Omega}(\theta, q) = \left| \frac{1}{4\pi} \sum_{J^\pi} \left[F_J^{(s)}(\boldsymbol{k}, \boldsymbol{k}') + F_J^{(p)}(\boldsymbol{k}, \boldsymbol{k}') \right] \right|^2 , \tag{1}$$

where $\boldsymbol{k}$, $\boldsymbol{k}'$, $\boldsymbol{q}$ are the momenta of incoming and outgoing pions and the momentum transfer during the DCX process, respectively. The partial amplitudes $F_J^{(s)}(\boldsymbol{k},\boldsymbol{k})$ and $F_J^{(p)}(\boldsymbol{k},\boldsymbol{k}')$ connected with s-wave and p-wave pion–nucleon effective interactions are given in the form [20, 21]:

$$h_p(\boldsymbol{q}) = -\sqrt{2}\, i\, \frac{f}{m_\pi} \sum_{pn,JM} \mathcal{F}_{pn}^{JM}(\boldsymbol{q}) \mathcal{R}_{pn}^{JM}\,; \tag{2}$$

$$h_s(\boldsymbol{q}) = -4\pi\, \frac{\lambda_1}{m_\pi} \sqrt{2}\, \omega_q \sum_{pn} (2j_p+1)^{\frac{1}{2}}\, \delta_{pn}\, \mathcal{R}_{pn}^{00}\,. \tag{3}$$

In both Eqs. (2) and (3) the charge transition density $\mathcal{R}_{pn}^{JM}$ is given by a formula

$$\begin{aligned}\mathcal{R}_{pn}^{JM} =& u_p v_n C^\dagger(pnJM) + v_p u_n \tilde{C}(pnJM) \\ &+ u_p u_n D(pnJM) - v_p v_n \tilde{D}^\dagger(pnJM)\,.\end{aligned} \tag{4}$$

The proton–neutron pair creation operator $C^\dagger(pnJM)$, the pair annihilation operator $\tilde{C}(pnJM)$ and additional one-body operators $\tilde{D}^\dagger(pnJM)$, $D(pnJM)$ appearing in eq. (4) are defined in the way keeping an usual phase convention under time-reversal. Definitions (2) and (3) also contain the function $\mathcal{F}_{pn}^{JM}$ which can be expressed in terms of the nuclear form-factor G_{pn}^J. Its explicit form can be found in Ref. [20]. The occupation amplitudes u's and v's are obtained by solving the BCS equation in some model space.

Using the above charge exchange operators (3) and (4) one can write the transition amplitudes $F_J^{(s)}$ and $F_J^{(p)}$ in second order perturbation theory as [21]:

$$F_J^{(r)}(\boldsymbol{k},\boldsymbol{k}') = \sum_m \frac{\langle f,0^+;\pi^-(\boldsymbol{k}')|\hat{O}^{(r)}|mJM\rangle\langle mJM|\hat{O}^{(r)}|i,0^+;\pi^+(\boldsymbol{k})\rangle}{D^{(s)}(E_i,\epsilon_k,E_m^J)}\,, \tag{5}$$

where $r = s$ or p for the s- and p-wave interactions (2) and (3), respectively. In Eq. (5) $|i,0^+;\ \pi^+(\boldsymbol{k})\rangle$ denotes the ground state of the initial nucleus (A,Z) and an incoming positive pion with the initial energy $\epsilon_k = (k^2+m_\pi^2)^{\frac{1}{2}}$. In analogy, $|f,0^+;\ \pi^-(\boldsymbol{k}')\rangle$ stands for a ground state of the final nucleus (A, Z+2) and an outgoing negative pion. In the case of p-wave interaction (2) the denominator $D^{(s)}(E_i,\epsilon_k,E_m^J)$ has a simple form $E_i+\epsilon_k-E_m^J$. A more complicated case of the double scattering of pion by two nucleons within sequential mechanism needs to involve a pion propagation between both acts of neutral pion absorption. In this picture there is impossible to separate nuclear and pion momenta and energies and one has to calculate the matrix elements in sum (5) using a pion propagator in the static approximation [22]

$$\frac{|\Phi_{\pi^0}(\boldsymbol{q})\rangle\langle\Phi_{\pi^0}(\boldsymbol{q})|}{|\boldsymbol{q}|^2+m_\pi^2}, \tag{6}$$

where $\Phi_{\pi^0}(\boldsymbol{q})$ is the neutral pion wave function and $\boldsymbol{q}=\boldsymbol{k}-\boldsymbol{k}'$ means the momentum transfer. Integration over intermediate pion momentum is understood.

Results discussed below were obtained with the plain wave approximation for the propagated neutral pion. The approximation is known as rough one [17, 23, 24], but to keep nuclear structure effects in pion–nucleus interaction as simple as possible we will use it through this paper.

The DCX amplitude (5) contains a sum over all excited states $|mJM\rangle$ of the intermediate nucleus and thus there is no need in the theory for the closure approximation. These intermediate states are populated in the particle-hole excitations and determined by solving the QRAP equation of motion. The detailed structure of this equation can be found in Refs. [20] and [21].

Using transition operators (2)–(3) and explicit form of the intermediate states one can find final expressions for the partial amplitudes in the ground state to ground state transition:

$$\begin{aligned} F_{\rm GS}^{(s)}(\boldsymbol{k},\boldsymbol{k}') &= \left(4\pi\frac{\lambda_s}{m_\pi}\right)^2 \sum_m \int \frac{\epsilon_k\epsilon_{k'}}{(2\pi)^3}d\boldsymbol{q}\frac{\Phi^\star_{\pi^0}(\boldsymbol{q})\Phi_{\pi^0}(\boldsymbol{q})}{D^{(s)}(E_i,\epsilon_q,E_m^0,\boldsymbol{q})} \\ &\times \left\{\left[2\sqrt{2}\sum_{pn}\delta(p,n)\hat{j}_p\left(\bar{X}^m_{(pn)0}\bar{v}_p\bar{u}_n+\bar{Y}^m_{(pn)0}\bar{u}_p\bar{v}_n\right)\right]\right. \\ &\times \left.\left[2\sqrt{2}\sum_{pn}\left(X^m_{(pn)0}u_pv_n+Y^m_{(pn)0}v_pu_n\right)\right]\right\}; \end{aligned} \tag{7}$$

$$\begin{aligned} F_{\rm GS}^{(p)}(\boldsymbol{k},\boldsymbol{k}') &= -\left(\frac{f}{m_\pi}\right)^2 \sum_{m,J}\frac{1}{E_i+\omega_k-E_m^J}P_J(cos\theta_{kk'}) \\ &\times \left\{\left[\sqrt{12}\sum_{pn}(-1)^{j_p+j_n+J}G^J_{pn}(k')\left(\bar{X}^m_{(pn)J}\bar{v}_p\bar{u}_n+\bar{Y}^m_{(pn)J}\bar{u}_p\bar{v}_n\right)\right]\right. \\ &\times \left.\left[\sqrt{12}\sum_{pn}G^J_{(pn)J}(k)\left(X^m_{(pn)J}u_pv_n+Y^m_{(pn)J}v_pu_n\right)\right]\right\}. \end{aligned} \tag{8}$$

In Eqs. (7)–(8) we introduced the abbreviation $\delta(p,n)=\delta_{n_pn_n}\delta_{j_pj_n}\delta_{l_pl_n}$. The quantity $P_J(\cos\theta_{kk'})$ is the Legendre polynomial depending on the scattering angle $\theta_{kk'}$. X's and Y's are forward- and backward-going amplitudes of the QRPA solution for the excited states in the intermediate nucleus.

3. Result and discussion

In this paper we look in some detail for a dependence of the differential cross section on structure of the intermediate states for the ground state transition on $^{56}Fe \to {}^{56}Ni$. This reaction has already been experimentally investigated [6, 11, 12, 25, 26]. Although the data for this transition are very limited, especially for non-analogue transitions one can conclude readily that the transition exhibits a well pronounced resonance-like behaviour if one is looking for energy dependence of the forward cross section [3, 26].

In the present calculations an inactive core is not taken into account. So, we assumed a model space consisting of all bound or quasi-bound states calculated with a Coulomb-corrected Woods–Saxon potential and with single particle energy lower than 5.0 MeV. We also allowed different shells to be occupied by protons and neutrons. Two-body matrix elements needed for construction of the QRPA equations for intermediate states are obtained from *the realistic* nuclear matter G-matrix by solving the Bethe–Goldstone equation with a one-boson-exchange potential of the Bonn group [27]. The two-body matrix elements calculated for nuclear matter are not specialized for a given nucleus. Thus and due to the finite Hilbert space used one has to renormalize them by multiplying with factors $g^{\rm n}_{\rm pair}$, $g^{\rm p}_{\rm pair}$, $g^{\rm pn}_{\rm pp}$ and $g^{\rm pn}_{\rm ph}$ slightly different from 1.0.

For the ground states of the parent and daughter nuclei one obtains uncorrelated vacuum states by solving the standard BCS equation in the above-mentioned model space. Two renormalization factors $g^{\rm n}_{\rm pair}$ and $g^{\rm p}_{\rm pair}$ multiplying the proton and neutron pairing matrix elements are fixed by adjusting the empirical pairing gaps [28] to the lowest quasiparticle energy obtained from the gap equation. Because the pairing gaps cannot be extracted for nuclei with a magic number of protons or neutrons we obtained the corresponding pairing strengths from the adjacent even-even nuclei. Their values are: ^{56}Fe — 0.938, 0.993; ^{56}Ni — 1.030, 0.908. As a result of such a procedure we evaluate the occupation amplitudes u's and v's needed not only for calculations of the ground state of the initial and final nuclei, but also used in the QRPA equation of motion of intermediate states. To determine the QRPA matrices fully one must also fix two additional renormalization factors: the strength of the particle–particle $g^{\rm pn}_{\rm pp}$ and the strength of the particle-hole $g^{\rm pn}_{\rm ph}$ interaction. For this purpose we used the isobaric analogue state (IAS) and the Gamow-Teller state in cobalt ^{56}Co which are known to be 3.65 MeV and 10.60 MeV, respectively [29].

Adjustment of these states to their experimental energies makes it possible to fix the strength $g_{\rm ph}^{\rm pn}$ approximately 0.9. Details of such a procedure can be found elsewhere [19]. The second factor $g_{\rm pp}^{\rm pn}$ can be treated as a free parameter of the theory and we decided to fix it at the bare nuclear matter value 1.0[1].

The calculated dependence of the DCX forward cross section on incident pion energy is shown in Fig. 1.

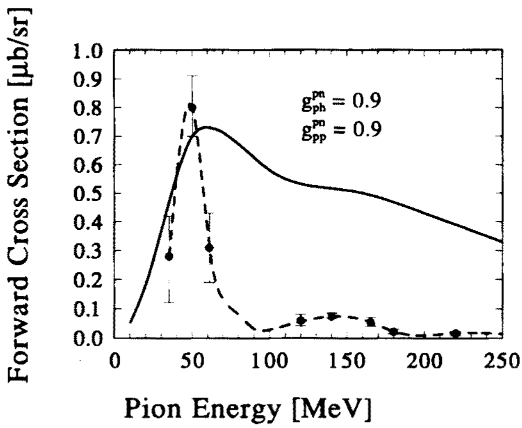

Fig. 1. Energy dependence of the forward-angle (5°) cross section for the ground to ground state transition on ^{56}Fe (solid line). Data indicated by error bars are taken from refs. 11, 12 and 25. (The drawn short-dashed curve is only to guide the eye.)

Although only semi-quantitative explanâtion of the resonance behaviour is obvious, our calculations point out a possibility of steep rise of the DCX cross section in the range energy of 10–60 MeV. The curve is not steep enough on the high energy side above 60 MeV, but a peak around this energy is clearly seen. So, even if dependence for higher energies is not so pronounced, one could argue that also the 2N-mechanism used in this paper gives an important contribution to the rise of the DCX cross section around pion energy 50–60 MeV. It is obvious, that if such a mechanism is responsible for the observed behaviour we must find some additional effects or/and reexamine approximations made in the approach to reduce magnitude of the DCX cross section in the higher than 60 MeV energy of the incident

[1] One can adjust this parameter using additional spectroscopic information, *e.g.* reduced transition probabilities between the excited states of the intermediate nucleus and its ground state.

pion. First of all what we have in mind is a replacement of the static pion–nucleus interactions (2) and (3) by their relativistic generalizations. An influence of such a change is expected to be not important for the p-wave amplitude but it could make more considerable alteration in the s-wave part of the DCX cross section. In the paper we have also assumed a zero-range character of the s-wave Hamiltonian (3). For more accurate approach one should consider the finite range of this interaction as well as finite size of the nucleon in a nucleus.

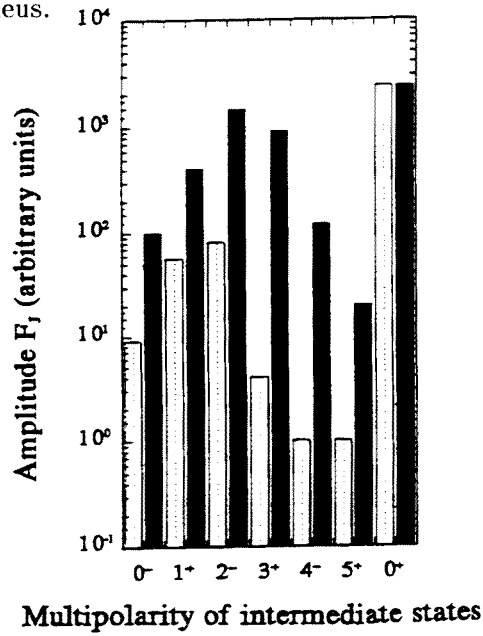

Fig. 2. The most important contributions to the DCX transition amplitude come from the intermediate 0^-, 1^+, 2^-, 3^+, 4^-, 5^+ and 0^+ states. The results for two pion energies: T_π = 10 MeV (empty bars) and T_π = 60 MeV (black bars) are shown. The particle–particle strength $g_{\rm pp}^{\rm pn}$ and the particle–hole strength $g_{\rm ph}^{\rm pn}$ are fixed to be 1.0 and 0.8, respectively. (For details see text.)

A careful analysis of the partial amplitudes (7)–(8) at pion energies, for which quite different behaviour of the forward cross section is observed may also shed some light on possible explanation of the behaviour of the DCX cross section. A magnitude of the DCX amplitudes for the intermediate states with the largest contribution is shown in Fig. 2 for the two energies of the incident pions: 10 and 60 MeV. All over increase of the amplitudes for each J^π-multipolarity comparing both energies can be seen. It is interesting to note that the "analogue" path through 0^+ intermediate states shows in both energy cases similar impact to the transition amplitude, whereas for the non-0^+ states the amplitudes display rise of 1–2 orders of the magnitude for 60 MeV. This increase is especially noticeable for higher angular momenta

$(3^+, 4^-, 5^+)$. As the non-analogue routs are important through short-range nucleon correlations [30, 31], present calculations may suggest also their role in a mechanism of the cross section rise around the incident pion energy 60 MeV. Investigations clearing this point are in progress.

5. Conclusion

In conclusion, we calculated the DCX cross section in the quasiparticle pn–QRPA using the realistic two-nucleon interaction and found that this model reproduced the gross features of the observed resonance-like shape of the cross section as a function of the pion energy. An increase of the transition rate magnitude for pion energy $T_\pi = 60$ MeV is mainly caused by strong increase of amplitudes for the non-0^+ intermediate state transitions. Due to approximations made it is impossible at this stage of the work to state definitely if the conventional $2N$ mechanism can exclusively provide an explanation of the phenomenon. Such a statement can not be drawn until calculations of the conventional sequential scattering at low energies are performed and confirmed by the results of corresponding single charge exchange measurements which at the moment are not available. Also particular attention has to be regarded to the role of nonanalogue intermediate states as we stressed above. We shall study these problems including transitions on ^{56}Fe to several individual excited states of nickel, predominantly to 0^+ and 2^+ states [3, 6, 32]. The latter makes it possible to settle the questions addressed in this paper more carefully.

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