# MAGNETIZATION OF THE PROTON CRYSTAL IN THE NEUTRON STAR MATTER*

M. KUTSCHERA

H. Niewodniczański Institute of Nuclear Physics
Radzikowskiego 152, 31-342 Kraków, Poland

AND

W. WÓJCIK

Institute of Physics, Cracow University of Technology
Podchorążych 1, 30-084 Kraków, Poland

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The permanent magnetization of the proton crystal immersed in the neutron background inside the neutron star core is studied. The magnetization is produced by the ferromagnetic ordering of spins of protons localized at the lattice sites. We calculate the magnetization of the crystal in a simple model based on the Skyrme forces in which we use variational wave functions for localized protons and Bloch neutrons in the Hartree-Fock approximation. The induced spin excess of the neutron Fermi sea is found and its contribution to the magnetization is included.

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## 1. Introduction

Magnetic properties of the dense matter, which play an important role in explaining the origin and/or evolution of the magnetic field of neutron stars, are determined by the behaviour of protons which form nuclear polarons [1]. At high densities the polarons become localized [2]. This triggers the spontaneous ferromagnetic spin ordering of protons which produces a permanent magnetization of the neutron star core [3].

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Protons, which are a dilute component of the neutron star core matter are likely to form a crystal structure at high densities [4]. This proton crystal has a single proton localized inside every elementary cell. The periodic potential, which localizes protons is due to a periodic density distribution of background neutrons [4]. The localization of protons in the neutron star matter is a result of the behaviour of the nuclear symmetry energy at high densities [5, 6].

In this paper magnetic properties of this proton crystal are studied. It is quite obvious that a single proton, which has a well-defined spin, will polarize the surrounding neutron matter, provided there exist any proton-neutron spin interactions. Propagation of polarized neutrons can in turn induce the polarization of neighbouring protons by the RKKY mechanism [7]. In this way, a long-range magnetic order can propagate in the proton crystal leading to the alignment of proton spins.

Here we calculate the magnetization of the crystal with all the proton spins ferromagnetically polarized. The magnetization consists of contributions due to localized protons and the induced polarization of the neutron Fermi sea [3]. To calculate the latter one we have chosen to work with the Skyrme forces which provide the spin interaction. The induced spin excess of the neutron Fermi sea is calculated variationally.

The Skyrme forces were used in an earlier calculation [4] of the proton crystal in the neutron star matter. However, the parametrization used in Ref.[4] is not well suited for the calculation of the neutron spin excess. The unwelcome feature of some earlier parametrizations is that pure neutron matter becomes unstable at higher densities [8]. The origin of this unphysical instability was discussed in Ref. [8]. Also, an improved parametrization applicable to spin excess calculations was proposed in Ref. [8]. The Skyrme force parameters from Ref. [8] are given in the Appendix.

The paper is organized as follows: In Section 2 we briefly discuss the proton and neutron trial wave functions which describe the proton crystal in Ref. [4]. In Section 3 the energy of the polarized proton crystal is calculated variationally and the net magnetization including the induced neutron spin excess is obtained. Results of our calculations are collected in Section 4. In the Appendix analytic formulae for the energy of polarized crystal are given together with parameters of Skyrme forces. For completeness, also energy of normal crystal [4] is given there.

## 2. Proton crystal: proton and neutron wave functions

In Ref. [4] a simple model of the proton crystal is constructed. We assume protons to form a simple cubic lattice. Single protons are localized

on lattice sites. The lattice constant $a$ is determined by the proton density,

$$a = n_{\mathrm{P}}^{-1/3} . \tag{1}$$

Here $n_{\mathrm{P}} = x n_B$, where $n_B$ is the baryon density and $x$ is the proton fraction.

The localized protons are described by a variational wave function:

$$\phi_{\mathrm{P}}(\boldsymbol{r}, \sigma) = C_{\mathrm{P}}[1 + \cos(q_{\mathrm{P}} x)][1 + \cos(q_{\mathrm{P}} y)][1 + \cos(q_{\mathrm{P}} z)]\chi_{\mathrm{P}}(\sigma) , \tag{2}$$

for

$$-\frac{\pi}{q_{\mathrm{P}}} \leq x \leq \frac{\pi}{q_{\mathrm{P}}}, \quad -\frac{\pi}{q_{\mathrm{P}}} \leq y \leq \frac{\pi}{q_{\mathrm{P}}}, \quad -\frac{\pi}{q_{\mathrm{P}}} \leq z \leq \frac{\pi}{q_{\mathrm{P}}} ,$$

and

$$\phi_{\mathrm{P}}(\boldsymbol{r}) = 0 \tag{3}$$

for

$$| x | > \frac{\pi}{q_{\mathrm{P}}}, \quad | y | > \frac{\pi}{q_{\mathrm{P}}}, \quad | z | > \frac{\pi}{q_{\mathrm{P}}} .$$

The normalization is

$$C_{\mathrm{P}} = \left(\frac{q_{\mathrm{P}}}{3\pi}\right)^{3/2} . \tag{4}$$

The wave function, Eq. (2), vanishes at the surface of the cube of the volume $(2\pi/q_{\mathrm{P}})^3$. For the proton to be localized inside the cubic cell of size $a$, the variational parameter $q_{\mathrm{P}}$ should satisfy the inequality

$$q_{\mathrm{P}} > 2\pi/a \equiv q . \tag{5}$$

One can generally write the wave function of the $j$-th proton localized on the site $\boldsymbol{R}_i$ as

$$\phi_i(\boldsymbol{r}_j, \sigma) = \delta_{ij}\phi_{\mathrm{P}}(| \boldsymbol{r}_i - \boldsymbol{R}_i |, \sigma) . \tag{6}$$

The proton spin is polarized in the z-direction.

The neutron wave functions are assumed to be the Bloch wave functions,

$$\psi_{\boldsymbol{k}_i}(\boldsymbol{r}_j, \sigma) = e^{i\boldsymbol{k}_i \boldsymbol{r}_j} u_{\boldsymbol{k}_i}(\boldsymbol{r}_j)\chi_{\mathrm{N}}(\sigma) . \tag{7}$$

The wave vector $\boldsymbol{k}_i$ is limited to the first Brillouin zone and the subscript $i$ denotes the i-th band. The functions $u_{\boldsymbol{k}_i}(\boldsymbol{r})$ have the symmetry of a simple cubic lattice. We choose them in the form

$$u_{\boldsymbol{k}_i}(\boldsymbol{r}) = C[1 + \alpha\cos(qx)][1 + \alpha\cos(qy)][1 + \alpha\cos(qz)] , \tag{8}$$

where $\alpha$ is a variational parameter. The wave vector $q$ for a crystal with a single proton per site is defined in Eq. (5).

The normalization of the wave function, Eq. (8), is

$$C = \frac{1}{\sqrt{V(1+0.5\alpha^2)^3}}. \tag{9}$$

The single-particle orbitals (7) are filled up to the Fermi level.

## 3. Spin excess calculations with skyrme forces

The Skyrme potential we use reads

$$\begin{aligned} v(\boldsymbol{r},\sigma) =& t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}) + \tfrac{1}{2}t_1[\boldsymbol{k}'^2\delta(\boldsymbol{r}) + \delta(\boldsymbol{r})\boldsymbol{k}^2] \\ &+ t_2(1+x_2P_\sigma)\boldsymbol{k}'\cdot\delta(\boldsymbol{r})\boldsymbol{k} + \tfrac{1}{6}t_3(1+x_3P_\sigma)n(\boldsymbol{R})\delta(\boldsymbol{r}). \end{aligned} \tag{10}$$

Here $\boldsymbol{r} = \boldsymbol{r}_i - \boldsymbol{r}_j$ and $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_i + \boldsymbol{r}_j)$. The operator $\boldsymbol{k} = \frac{i}{2}(\nabla_i - \nabla_j)$ acts on the right of the delta-function and $\boldsymbol{k}'$ is the same operator acting on the left of $\delta(\boldsymbol{r})$. $P_\sigma$ is the spin exchange operator. This parametrization employs the exchange parameter $x_2$ which is commonly neglected in earlier parametrizations [8]. A suitably chosen value of $x_2$ removes the unphysical instability of the neutron matter at higher densities [8]. Parameters of this potential from Ref. [8] which we use here are given in the Appendix.

The Skyrme potential depends on the spin variables of the particles through the spin exchange operator $P_\sigma = (1 + \vec{\sigma}_1\vec{\sigma}_2)/2$. The spin interactions of polarized protons with the neutrons will induce the neutron spin excess

$$s = \tfrac{1}{2}(n_{\mathrm{N}}^u - n_{\mathrm{N}}^d), \tag{11}$$

where $n_{\mathrm{N}}^u$ and $n_{\mathrm{N}}^d$ is the density of neutrons with spin up and spin down, respectively.

### *3.1. The normal state*

The variational wave function of the whole system, $\Psi$, is a Slater determinant of proton and neutron single-particle orbitals (6) and (7). It contains two variational parameters $\alpha$ and $q_{\mathrm{P}}$ which will be chosen to minimize the energy of the crystal at a given baryon density $n_B$ and for the proton fraction $x$. There is no spin excess in the normal state, which is unpolarized.

In the case of contact interactions (10) the energy

$$E = \langle \Psi \mid H \mid \Psi \rangle \tag{12}$$

of the crystal with localized protons, whose wave functions do not overlap, is the sum of single cell energies $E_{cell}$,

$$E = N_c E_{\text{cell}} . \tag{13}$$

Here $N_c$ is the total number of cells in the crystal which for a single proton per site is equal to the total number of protons, $N_c = B_{\text{P}} = V n_{\text{P}}$.

The cell energy consists of the neutron background energy including neutron-neutron interactions, the proton kinetic energy and the proton-neutron interaction energy:

$$E_{\text{cell}} = E_{\text{neut}} + E_{\text{kin}} + E_{\text{PN}} . \tag{14}$$

The neutron energy is

$$E_{\text{neut}} = a^3 \epsilon_{\text{N}}(\alpha) , \tag{15}$$

where the mean neutron-background energy density is

$$\epsilon_{\text{N}}(\alpha) = \frac{1}{V}\Big[\sum_{\boldsymbol{k}}^{k_F} \langle \psi_{\boldsymbol{k}} \mid -\frac{1}{2m_{\text{N}}}\nabla^2 \mid \psi_{\boldsymbol{k}} \rangle + \frac{1}{2}\sum_{\boldsymbol{k}}^{k_F}\sum_{\boldsymbol{l}}^{k_F} \langle \psi_{\boldsymbol{k}}\psi_{\boldsymbol{l}} \mid v(\boldsymbol{x},\boldsymbol{y}) \mid \psi_{\boldsymbol{k}}\psi_{\boldsymbol{l}} - \psi_{\boldsymbol{l}}\psi_{\boldsymbol{k}} \rangle\Big] . \tag{16}$$

One can obtain an analytic expression for $\epsilon_{\text{N}}$ which is given in Ref. [4] for $x_2 = 0$. In the Appendix we give the modified formula valid for $x_2 \neq 0$.

The proton kinetic energy for the wave function, Eq. (2), is

$$E_{\text{kin}} \equiv E_{\text{kin}}(q_{\text{P}}) = \frac{q_{\text{P}}^2}{2m_{\text{P}}} . \tag{17}$$

The proton-neutron interaction energy is

$$E_{\text{PN}} \equiv E_{\text{PN}}(\alpha, q_{\text{P}}) = \sum_{\boldsymbol{k}}^{k_F} \langle \psi_{\boldsymbol{k}}\phi_{\text{P}} \mid v(\boldsymbol{x},\boldsymbol{y}) \mid \psi_{\boldsymbol{k}}\phi_{\text{P}} \rangle . \tag{18}$$

For our choice of trial wave functions also this expression can be calculated analytically and is given in the Appendix.

The results of minimization of the cell energy, Eq. (14), are discussed in detail in Ref. [4].

### 3.2. Ferromagnetic state

When the proton spins are ferromagnetically ordered the proton-neutron spin interactions induce the spin excess, $s \neq 0$, in the neutron Fermi sea. The cell energy of the ferromagnetic crystal is a sum of the polarized neutron Fermi sea energy, $E_{\mathrm{neut},m}$, the proton kinetic energy, $E_{\mathrm{kin}}$, and the interaction energy of the proton with the polarized neutron Fermi sea, $E_{\mathrm{PN}m}$,

$$E^m_{\mathrm{cell}} = E_{\mathrm{neut},m} + E_{\mathrm{kin}} + E_{\mathrm{PN}m}\,. \tag{19}$$

It depends on the neutron spin excess $s$ as now both the neutron background energy and the proton-neutron interaction energy depend on $s$.

The neutron energy is

$$E_{\mathrm{neut},m} = a^3 \epsilon_{Nm}(\alpha, s), \tag{20}$$

where the mean energy density of the polarized neutron Fermi sea is

$$\epsilon_{\mathrm{N}m}(\alpha, s) = \frac{1}{V}\left[\sum_{i=u,d}\sum_{k}^{k_i}\langle\psi_k \mid -\frac{1}{2m_{\mathrm{N}}}\nabla^2 \mid \psi_k\rangle\right.$$
$$\left. + \frac{1}{2}\sum_{i=u,d}\sum_{j=u,d}\sum_{k}^{k_i}\sum_{l}^{k_j}\langle\psi_k\psi_l \mid v(\boldsymbol{x},\boldsymbol{y}) \mid \psi_k\psi_l - \psi_l\psi_k\rangle\right]. \tag{21}$$

Here $k_u, k_d$ is the Fermi momentum of spin up and spin down neutron Fermi sea, respectively. We give the analytic expression for $\epsilon_{Nm}$ in the Appendix.

The proton-neutron interaction energy is

$$E_{\mathrm{PN}m} \equiv E_{\mathrm{PN}m}(\alpha, q_{\mathrm{P}}, s) = \sum_{i=u,d}\sum_{k}^{k_i}\langle\psi_k\phi_{\mathrm{P}} \mid v(\boldsymbol{x},\boldsymbol{y}) \mid \psi_k\phi_{\mathrm{P}}\rangle\,. \tag{22}$$

For our choice of trial wave functions also this expression can be calculated analytically, see the Appendix.

To determine the values of variational parameters $\alpha$ , $q_{\mathrm{P}}$ and $s$ we minimize the energy per particle in the cell

$$W = \frac{E^m_{\mathrm{cell}}}{B_{\mathrm{cell}}}\,, \tag{23}$$

where the total baryon number of the cell is

$$B_{\text{cell}} = a^3(1-x)n_B + 1\,, \tag{24}$$

for a given total baryon density $n_B$ and the proton fraction $x$.

Having the neutron spin excess $s$, which is a function of the baryon density $n_B$, we can calculate the magnetic moment of the cell

$$\mu_{\text{cell}} = a^3 \mu_{\text{N}} g s + \mu_{\text{P}}\,, \tag{25}$$

where $\mu_{\text{N}}$ and $\mu_{\text{P}}$ is the neutron and proton magnetic moment, respectively, and $g = 2$. The magnetization is

$$M = \mu_{\text{cell}}/a^3\,. \tag{26}$$

## 4. Results and discussion

The results of our variational calculations are presented in Figs. 1–4. In Fig. 1 we show, as an example, the energy per particle of the ferromagnetically polarized crystal, Eq. (23), as a function of the neutron spin excess density $gs$, for the baryon density $n_B = 0.5fm^{-3}$ and the proton fraction $x = 0.01$, and for appropriate parameters $\alpha$ and $q_{\text{P}}$. The energy displays a well defined minimum determining the induced spin excess $s$.

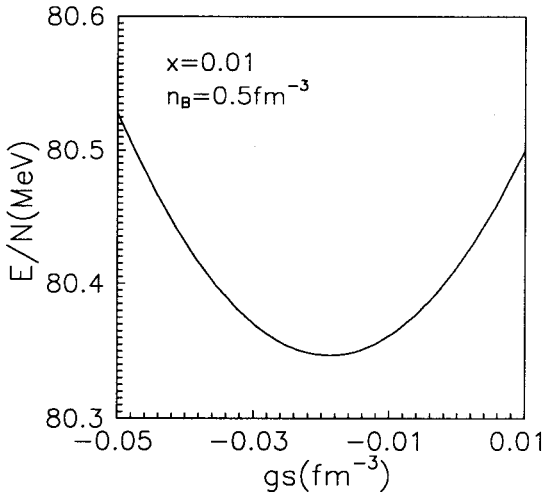


Fig. 1. The energy per particle of the polarized crystal as a function of the spin excess of the neutron Fermi sea. The minimum of the curve determines the induced neutron spin excess.

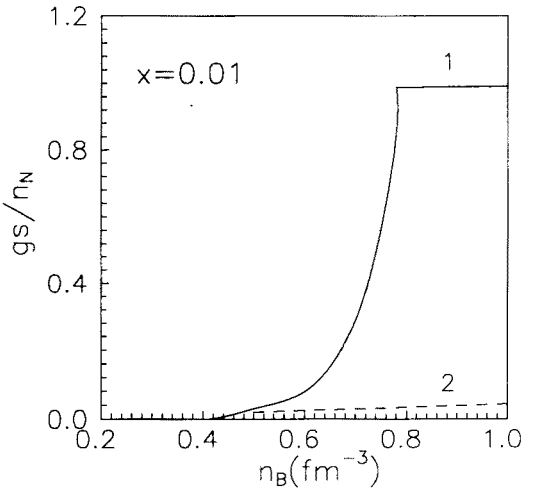


Fig. 2. The induced spin asymmetry of the neutron Fermi sea as a function of baryon density. The solid curve corresponds to the first set of the Skyrme force parameters. The curve displays the ferromagnetic ordering of the neutron spins above the density $n_f = 0.78$ fm$^{-3}$. The dashed curve corresponds to the second set of the Skyrme force parameters.

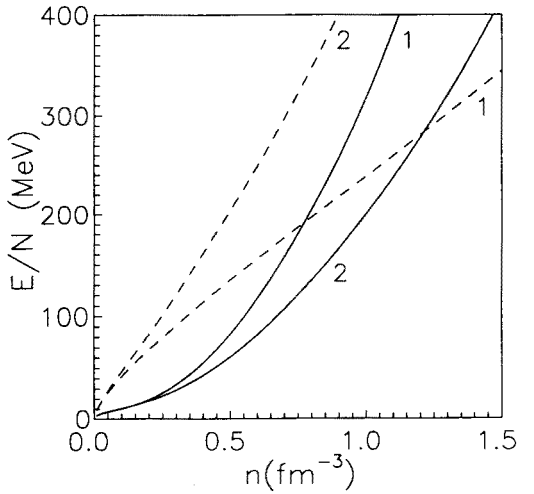


Fig. 3. The energy per particle of normal neutron matter (solid curves) and ferromagnetic neutron matter (dashed curves) as functions of the neutron density. The curves 1 and 2 correspond, respectively, to the first and the second set of the Skyrme force parameters. The curves 1 cross at $n_f = 0.78$ fm$^{-3}$.

In Fig. 2 we show the degree of polarization of the neutron Fermi sea corresponding to the neutron spin excess $s$,

$$\alpha_{\rm spin} = \frac{gs}{n_{\rm N}} = \frac{n_{\rm N}^u - n_{\rm N}^d}{n_{\rm N}^u + n_{\rm N}^d}\,. \tag{27}$$

as a function of the baryon density for the proton fraction $x = 0.01$. The spontaneous polarization occurs above the critical density which is the same as the critical density for the proton localization. The spin asymmetry, Eq. (27), generally increases with the baryon density. In Fig. 2 we show curves corresponding to two sets of Skyrme force parameters from the Appendix. The induced spin asymmetry $\alpha_{\rm spin}$ depends on the parametrization of Skyrme forces. For the parameters, fitting the $UV14 + TNI$ equation of state for the neutron matter, we find that $\alpha_{\rm spin} = 1$ for densities $n_B > 0.78{\rm fm}^{-3}$. This means that not only protons are fully polarized but also the neutron matter is ferromagnetically polarized. To explain this behaviour we plot in Fig. 3 the energy per particle of normal and ferromagnetic neutron matter, for two parametrizations from the Appendix. For $x_2 = -1.02$ the ferromagnetic state becomes the ground state above $n_f = 0.78{\rm fm}^{-3}$. This is why the spin asymmetry $\alpha_{\rm spin} = 1$ for $n_B \geq n_f$. For $x_2 = -1.08$, from the second set of parameters, the ground state is unpolarized at higher densities. As a result, the spin asymmetry $\alpha_{\rm spin} \ll 1$. We regard the small values of $\alpha_{\rm spin}$ as more realistic.

The net magnetization of the proton crystal, Eq. (26), is shown in Fig. 4. A magnetized neutron star core contributes to the magnetic moment of the neutron star [9, 10]. The magnetization required to explain the observed value of the magnetic field of neutron stars is of order $10^{13}Gs$. Here we obtain such values for the second set of Skyrme force parameters, which correspond to the neutron matter which is far from instability with respect to spin fluctuations. In case of ferromagnetically unstable neutron matter, the curve labelled 1 in Fig. 4, we obtain too large values of the magnetization.

## APPENDIX

In the calculations reported here we use two sets of the Skyrme force parameters. For both sets we use the same parameters $t_0, t_1, t_2$ and $t_3$: $t_0 = -1057.3$ MeV fm$^3$, $t_1 = 235.9$ MeV fm$^5$, $t_2 = -100.0$ MeV fm$^5$, $t_3 = 14463.5$ MeV fm$^6$. The exchange parameters for the first set are adjusted to fit the neutron matter equation of state with $UV14 + TNI$ interaction from Ref. [11]: $x_0 = 0.1865$, $x_2 = -1.08$ and $x_3 = 1.0$. The second set consists of exchange parameters fitting the $UV14 + UVII$ equation of state for neutron matter from Ref. [11]: $x_0 = 0.17$, $x_2 = -1.02$ and $x_3 = 0.64$.

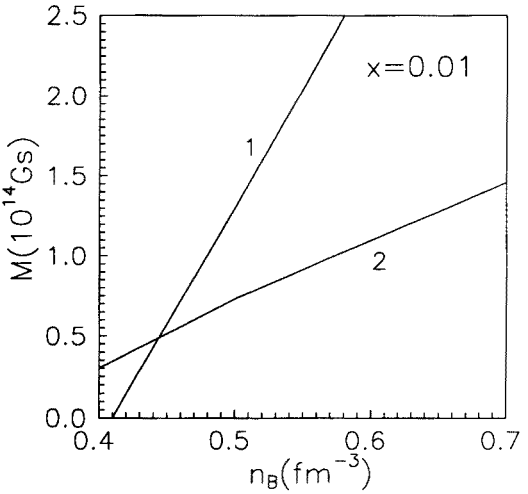


Fig. 4. The magnetization of the proton crystal as a function of baryon density for both sets of the Skyrme force parameters. The proton fraction is $x = 0.01$.

*Energy of the uniform nucleon matter*

With the effective interaction (10) one finds the energy density of a uniform nucleon matter of density $n_B$ and proton fraction $x$ in the form [12]:

$$\varepsilon_{\mathrm{un}} = \left(\frac{1}{2m_{\mathrm{N}}} + B_{\mathrm{N}}\right)\tau_{\mathrm{N}} + \left(\frac{1}{2m_{\mathrm{P}}} + B_{\mathrm{P}}\right)\tau_{\mathrm{P}} + n_B^2 \times [b + dn_B - (\tfrac{1}{2} - x)^2(a_3 + a_4 n_B)], \tag{A1}$$

where

$$B_i = \tfrac{1}{4}[(t_1 + t_2(1 + 0.5x_2))n_B + \tfrac{1}{2}(t_2(1 + 2x_2) - t_1)n_i], \quad i = \mathrm{N}, \mathrm{P} \tag{A2}$$

and

$$b = \tfrac{3}{8}t_0, \quad d = \tfrac{1}{16}t_3, \quad a_3 = (\tfrac{1}{2} + x_0)t_0, \quad a_4 = \tfrac{1}{6}t_3(\tfrac{1}{2} + x_3). \tag{A3}$$

*Single-proton cell energy for the normal state*

The mean energy density of the neutron background is

$$\varepsilon_{\mathrm{N}}(\alpha) = n_{\mathrm{N}}[e_0^{(\mathrm{N})} + T_0^{\mathrm{NN}} + T_1^{\mathrm{NN}} + T_2^{\mathrm{NN}} + T_3^{\mathrm{NNN}} + q^2(w + u)], \tag{A4}$$

where the kinetic energy density is

$$n_N \epsilon_0^{(N)} = \frac{3}{10 m_N} k_N^2 n_N , \tag{A5}$$

where $k_N = (3\pi^2 n_N)^{\frac{1}{3}}$ is the Fermi momentum of uniform neutron matter. The interaction terms are

$$n_N T_0^{NN} = \frac{1}{32} t_0 (1 - x_0) n_N^2 \gamma^3 \frac{1}{\beta^6} , \tag{A6}$$

$$n_N T_1^{NN} = \frac{3}{320} t_1 k_N^2 n_N^2 \gamma^3 \frac{1}{\beta^6} , \tag{A7}$$

$$n_N T_2^{NN} = \frac{9}{320} t_2 (1 + x_2) k_N^2 n_N^2 \gamma^3 \frac{1}{\beta^6} , \tag{A8}$$

$$n_N T_3^{NNN} = \frac{1}{192} t_3 (1 - x_3)(16 + 120\alpha^2 + 90\alpha^4 + 5\alpha^6)^3 n_N^3 \frac{1}{\beta^9} . \tag{A9}$$

The $q$-dependent terms are

$$n_N w = \frac{3}{2 m_N} \alpha^2 n_N \frac{1}{\beta^2} , \tag{A10}$$

$$n_N u = \frac{3}{16} t_1 n_N^2 \alpha^2 (4 + \alpha^2) \gamma^2 \frac{1}{\beta^6} . \tag{A11}$$

In these formulae $\beta$ and $\gamma$ are

$$\beta = 2 + \alpha^2 , \tag{A12}$$

$$\gamma = 8 + 24\alpha^2 + 3\alpha^4 . \tag{A13}$$

The proton-neutron interaction energy for the normal state is

$$E_{PN}(\alpha, q_P) = T_0^{NP} + T_1^{NP} + T_2^{NP} + T_3^{NNP} , \tag{A14}$$

where

$$T_0^{NP} = \frac{64}{27} t_0 (1 + 0.5 x_0) n_N A_1^3 \frac{1}{\beta^3} , \tag{A15}$$

$$T_1^{NP} = \frac{32}{27} t_1 A_1^2 \left( \frac{3}{10} k_N^2 n_N A_1 + \frac{3}{2} n_N \left( \alpha q^2 A_4 + q_P^2 A_5 + 2\alpha q q_P A_6 \right) \right) \frac{1}{\beta^3} , \tag{A16}$$

$$T_2^{NP} = \frac{32}{27} t_2 (1 + 0.5 x_2) A_1^2 \times \left( \frac{3}{10} k_N^2 n_N A_1 + \frac{3}{2} n_N \left( \alpha^2 q^2 A_2 + q_P^2 A_3 - 2\alpha q q_P A_6 \right) \right) \frac{1}{\beta^3} , \tag{A17}$$

$$T_3^{\mathrm{NNP}} = \frac{64}{81} t_3 (5 + x_3) n_{\mathrm{N}}^2 A_8^3 \frac{1}{\beta^6} . \tag{A18}$$

*Single-proton cell energy for the polarized state*

The mean energy density of the polarized neutron background is

$$\varepsilon_{\mathrm{N}m}(\alpha, s) = e_{0m}^{(N)} + T_{0m}^{\mathrm{NN}} + T_{1m}^{\mathrm{NN}} + T_{2m}^{\mathrm{NN}} + T_{3m}^{\mathrm{NNN}} + q^2 (w_m + u_m) , \tag{A19}$$

where the free kinetic energy density is

$$e_{0m}^{(\mathrm{N})} = \frac{3}{10 m_{\mathrm{N}}} (k_u^2 n_{\mathrm{N}}^u + k_d^2 n_{\mathrm{N}}^d) . \tag{A20}$$

Here $k_i = (6\pi^2 n_{\mathrm{N}}^i)^{\frac{1}{3}}$, $i = u, d$, is the Fermi momentum of spin up and spin down Fermi sea, respectively. The interaction terms are

$$T_{0m}^{\mathrm{NN}} = \frac{1}{8} t_0 (1 - x_0) n_{\mathrm{N}}^u n_{\mathrm{N}}^d \gamma^3 \frac{1}{\beta^6} , \tag{A21}$$

$$T_{1m}^{\mathrm{NN}} = \frac{3}{160} t_1 \left(k_u^2 + k_d^2\right) n_{\mathrm{N}}^u n_{\mathrm{N}}^d \gamma^3 \frac{1}{\beta^6} , \tag{A22}$$

$$T_{2m}^{\mathrm{NN}} = \frac{3}{160} t_2 (1 + x_2)(2 k_u^2 (n_{\mathrm{N}}^u)^2 + 2 k_d^2 (n_{\mathrm{N}}^d)^2 + (k_u^2 + k_d^2) n_{\mathrm{N}}^u n_{\mathrm{N}}^d) \gamma^3 \frac{1}{\beta^6} , \tag{A23}$$

$$T_{3m}^{\mathrm{NNN}} = \frac{1}{48} t_3 (1 - x_3)(16 + 120\alpha^2 + 90\alpha^4 + 5\alpha^6)^3 n_{\mathrm{N}} n_{\mathrm{N}}^u n_{\mathrm{N}}^d \frac{1}{\beta^9} . \tag{A24}$$

The $q$-dependent terms are

$$w_m = n_{\mathrm{N}} w , \tag{A25}$$

$$u_m = \frac{3}{4} t_1 n_{\mathrm{N}}^u n_{\mathrm{N}}^d \alpha^2 (4 + \alpha^2) \gamma^2 \frac{1}{\beta^6} . \tag{A26}$$

The proton-neutron interaction energy for the polarized state is

$$E_{\mathrm{PN}m}(\alpha, q_{\mathrm{P}}, s) = T_{0m}^{\mathrm{NP}} + T_{1m}^{\mathrm{NP}} + T_{2m}^{\mathrm{NP}} + T_{3m}^{NNP} , \tag{A27}$$

where

$$T_{0m}^{\mathrm{NP}} = \frac{64}{27} t_0 (n_{\mathrm{N}} + n_{\mathrm{N}}^u x_0) A_1^3 \frac{1}{\beta^3} , \tag{A28}$$

$$T_{1m}^{\mathrm{NP}} = \frac{16}{27} t_1 A_1^2 \Big( \frac{3}{5} (k_u^2 n_{\mathrm{N}}^u + k_d^2 n_{\mathrm{N}}^d) A_1 + 3 n_{\mathrm{N}} (\alpha q^2 A_4 + q_{\mathrm{P}}^2 A_5 + 2\alpha q q_{\mathrm{P}} A_6) \Big) \frac{1}{\beta^3} , \tag{A29}$$

$$T_{2m}^{\mathrm{NP}} = \frac{16}{27} t_2 A_1^2 \left( \frac{3}{5} ((1+x_2) k_u^2 n_{\mathrm{N}}^u + k_d^2 n_{\mathrm{N}}^d) A_1 \right.$$

$$\left. + 3(n_{\mathrm{N}} + x_2 n_{\mathrm{N}}^u)(\alpha^2 q^2 A_2 + q_{\mathrm{P}}^2 A_3 - 2\alpha q q_{\mathrm{P}} A_6) \right) \frac{1}{\beta^3} , \quad \text{(A30)}$$

$$T_{3m}^{\mathrm{NNP}} = \frac{256}{81} t_3 \left( (n_{\mathrm{N}} + n_{\mathrm{N}}^u x_3) n_{\mathrm{N}} + (1-x_3) n_{\mathrm{N}}^u n_{\mathrm{N}}^d \right) A_8^3 \frac{1}{\beta^6} . \quad \text{(A31)}$$

In all above formulae the coefficients $A_1 .... A_8$ are the following integrals:

$$A_1 = \frac{q_{\mathrm{P}}}{2\pi} \int_{\frac{-\pi}{q_{\mathrm{P}}}}^{\frac{\pi}{q_{\mathrm{P}}}} (1 + \cos(q_{\mathrm{P}} x))^2 (1 + \alpha \cos(qx))^2 \, dx , \quad \text{(A32)}$$

$$A_2 = \frac{q_{\mathrm{P}}}{2\pi} \int_{\frac{-\pi}{q_{\mathrm{P}}}}^{\frac{\pi}{q_{\mathrm{P}}}} (1 + \cos(q_{\mathrm{P}} x))^2 \sin(qx)^2 \, dx , \quad \text{(A33)}$$

$$A_3 = \frac{q_{\mathrm{P}}}{2\pi} \int_{\frac{-\pi}{q_{\mathrm{P}}}}^{\frac{\pi}{q_{\mathrm{P}}}} \sin(q_{\mathrm{P}} x)^2 (1 + \alpha \cos(qx))^2 \, dx , \quad \text{(A34)}$$

$$A_4 = \frac{q_{\mathrm{P}}}{2\pi} \int_{\frac{-\pi}{q_{\mathrm{P}}}}^{\frac{\pi}{q_{\mathrm{P}}}} (1 + \cos(q_{\mathrm{P}} x))^2 \cos(qx) \, (1 + \alpha \cos(qx)) \, dx , \quad \text{(A35)}$$

$$A_5 \frac{q_{\mathrm{P}}}{2\pi} \int_{\frac{-\pi}{q_{\mathrm{P}}}}^{\frac{\pi}{q_{\mathrm{P}}}} \cos(q_{\mathrm{P}} x) \, (1 + \cos(q_{\mathrm{P}} x)) \, (1 + \alpha \cos(qx))^2 \, dx , \quad \text{(A36)}$$

$$A_6 = \frac{q_{\mathrm{P}}}{2\pi} \int_{\frac{-\pi}{q_{\mathrm{P}}}}^{\frac{\pi}{q_{\mathrm{P}}}} \sin(q_{\mathrm{P}} x) \, (1 + \cos(q_{\mathrm{P}} x)) \sin(qx) \, (1 + \alpha \cos(qx)) \, dx , \quad \text{(A37)}$$

$$A_7 = \frac{q_{\mathrm{P}}}{2\pi} \int_{\frac{-\pi}{q_{\mathrm{P}}}}^{\frac{\pi}{q_{\mathrm{P}}}} (1 + \cos(q_{\mathrm{P}} x))^4 (1 + \alpha \cos(qx))^2 \, dx , \quad \text{(A38)}$$

$$A_8 = \frac{q_P}{2\pi} \int_{\frac{-\pi}{q_P}}^{\frac{\pi}{q_P}} (1 + \cos(q_P x))^2 (1 + \alpha \cos(qx))^4 \, dx \,. \tag{A39}$$

## REFERENCES


[1] M. Kutschera, W. Wójcik, *Phys. Rev.* **C47**, 1077 (1993).
[2] M. Kutschera, W. Wójcik, *Acta Phys. Pol.* **B21**, 823 (1990).
[3] M. Kutschera, W. Wójcik, *Phys. Lett.* **223B**, 11 (1989).
[4] M. Kutschera, W. Wójcik, *Nucl. Phys.* **A581**, 706 (1995).
[5] M. Kutschera, *Z. Phys.* **A348**, 263 (1994).
[6] M. Kutschera, *Phys. Lett.* **B340**, 1 (1994).
[7] M. Kutschera, W. Wójcik, *Acta Phys.Pol.* **A**, in press.
[8] M. Kutschera, W. Wójcik, *Phys. Lett.* **B325**, 271 (1994).
[9] M. Kutschera, W. Wójcik, *Acta Magnetica* **VIII**, 3 (1991).
[10] M. Kutschera, W. Wójcik, *Acta Phys. Pol.* **B23**, 947 (1992).
[11] R.B. Wiringa, V. Fiks, A. Fabrocini, *Phys. Rev.* **C38**, 1010 (1988).
[12] J.M. Lattimer, *Ann. Rev. Nucl. Part. Sci.* **31**, 337 (1981).