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SHELL-MODEL STUDIES OF NUCLEI AT THE PROTON DRIP-LINE *

W.E. Ormand**

I.N.F.N. Sezione di Milano, via Celoria 16, 21033 Milano, Italy
and
Oak Ridge National Laboratory, Oak Ridge, TN 37831-6373, USA †

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Binding energies for nuclei with $A \leq 70$ can be predicted by exploiting analog symmetry. This is accomplished by computing the Coulomb energy difference between mirror nuclei and adding the Coulomb shift to the experimental binding energy of the neutron-rich mirror. The location of the proton drip-line is investigated and candidates for the detection of the exotic decay mode known as di-proton emission are identified. The consequences of analog-symmetry breaking on the calibration of solar neutrino detectors are discussed.

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1. Introduction

The structure of exotic nuclei is one of the most exciting challenges in low-energy nuclear physics today. Detailed theoretical studies of exotic nuclei, when confronted with experiments, will yield important information about the interaction between nucleons in the nucleus and the validity of our models for the structure of nuclei. In addition, the study of exotic nuclei is essential to many fundamental issues in physics today. For example, it is believed that many of the heavy elements in the universe are produced by the radiative capture of neutrons or protons on unstable nuclei [1]. The competition between β decay and particle capture traces out a path that

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** Present address: Department of Physics and Astronomy, 202 Nicholson Hall, Louisiana State University, Baton Rouge, LA 70803-4001, USA

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synthesizes the known elements. The details of this path depend on the temperature of the site, as well as nuclear properties such as binding energies, level densities, spectroscopic factors, and β-decay lifetimes.

A new feature of proton-rich nuclei is the possibility of a new decay mode known as di-proton emission. Because of the pairing interaction, a nucleus with an even number of protons (Z, N) is generally more tightly bound than a $(Z-1, N)$ nucleus, but, because of the symmetry energy and Coulomb repulsion, it may be unbound relative to the $(Z-2, N)$ system. Here, it will be shown that the possiblity of observing this new decay mode is extremely sensitive to the two-proton separation energies.

Proton-rich nuclei can also be used to calibrate solar neutrino detectors. In particular, under the assumption of analog symmetry, the Gamow–Teller matrix elements needed to determine the response of ^{40}Ar, the medium for the ICARUS detector, can be obtained by studying the β-decay of the analog nucleus ^{40}Ti [2]. It will be seen, however, that corrections due to isospin breaking can be significant.

2. Coulomb energy differences

If the nuclear Hamiltonian is composed of at most two-body parts, it may be separated into three components. The dominant part, which is also responsible for most of the nuclear binding, is isoscalar and is due to the strong interaction. The other two components are due to the Coulomb interaction and charge non-symmetric parts of the nucleon–nucleon interaction, and are isovector and isotensor in character. If the isovector and isotensor components are weak, then the binding energies for the members of an isospin multiplet may be obtained within the context of the isobaric mass multiplet equation (IMME) [3]

$$BE(A,T,T_z,i) = a(A,T,i) + b(A,T,i)T_z + c(A,T,i)T_z^2, \tag{1}$$

where T and $T_z = (Z-N)/2$ denote the isospin and its third component for the members of the isospin multiplet. The coefficients a, b, and c separately depend on the isoscalar, isovector, and isotensor components of the nuclear Hamiltonian, respectively.

From Eq. (1), the binding energy difference between iosbaric analogs with $T_z = \pm T$ is given by

$$BE(A,T,T_z=T,i) - BE(A,T,T_z=-T,i) = 2b(A,T,i)T. \tag{2}$$

Therefore, the most accurate way to predict absolute binding energies for proton-rich nuclei whose analog has an experimentally measured mass is to compute the b-coefficient for the multiplet and add $2bT$ to the experimental binding energy, $BE_{\rm exp}(A,T,T_z=-T,i)$, of the neutron-rich analog.

Shell-model calculations for the Coulomb energy difference have recently been carried out using empirical "Coulomb" interactions [4] for 37 nuclei in the mass region $36 \leq A \leq 48$ with active particles in the $0d_{3/2}$ and $0f_{7/2}$ orbits [5] and for 75 nuclei with $46 \leq A \leq 70$ using the $0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$, and $1p_{1/2}$ orbits [6]. For the most part, empirical "Coulomb" interactions were found to reproduce experimental *b*-coefficients at level of 30–45 keV [4], and, consequently, the binding energies obtained using Eq. (2) for unknown nuclei have an accuracy of approximately $40|Z-N|$ keV.

A prominent feature of all the particle-stable nuclei studied in Refs. [5, 6] is that the Q-value for β^+ decay is generally 12 MeV or more. Hence, the β-decay halflives for these nuclei are of the order 1–100 ms.

3. Di-proton emission

With the predicted binding energies, candidates for di-proton emission may be identified. The most important observation is that the number of candidates for which experimental detection is feasible is sharply limited by the two-proton separation energy. This is in part due to the fact that β^+ emission is a competing decay mechanism with lifetimes of the order 1-100 ms. In addition, a lower limit of approximately 1 ns is often imposed by the experimental apparatus. On the other hand, the decay rate for diproton emission is determined by the probability to penetrate through the Coulomb barrier, which is exponentially dependent on the two-proton separation energy. Because of this, the number of candidates for which the observation of diproton decay is practical, is limited to nuclei with two-proton separation energies between 0.9 and 1.4 MeV.

TABLE I

Range of halflives for di-proton emitter candidates. Also listed are the predictions for the one- and two-proton separation energies (S_p and S_{2p}).

^{A}Z	S_p (MeV)	S_{2p} (MeV)	$t_{1/2}$ (s)	$t_{1/2}^{\text{min}}$ (s)	$t_{1/2}^{\text{max}}$ (s)
^{38}Ti	0.438(164)	-2.432(132)	9×10^{-16}	4×10^{-16}	2×10^{-15}
^{45}Fe	-0.010(198)	-1.279(181)	10^{-6}	10^{-8}	10^{-4}
^{48}Ni	0.505(351)	-1.290(330)	4×10^{-6}	5×10^{-9}	0.09
^{59}Ge	0.058(211)	-1.343(192)	10^{-3}	10^{-5}	0.3
^{63}Se	0.069(288)	-1.530(262)	6×10^{-5}	3×10^{-7}	5×10^{-2}
^{66}Kr	-0.001(351)	-2.832(325)	3×10^{-12}	2×10^{-13}	6×10^{-11}
^{67}Kr	0.155(288)	-1.538(262)	2×10^{-3}	10^{-5}	0.2

Listed in Table I are nuclei with di-proton halflives (estimated via the WKB approximation) that are predicted to be of the order 1 ms or shorter. Note that since the uncertainties in the two-proton separation energies are of the order 200 keV, the exponential sensitivty in the halflife on the separation energy leads to a fairly wide range of halflives, as is indicated by $t_{1/2}^{\mathrm{min}}$ and $t_{1/2}^{\mathrm{max}}$. From Table I, the best candidates for experimental observation are ^{45}Fe, ^{48}Ni, and ^{66}Se.

4. Calibration of neutrino detectors and analog symmetry

Assuming analog symmetry, the Gamow–Teller (GT) matrix elements needed to calibrate the response of a solar neutrino detector can be determined from the β-decay of the proton-rich analog, *e.g.*, ^{40}Ti for ^{40}Ar and ^{37}Ca for ^{37}Cl. The primary source of analog-symmetry breaking (isospin mixing) is the Coulomb force, and although the effect on Fermi transitions is small ($\sim$ 1%) [7], it can be quite large for GT transitions. In addition, the final state in the β decay is often proton unbound, which may lead to a further analog-symmetry breaking. These concepts can be tested experimentally by examining the beta decays of the analog nuclei ^{13}O and ^{13}B to the first excited states in ^{13}N and ^{13}C, respectively. In the case of ^{13}N, this state is proton unbound, whereas in ^{13}C it is neutron bound. A measure of the deviation of analog symmetry may be provided by the ratio of the ft values for the β^+ and β^- decays, and is defined by $R = 1 - ft^+/ft^-$. Experimental data [8] give $R = -0.10(2)$ for the decay to the ground state and $R = -0.21(8)$ for the first excited state. Hence, corrections of the order of 20% may have to be applied to the calibration of neutrino detectors. Using the methods developed in Ref. [7] to compute the isospin-mixing effects on beta-decay, theoretical estimates for R are -0.5 and -0.13 for the ground and first excited states, respectively.

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