NEUTRINO MASSES*

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(Received October 26, 1997)

67 years after neutrinos had been introduced into physics many of their properties remain unknown. An open question of whether neutrinos have masses is addressed in the present lecture. After reviewing briefly limits on neutrino masses obtained in direct experiments, indirect searches for neutrino masses are discussed. Particular attention is paid to the search for neutrino oscillations. The Sun is a particularly strong source of neutrinos and for this reason has been used for pertinent experiments. The different solar neutrino experiments are compared in the lecture.

PACS numbers: 14.60. Pq

1. Introduction

In spite of the observation of numerous composed particles, we have today a rather simple picture of elementary particles, consisting of six quarks (u, d, c, s, b, t) and of six leptons (ν_e, e^-, ν_μ, μ^-, ν_τ, τ^-) as well as of their associated antiparticles. Neutrinos thus comprise 25% of our presently known elementary particles, this being the prime reason for the particular interest which is extended these days to neutrinos. Neutrinos had been instrumental in the acceptance of the theory of weak interactions, because of the first observation of neutrino induced weak neutral currents at CERN in the Gargamelle Bubble chamber in 1973 [1]. Likewise, neutrinos had been instrumental in the experimental verification of the existence of quarks within nuclei, where both electrons and neutrinos had been used in deep inelastic scattering experiments to demonstrate the existence of point-like objects within nuclei.

* Presented at the XXI School of Theoretical Physics "Recent Progress in Theory and Phenomenology of Fundamental Interactions", Ustroń, Poland, September 19–24, 1997.

2. Neutrino properties

While we experimentally know a lot about quarks and about charged leptons, we know very little about neutrinos. We know today, that neutrinos are fermions, that they are merely subject to the weak interaction (apart from the very much smaller gravitational interaction) and that there exist only three types of light ($m_\nu < 1$ MeV) neutrinos [2]. Table I shows some of the unknown properties of neutrinos.

TABLE I

Some of the unknown properties of neutrinos

- what is their mass?
- are they stable particles?
- are neutrinos Dirac or Majorana particles?
- is there flavor mixing in vacuum? (analogy to quarks)
- is there flavor mixing in matter?
- is the total lepton number conserved?
- are neutrinos contributing to dark matter?
- are there relic neutrinos?
- is there coherent elastic scattering of neutrinos?
- is there an anomaly in the solar neutrino flux?
- are there right-handed neutrinos?
- why are there just 3 neutrino flavors?
- are there (static or transition) neutrino magnetic moments?
- are there heavy neutrinos (in excess of present limits given by LEP)?

Before discussing neutrino masses, we shall briefly mention the sources and detection methods available for experiments.

3. Neutrino sources

The usual radioactive sources with strengths even in the range of Ci's are much too weak for studying neutrinos. Table II shows the prime sources available for measurements at nuclear energies.

TABLE II

Sources of neutrinos available for experiments at nuclear energies

1) Nuclear fission (nuclear reactors)	Basic reaction: $n \to p + e^- + \bar{\nu}_e$
2) Nuclear fusion (Sun)	Basic reaction: $p \to n + e^+ + \nu_e$
3) Supernova explosions	Production of $\nu_e \bar{\nu}_e \nu_\mu \bar{\nu}_\mu \nu_\tau \bar{\nu}_\tau$

4. Neutrino detection

The detection of electron neutrinos can be achieved by a measurement of the inverse β-decay as shown in Fig. 1, by neutral current reactions and by elastic scattering off electrons. One may also use the Cherenkov radiation produced by scattered electrons for detection.

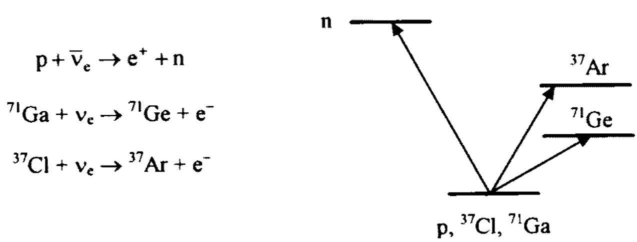

Fig. 1. Representative examples for the detection of electron-neutrinos by means of the inverse β-decay (charged current processes). Note the presence of threshold energies.

5. Neutrino masses

Theory is unable to predict neutrino masses for actual observations. Cosmological arguments, attributing the entire mass of the Universe to neutrinos only, provide an upper limit on the mass of stable neutrinos at low energies, which within a factor of two is given by

$$\sum m_i c^2 < 70\,\text{eV} \tag{1}$$

with the summation extending over all stable light neutrinos, *i.e.* over the known three flavors. Neutrino masses in the range of few eV might indeed be suited to close the Universe and might well be suited as a source of the dark matter. Additional sources of a less relativistic nature are presumably required to arrive at the actual situation.

5.1. Direct searches for neutrino masses

All experiments performed up to now are consistent with a zero rest mass of neutrinos, but this may be a consequence of the fact, that neutrino energies had been rather large compared to their masses. Direct measurements of neutrino masses have yielded only limits. In the case of electron neutrinos, such measurements were performed by employing the 18.6 keV transition in tritium, which exhibits a particular low maximum energy, therefore making an influence of a neutrino mass on the β-spectrum more easily detectable. The principle of such a detection is shown in Fig. 2. Most such measurements for unknown reasons yield negative values of m_ν^2 which then are used to deduce error bars and therefore upper limits on neutrino masses. Such upper limits on the masses of the three neutrino types are shown in Table III. Substantially lowering these mass limits appears hardly feasible with present techniques.

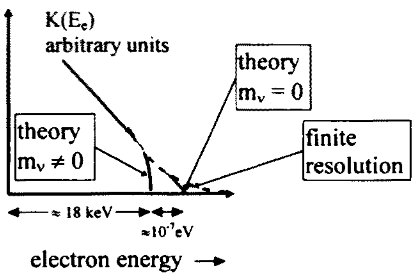

Fig. 2. Direct search for the mass of electron neutrinos by means of the low energy radioactive decay of tritium according to the reaction ^{3}H$\rightarrow^3$He$+e^-$ $+ \bar{\nu}_e$.

TABLE III

Limits for neutrino masses obtained in direct experiments

neutrino	mass limit	confidence level	decay	Ref.
ν_e	< 4.4 eV	95%	tritium decay	[3]
ν_μ	< 0.17 MeV	90%	$\pi^+ \rightarrow \mu^+\nu_\mu$	[4]
ν_τ	< 24 MeV	95%	$\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$	[5]

5.2. Indirect searches for neutrino masses

Indirect experiments, in particular the search for neutrino oscillations, seem to provide a much higher sensitivity to neutrino masses, which presumably are very small. Neutrino oscillations might occur either in vacuum (for instance by employing reactor neutrinos or by observing neutrinos in the space between Sun and Earth) or in matter, such as existing within the Sun.

Neutrino oscillations should occur, if at least one of the mass parameters of the neutrinos, $\Delta m_{ij}^2 = m_j^2 - m_i^2$, differs from zero and if in addition the mass eigenstates of neutrinos (responsible for the propagation in vacuum) and the weak interaction eigenstates (responsible for weak decays) would be different. The latter connection should be linear, as required by the basic laws of quantum mechanics. The linear relation between weak interaction eigenstates and energy (mass) eigenstates of neutrinos is given by Eq. (2), where we have confined ourselves to a two neutrino approximation, which involves only one mixing parameter.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_3 \end{pmatrix}$$

weak interaction eigenstates — mixing matrix — mass eigenstates (2)

In the case of a two neutrino approximation, one obtains only one mixing angle. In the case of a realistic three neutrino description, four mixing parameters would result.

We have in the two neutrino approximation:

$$\begin{aligned} \langle\nu_e(0)| &= \cos\theta\cdot\nu_1(0) + \sin\theta\cdot\nu_2(0) = \nu_1\cdot\cos\theta + \nu_2\cdot\sin\theta\,, \\ \langle\nu_e(t)| &= \nu_1\cdot e^{-iE_1t}\cdot\cos\theta + \nu_2\cdot e^{-iE_2t}\cdot\sin\theta\,, \\ \langle\nu_\mu(t)| &= -\nu_1\cdot e^{-iE_1t}\cdot\sin\theta + \nu_2\cdot e^{-iE_2t}\cdot\cos\theta\,. \end{aligned}$$

Assuming the normalization conditions $\langle\nu_i|\nu_j\rangle = \delta_{ij}$ one obtains for the $\nu_e \to \nu_\mu$ transition matrix element $\langle\nu_\mu(t)|\nu_e(0)\rangle = \frac{1}{2}\sin 2\theta\Big(e^{-iE_2t} - e^{-iE_1t}\Big)$. The oscillation probability is given by

$$\begin{aligned} P(\nu_e \to v_\mu) &= P(\bar\nu_e \to \bar\nu_\mu) = |\langle\nu_\mu(t)|\nu_e(0)\rangle|^2 \\ &= \tfrac{1}{2}\sin^2 2\theta[1-\cos(E_2t - E_1t)] = \tfrac{1}{2}\sin^2 2\theta[1-\cos(\Delta m^2/2p_\nu)] \\ &= \tfrac{1}{2}\sin^2 2\theta\cdot\sin^2\left[\frac{1.27\cdot(\Delta m^2/\mathrm{eV}^2)\cdot(L/m)}{E_\nu/\mathrm{MeV}}\right]. \end{aligned} \quad (3)$$

The probability that there is no oscillation is given by $P(\nu_e \to \nu_e) = P(\bar\nu_e \to \bar\nu_e) = 1 - P(\nu_e \to \nu_\mu)$. For the energies we have $E_2 - E_1 = \sqrt{p_2^2 + m_2^2} - \sqrt{p_1^2 + m_1^2} \approx \sqrt{p_\nu^2 + m_2^2} - \sqrt{p_\nu^2 + m_1^2} \approx \frac{m_2^2 - m_1^2}{2p} \approx \frac{\Delta m^2}{2E_\nu}$, defining the mass parameter Δm^2. We should note, that small values of Δm^2 require larger values of the distance L, such as to keep the argument of the trigonometric function below 2π. Disappearance (inclusive) experiments over short distances, for example experiments of the type $\bar{\nu}_e \to \bar{\nu}_\mu$, are often performed with reactor neutrinos. The energy of the neutrino generated in such an oscillation, *e.g.* $\bar{\nu}_\mu$ in the example given, is often too low to create charged leptons by means of charged current reactions., *e.g.* reactions of the type $\bar{\nu}_\mu \to \mu^+$. One observes instead mere changes in the intensity of one and the same neutrino (disappearance experiment), a procedure which is associated with only a low sensitivity to mixing angle ($sin^2 2\theta > 0.05$), but due to the low energies involved exhibits a relatively high sensitivity to the mass parameter ($\Delta m^2 > 0.02\,\mathrm{eV}^2$). The disadvantage of a low sensitivity to mixing angle could be avoided by applying energies, which are sufficiently high to generate new types of neutrinos (appearance experiment), but such a procedure would result in a low sensitivity to mass parameters. Both disadvantages can be avoided by performing experiments at very large distances, where according to Eq. (3) large values of the neutrino energy E_ν can be compensated for by large values of L. Experiments of this kind are in progress at the Fermi-Lab [6]. An alternative would be the detection of solar neutrinos at very large distances, yielding due to the small energies involved a high sensitivity to small mass parameters, while at the same time yielding a high sensitivity also to mixing angles through the MSW- (Mikheyev-Smirnov-Wolfenstein)-effect [7,8], a resonant conversion of electron-neutrinos into other types of neutrinos occuring within the Sun. This MSW effect is based upon the fact, that electron neutrinos are subject to both charged current (CC) and neutral current (NC) interactions, while muon and tauon neutrinos are subject only to NC interactions, as illustrated in Fig. 3.

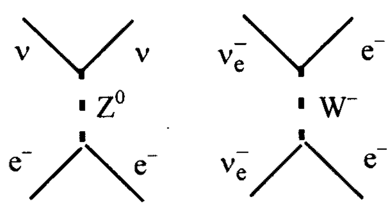

Fig. 3. Neutral current (NC) and charged current (CC) interactions of neutrinos.

The NC interactions between neutrinos and quarks because of the weak interaction universality is identical for all types of neutrinos, therefore giving rise only to a common phase factor, which will be left out in our considerations. The CC interaction in the framework of the two- neutrino approximation yields a dependence of the solar neutrino flux on the solar electron density ρ_e and on the matter parameter θ_m involving only quantities already introduced and given by the relation

$$\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta \pm \frac{2\sqrt{2}G_F \rho_e E_\nu}{\Delta m^2}}\,.$$

Apparently, there arises, in case of a mass hierarchy $\Delta m^2 = m_2^2 - m_1^2 > 0$, a resonance condition, given by $\cos 2\theta = (2\sqrt{2}G_F \rho_e E_\nu)/\Delta m^2$. Whether or not such a resonance does exist cannot be predicted, because of the unknown value of the mass parameter Δm^2. The consequences of the MSW effect can be interpreted as follows: Electron neutrinos, created in the solar core by means of nuclear fusion processes, propagate to the solar surface. A resonance condition caused by proper vacuum parameters Δm^2 (or θ) would lead to a conversion of these propagating neutrinos into neutrinos of other flavors, if the processes proceed in an adiabatic manner. In consequence, one would observe a reduced flux of neutrinos in a terrestrial detector, because such a detector responds only to electron neutrinos. Non-adiabatic solar processes instead would lead to a conservation of neutrino flavor, with a corresponding preservation of flux in a terrestrial detector. Adiabatic and non-adiabatic processes generated by the MSW-effect are illustrated in Fig. 4. Measurements of solar neutrinos are presumably involving the MSW-effect. The Sun, a principal series star, is a source of photons and of neutrinos, with the neutrino luminosity equal to some 2.3% of the photon luminosity. Photons need several hundred thousand years to arrive at the solar surface. They undergo numerous scattering processes along their way within the Sun and thus contain no memory of their origin. Neutrinos, by contrast, arrive 8.3 minutes after their production in the Sun. With neutrinos subject only to weak interactions, *i.e.* with the Sun being practically transparent to neutrinos, these particles provide a nearly instantaneous picture of the neutrino reactions proceeding inside the Sun. The Sun is the only star which is sufficiently close to permit measurements of such stellar neutrinos. These neutrinos are produced during solar fusion reactions occuring in the inner 25 % of the solar radius, obeying the overall reaction $4p \to ^4\mathrm{He} + 2e^+ + 2\nu_e$, where two neutrinos are liberated together with an energy of 26.1 MeV. This overall reaction is composed of a number of detailed processes, with the composite solar neutrino spectrum shown in Fig. 5.

Most important amongst the solar neutrino processes is the introductory reaction,

$$p + p \to d + e^+ + \nu_e \qquad (E_\nu < 0.42\mathrm{MeV})$$

which is responsible for our existence, because the Sun shines over such a long period only, since the introductory reaction of solar fusion corresponds to a weak process. Another experimentally important process is given by

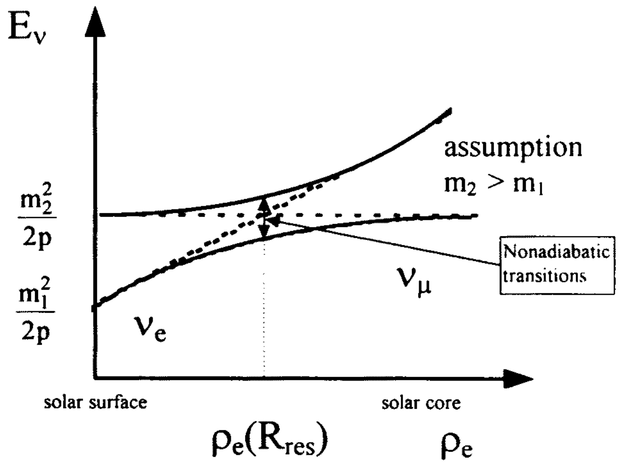

Fig. 4. Energy eigenvalues for solar neutrinos, passing from a region of high electron density $\rho_e(0)$ within the solar core (at the right of the figure) to the solar surface with electron density zero (at the left of the figure). The figure assumes a mass hierarchy $\Delta m^2 = m_2^2 - m_1^2 > 0$, attributing a lower mass value to the electron neutrinos at the solar surface. The dashed lines hold for the absence of mixing, with the energy of the electron neutrino depending linearly on the electron density due to its charged current interaction, and with the energy of the muon neutrino staying constant due to the absence of a charged current interaction with the electrons. The solid lines hold in the presence of mixing, with the minimum separation appearing in the resonance region. Only electron neutrinos of sufficiently high energy have a chance to pass through such a region. Under pure adiabatic conditions, such neutrinos will then change their flavor from ν_e produced within the solar core to another flavor such as ν_μ or ν_τ on the solar surface. Under non-adiabatic conditions, where the neutrino passage through the resonance region is rather rapid, we may even have the indicated transitions between the two branches in the resonance region, leading to a reduction in the conversion rate.

the reaction

$$^8\mathrm{B} \to 2 \times {}^4\,\mathrm{He} + e^+ + \nu_e \qquad (E_\nu < 14\,\mathrm{MeV})\,.$$

This reaction is completely uninteresting as concerns energy production (photon luminosity), but because of a neutrino cross section growing near the energy threshold in proportion to E_ν^2 the reaction has been most interesting for measuring solar neutrinos. In fact, the pioneer of solar neutrino research, Ray Davis, has made his first solar neutrino observations by measuring neutrinos from the ^{8}B reaction. His energy threshold of about 0.81

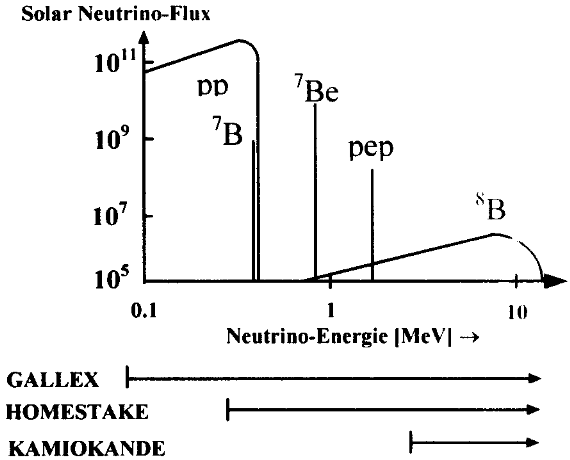

Fig. 5. Pertinent components of the solar proton-cycle neutrino spectrum according to the Standard Solar Model. Shown are line spectra in [$cm^{-2}s^{-1}$] and continuous spectra in [$cm^{-2}s^{-1}MeV^{-1}$]. Different thresholds of various experiments are also indicated.

MeV was obtained by means of the Chlorine reaction, also indicated in Fig. 5, employing a detector of some 630 t of C_2Cl_4, which within its 10^{29} nuclei gave rougly every two days one solar neutrino absorption. This number represents in view of some $6 \cdot 10^{10}$ neutrinos/cm^2/sec incident on our Earth another drastic manifestion of the meaning of "weak interaction". Cosmic ray induced events, which are indistinguishable from solar induced neutrino events, require such observations to be performed deep underground. Of particular experimental interest is the low energy solar pp- neutrino reaction which (compare Fig. 5) requires the particularly expensive ^{71}Ga for an efficient absorption of solar neutrinos. The interest for this particular absorption arises from the fact, that the introductory pp-reaction is responsible for most of the solar energy production, as measured by the well-known solar luminosity. It is for this reason, that this reaction is very well understood, in contrast to the very rare ^{8}B reaction with its more poorly understood cross section. One corresponding experiment, known as GALLEX (= Gallium experiment), has in the meantime been performed in the Gran Sasso underground neutrino laboratory 180 km northeast of Rome. The various laboratories participating in this experiment are shown in Table IV, while a summary of performed neutrino experiments is given in Table V.

TABLE IV

Laboratories participating in GALLEX

MPI	Heidelberg/Germany (T.Kirsten, spokesman of the collaboration)
KFK	Karlsruhe/Germany
TU	München/Germany
INFN	Milano/Italy
INFN	Rome/Italy
DAPNIA	Saclay/France
CEN	Grenoble/France
U	Nice/France
WIS	Rehovoth/Israel
BNL	Brookhaven/USA

TABLE V

Measurements and predictions for solar neutrino experiments; errors are usually stated as sequence of statistical and systematic errors, except for Kamioka, where only relative numbers apply. Predictions based on the Standard Solar Model are taken from Bahcall and Pinsonneault [13].

EXPERIMENT			Prediction [SNU]*	
collaboration	measurement [SNU]*	threshold energy	Ref.	Bahcall [3σ]
HOMESTAKE $^8B + (^7Be)$	$2.55 \pm 0.17 \pm 0.18$	0.814 MeV	[9]	8.0 ± 3.0
KAMIOKA ^8B	$(0.51 \pm 0.07) \times$pred.	7.5 MeV	[10]	$(1.0 \pm 0.14) \times$pred.
SAGE $E_\nu > 233$ keV	$69 \pm 10^{+5}_{-7}$	233 keV	[11]	132^{+21}_{-17}
GALLEX $E_\nu > 233$ keV	$69.7 \pm 6.7^{+3.9}_{-4.5}$	233 keV	[12]	132^{+21}_{-17}

*1 [SNU] = 1 [Solar Neutrino Unit] = 10^{-36} [captures/target atom/s]

Characteristic feature of all these solar experiments is a deficit of neutrinos according to the prediction of the Standard Solar Model. It is at present not entirely obvious, whether this deficit is caused by a wrong interpretation of the neutrino production in the Sun (astrophysics) or by neutrino oscillations (particle physics), though indications point to the latter case. If the MSW-effect would be the reason for the neutrino deficit, then masses of the neutrinos could be worked out based on the experimental results. One alternative would be to wait for the results of the Superkamiokande experiment, which would give a deviation of the Curie plot for the ^{8}B-spectrum. Another alternative would be to wait for the results of the Canadian SNO (Sudbury-Neutrino-Observatory)-experiment, which might be able to measure the NC interaction, an interaction, which would be independent of neutrino oscillations. Such an experiment would yield the entire neutrino flux produced in the Sun and thus would permit a direct distinction between astrophysical or particle physical reasons for the observed solar neutrino deficit.

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