ON THE POSSIBILITIES OF DISTINGUISHING DIRAC FROM MAJORANA NEUTRINOS* **

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The problem if existing neutrinos are Dirac or Majorana particles is considered in a very pedagogical way. After a few historical remarks we recall the theoretical description of neutral spin 1/2 particles, emphasizing the difference between chirality and helicity which is important in our discussion. Next we describe the properties of neutrinos in the cases when their interactions are given by the standard model and by its extensions (massive neutrinos, right-handed currents, electromagnetic neutrino interaction, interaction with scalar particles). Various processes where the different nature of neutrinos could in principle be visible are reviewed. We clear up misunderstandings which have appeared in last suggestions how to distinguish both types of neutrinos.

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1. Introduction

The main problem in neutrino physics is the one of the neutrino mass and mixing between different neutrino flavours. There are many indications that neutrinos are really massive particles (LSND experiment, problem of the solar and atmospheric neutrinos, dark matter).

If neutrinos are massive, the next problem is connected with their nature. Charged fermions are Dirac particles and it is a consequence of the electric charge conservation. Lepton number conservation is decidedly less

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fundamental than charge conservation and it does not govern the dynamics. Total lepton number can be broken, as it is predicted by many extensions of the Standard Model (SM). Then neutrinos do not hold any additive internal quantum numbers and can be identical to their own antiparticles. Such fermions are now generally known (not only for spin 1/2) as Majorana particles. The dilemma whether existing neutrinos are Dirac or Majorana particles is the subject of this paper. We would like to stress that it is not the point if Dirac and Majorana neutrino differ or not. Of course, they do. Majorana neutrinos are their own antiparticles, which is not the case for Dirac neutrino. The problem is whether there is some chance to distinguish them experimentally (within the Standard Model or beyond the SM neutrino interactions). These questions can also be divided into two parts. Firstly we can consider if they are distinguishable in principle and secondly what are technical possibilities to see different effects in real experiment for both types of neutrino. We would like to present a critical review of various efforts and suggestions how to distinguish Dirac from Majorana nature. It is still a "hot" problem and there are many answers emerging here, both correct and wrong. After short historical remarks (Chapter 2) we remind the theoretical description of massless and massive, neutral spin 1/2 fermions (Chapter 3). Next, in Chapter 4, we describe the standard model interaction of neutrinos and analyze the others, beyond the standard model, neutrino properties which can give better chance to distinguish their nature. In Chapter 5 we give a review of various processes where it seems to be possible for light neutrinos to find some specific signal different for both characters of neutrinos. In case of wrong suggestions we indicate the place of errors. In Chapter 6 we summarize our main conclusions.

2. Historical remarks

After Wolfgang Pauli hypothesis [1] in 1930 neutrino was born as a Dirac fermion described by Paul Dirac equation known from 1927. Neutrino and antineutrino were distinct particles. Such Dirac particles were used in 1934 by Enrico Fermi in his model of neutrino interactions with nucleons in β^- and β^+ decay [2]. Three years later, in 1937, Etore Majorana wrote his famous equation [3] in which neutrino was a neutral object, the same as its own antiparticle. Two years before, Maria Goeppert Meyer noticed that single β decay was not allowed for even-even nuclei, but decay for such nuclei with emission of two electrons

$$(A, Z) \longrightarrow (A, Z+2) + 2\,\mathrm{e}^- + 2\bar{\nu}_e, \tag{1}$$

was possible.

Already in 1939 Wendell Furry realized [4] that (if neutrinos have Majorana character) the neutrinoless double β decay

$$(A, Z) \longrightarrow (A, Z+2) + 2\,e^- \tag{2}$$

would be possible, too.

We will see that this process is also nowadays the best place where the nature of neutrinos is tested. In 1952 Raymond Davis found no evidence that antineutrinos from the reactor were absorbed in the chlorine detector by the reaction [5]

$$\bar{\nu}_e + ^{37}_{17}\mathrm{Cl} \longrightarrow e^- + ^{37}_{18}\mathrm{Ar}. \tag{3}$$

Four years later in 1956 (after neutrino discovering [6]) it was known that only neutrino ν_e could produce electron

$$\nu_e + ^{37}_{17}\mathrm{Cl} \longrightarrow e^- + ^{37}_{18}\mathrm{Ar}. \tag{4}$$

The results of both observations indicated that neutrino (ν_e) and antineutrino ($\bar{\nu}_e$) were distinct particles and to describe the difference the electron lepton number was introduced ($L_{\nu_e} = 1, L_{\bar{\nu}_e} = -1$) . After this discovery it was obvious that the neutrinos should be treated as the Dirac particle and not as the Majorana one.

In 1956 parity violation was discovered by Tsung Dao Lee and Chen–Ning Yang [7] and experimentally supported one year later by Chien–Shiung Wu *et al.* [8]. Immediately it was realized that breaking of the mirror symmetry is easy to understand if we assumed that neutrinos were massless particles [9]. Four component spinor, resolution of the Dirac equation with vanishing mass, decoupled for two independent two component spinors. The first one, which described particle with negative and antiparticle with positive helicity ($\nu_{\mathrm{L}}, \bar{\nu}_{\mathrm{R}}$) , and the second one with opposite helicities for particle and antiparticle ($\nu_{\mathrm{R}}, \bar{\nu}_{\mathrm{L}}$) . If the neutrino interaction was of V-A type then only one particle should be visible, and experiments should decide which one. Such an experiment had been done in 1958 by Maurice Goldhaber *et al.* in Brookhaven [10]. The answer was clear. The neutrinos from β^+ decay had negative helicity and that ones from β^- were positive helicity states. Only the first ($\nu_{\mathrm{L}}, \bar{\nu}_{\mathrm{R}}$) resolution of massless Dirac equation (known as Weyl equations) was realized in nature. After this discovery the Davis' result could be interpreted in an alternative way. The chlorine experiment could only distinguish negative from positive helicity particle states; it couldnot tell the difference between Dirac and Majorana neutrinos. So, from the experimental point of view there was no way to distinguish

$$\begin{aligned} \nu_{\mathrm{L}} &\Leftrightarrow \nu\,(-)\,, \\ \bar{\nu}_{\mathrm{R}} &\Leftrightarrow \bar{\nu}\,(+)\,. \end{aligned} \tag{5}$$

In 1957-58 several papers appeared [11] which had shown that there was equivalence between Weyl and massless Majorana fermions. Then, for almost twenty years, there was practically no discussion in literature about neutrino's nature. In the seventies unification theories appeared with massive neutrinos [12]. The so called "see-saw" mechanism made it possible to understand why the mass of neutrinos was very small [13]. After first observations of the solar neutrino anomaly [14] the problem of neutrino mass became one of the most important subjects in particle physics (and later in astrophysics and cosmology). For massive neutrinos the problem of their nature once more began to be very important. Fifty years later the Majorana paper has become again famous, as it poses what Pontecorvo calls "the central problem in neutrino physics": is neutrino identical to its own antiparticle? From the beginning of eighties papers with different suggestions how to resolve this problem have been appearing continuously. Unfortunately, the very pessimistic observation made in 1982 [15] stating that all observable effects which differentiate Dirac and Majorana neutrinos disappear if neutrino mass goes to zero is still valid.

3. Dirac, Majorana, Weyl neutrinos their helicity, chirality and all that

For the future discussion it is worth presenting a short reminder of definitions of the basic properties of spin 1/2 fermion.

It is well known that Lorentz group $L_+^\uparrow$ has two nonequivalent two-dimensional representations. The objects which transform under Lorentz transformation are known as the van der Waerden spinors [16], right Ψ_R and left Ψ_L.

$$\Psi_R \stackrel{\text{Lorentz transformation}}{\longrightarrow} \Psi_R' = e^{\frac{i}{2}\theta\vec{n}\vec{\sigma}}\, e^{-\frac{\lambda}{2}\vec{m}\vec{\sigma}}\, \Psi_R \,, \tag{6}$$

and

$$\Psi_L \stackrel{\text{Lorentz transformation}}{\longrightarrow} \Psi_L' = e^{\frac{i}{2}\theta\vec{n}\vec{\sigma}}\, e^{\frac{\lambda}{2}\vec{m}\vec{\sigma}}\, \Psi_L \,, \tag{7}$$

where $\vec{n}, \vec{m}, \theta$ and λ are proper characteristics of Lorentz transformation and $\vec{\sigma}$ are Pauli matrices [17]. For zero mass objects these spinors satisfy the Weyl equations [18].

$$(\widehat{\sigma}^\mu \partial_\mu)\, \Psi_R = 0, \ \widehat{\sigma}^\mu = \left(\sigma^0, \vec{\sigma}\right), \tag{8}$$

$$(\sigma^\mu \partial_\mu)\, \Psi_L = 0, \ \sigma^\mu = \left(\sigma^0, -\vec{\sigma}\right), \tag{9}$$

and describe particle with positive (Ψ_R) and negative (Ψ_L) helicities. For massless particle the spin projection on momentum is Lorentz invariant. For particles with mass the Weyl equations are not satisfied and there are two possibilities. The first one, more fundamental was discovered by Majorana [3]. The fields $\Psi_{\rm R(L)}$ satisfy the Majorana equations

$$i\,(\widehat{\sigma}^{\mu}\partial_{\mu})\,\Psi_{\rm R} \;-m\varepsilon\Psi_{\rm R}^{*} \;=0, \tag{10}$$

and

$$i\,(\sigma^{\mu}\partial_{\mu})\,\Psi_{\rm L} \;+m^{'}\varepsilon\Psi_{\rm L}^{*} \;=0, \tag{11}$$

where $m, m^{'}$ are particle masses and $\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Equations (10), (11) describe two completely different objects with masses m and $m^{'}$ which do not possess any additive quantum numbers and particles are their own antiparticles.

The second possibility of the field equation for massive fermion had been known before Majorana as the Dirac equation [19]

$$\begin{aligned} i\,(\widehat{\sigma}^{\mu}\partial_{\mu})\,\Psi_{\rm R} \;- m\Psi_{\rm L} \;&=\; 0, \\ i\,(\sigma^{\mu}\partial_{\mu})\,\Psi_{\rm L} \;- m\Psi_{\rm R} \;&=\; 0, \end{aligned} \tag{12}$$

and had described only one fermion with some additive quantum number (*e.g.* charge). Usually this equation is presented in four dimensional Dirac bispinor formalism as

$$(i\gamma^{\mu}\partial_{\mu} - m)\,\Psi = 0, \tag{13}$$

where

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \widehat{\sigma}^{\mu} & 0 \end{pmatrix}, \qquad \text{and} \qquad \Psi = \begin{pmatrix} \Psi_{\rm R} \\ \Psi_{\rm L} \end{pmatrix},$$

which is known as Weyl representation for Dirac γ matrices. In this representation let us define

$$\gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\; {\rm P_L} \;=\; \frac{1}{2}\,(1-\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\text{and } P_{\rm R} = \frac{1}{2}\,(1-\gamma_5) \;=\; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{14}$$

Then

$$\Psi_{\rm R} \;\equiv \begin{pmatrix} \Psi_{\rm R} \\ 0 \end{pmatrix} \equiv P_{\rm R}\Psi, \qquad \Psi_{\rm L} \;\equiv \begin{pmatrix} 0 \\ \Psi_{\rm L} \end{pmatrix} \equiv P_{\rm L}\Psi.$$

The spinors $\Psi_{\rm R(L)}$ are eigenvectors of γ_5

$$\gamma_5\Psi_{\rm R} = \Psi_{\rm R},\; \gamma_5\Psi_{\rm L} = -\Psi_{\rm L}, \tag{15}$$

and are known as chiral eigenvectors with eigenvalues + and − which have the name "chirality". For massless particles the chirality "±" coincide with the helicity, $\pm\frac{1}{2}$. For massive particles the chirality and helicity decouple. As we know from Eqs. (6), (7) the chirality is Lorentz invariant, irrespective of whether particle is massive or massless. The helicity is Lorentz invariant only for massless particle. For a massive particle there always exist Lorentz frames in which the particle has opposite momentum. This means that helicity of this particle changes sign and cannot be a Lorentz invariant object. It is instructive to decompose the free fields $\Psi_{\mathrm{L(R)}}$ for different kinds of particles in the helicity representation.

This decomposition is shown in Table I where we use the following denotations:

$$\begin{aligned} \vec{k} &= k\,(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)\,, \\ E &= \sqrt{m^2+k^2} \end{aligned} \tag{16}$$

are momentum and energy of the particle;

$$\chi\left(\vec{k},+\right) = \begin{pmatrix} \mathrm{e}^{-i\varphi/2} & \cos\theta/2 \\ \mathrm{e}^{i\varphi/2} & \sin\theta/2 \end{pmatrix}, \; \chi\left(\vec{k},-\right) = \begin{pmatrix} -\,\mathrm{e}^{-i\varphi/2} & \sin\theta/2 \\ \mathrm{e}^{i\varphi/2} & \cos\theta/2 \end{pmatrix} \tag{17}$$

are Pauli spinors for helicity $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively, and the $A^{\dagger}\,(A)$, $B^{\dagger}\,(B)$ are creation (annihilation) operators for Dirac, Weyl particle and antiparticle respectively, and the $a\,(a^{+})$ are suitable operators for Majorana particles.

The careful analysis of the Table I is very instructive and it is worth making some comments.

1. For the Dirac fields there are two distinct operators, one for particle $A^{\dagger}\,(A)$, and the other one for antiparticle $B^{\dagger}\,(B)$. We see that for $E \gg m \neq 0, \sqrt{E-k} \approx \frac{m}{\sqrt{2E}} + \mathcal{O}\,(m^2)$ and for definite chirality L or R there are two helicity states, $h = \pm\frac{1}{2}$. This fact is a consequence of the Lorentz invariance. However in that case both helicities have different weights; $\sqrt{E+k} \approx \sqrt{2E}$ for "good helicity" and $\sqrt{E-k} \approx \frac{m}{\sqrt{2E}}$ for "wrong helicity" states. For a pure left-handed interaction particles in the mixed helicity states will be produced. If helicity is not measured then the chiral particle state with energy E will be an incoherent superposition of two helicity states described by the statistical operator $\rho(E)$

$$\begin{aligned} \rho_{\text{particle}}\,(E) = {} & \left(\frac{E+k}{2E}\right) \left|\vec{k}, h=-\tfrac{1}{2}\right\rangle_p \left\langle \vec{k}, h=-\tfrac{1}{2}\right| \\ & + \left(\frac{E-k}{2E}\right) \left|\vec{k}, h=+\tfrac{1}{2}\right\rangle_p \left\langle \vec{k}, h=+\tfrac{1}{2}\right|. \end{aligned} \tag{18}$$

TABLE I

The fields $\Psi_{L(R)}$ for massive Dirac, massless Weyl and for both massive and massless Majorana neutrinos. See text for all denotations used in the Table. The integration is over three momentum: $\int = \int \frac{d^3k}{(2\pi)^3 2E}$.

	$m \neq 0$	m=0
Dirac and Weyl fields	$\Psi_R(x) = \int \left[A(+)\,e^{-ikx} - B^\dagger(-)\,e^{ikx}\right] \chi(+)\sqrt{E+k} + \int \left[A(-)\,e^{-ikx} + B^\dagger(+)\,e^{ikx}\right] \chi(-)\sqrt{E-k}$	$\Psi_R(x) = \int \left[A(+)\,e^{-ikx} - B^\dagger(-)\,e^{ikx}\right] \chi(+)\sqrt{2E}$
	$\Psi_L(x) = \int \left[A(-)\,e^{-ikx} - B^\dagger(+)\,e^{ikx}\right] \chi(-)\sqrt{E+k} + \int \left[A(+)\,e^{-ikx} + B^\dagger(-)\,e^{ikx}\right] \chi(+)\sqrt{E-k}$	$\Psi_L(x) = \int \left[A(-)\,e^{-ikx} - B^\dagger(+)\,e^{ikx}\right] \chi(-)\sqrt{2E}$
Majorana fields	$\Psi_R(x) = \int \left[a(+)\,e^{-ikx} - a^\dagger(-)\,e^{ikx}\right] \chi(+)\sqrt{E+k} + \int \left[a(-)\,e^{-ikx} + a^\dagger(+)\,e^{ikx}\right] \chi(-)\sqrt{E-k}$	$\Psi_R(x) = \int \left[a(+)\,e^{-ikx} - a^\dagger(-)\,e^{ikx}\right] \chi(+)\sqrt{2E}$
	$\Psi_L(x) = \int \left[a(-)\,e^{-ikx} - a^\dagger(+)\,e^{ikx}\right] \chi(-)\sqrt{E+k} + \int \left[a(+)\,e^{-ikx} + a^\dagger(-)\,e^{ikx}\right] \chi(+)\sqrt{E-k}$	$\Psi_L(x) = \int \left[a(-)\,e^{-ikx} - a^\dagger(+)\,e^{ikx}\right] \chi(-)\sqrt{2E}$

In such a state the neutrino *e.g.* in $\pi^+ \longrightarrow \mu^+\nu_\mu$ decay will be produced. It is opposite for antiparticle

$$\rho_{\rm antiparticle}(E) = \left(\frac{E+k}{2E}\right)\left|\vec{k}, h=+\tfrac{1}{2}\right\rangle_a\left\langle \vec{k}, h=+\tfrac{1}{2}\right| + \left(\frac{E-k}{2E}\right)\left|\vec{k}, h=-\tfrac{1}{2}\right\rangle_a\left\langle \vec{k}, h=-\tfrac{1}{2}\right| \qquad (19)$$

as *e.g.* for the neutrino in $\pi^- \longrightarrow \mu^-\bar{\nu}_\mu$ decay. For relativistic particles the wrong helicity states $|\vec{k}, h=+\frac{1}{2}\rangle_p$ and $|\vec{k}, h=-\frac{1}{2}\rangle_a$ have very small weight $(\frac{m}{2E})^2$ and even if, in principle, they can be produced, they have never been visible. Sometimes they are called "sterile neutrino".

2. In the Majorana case there is only one operator which creates particle and its own antiparticle. Both states (18) and (19) describe the same object - the Majorana neutrino. There is no sterile neutrino; both helicity states can be produced with equal weights. The left-handed $\Psi_{\rm L}$ and the right-handed $\Psi_{\rm R}$ fields are connected $\Psi_{\rm R}(x) = -\varepsilon\Psi_{\rm L}^*(x)$. The Majorana fields can be also written in the four component form

$$\Psi(x) = \begin{pmatrix} -\varepsilon\Psi_{\rm L}^*(x) \\ \Psi_{\rm L}(x) \end{pmatrix}, \qquad (20)$$

which satisfies the condition

$$i\gamma^2\Psi^*(x) = \Psi(x), \quad \text{where} \quad i\gamma^2 = \begin{pmatrix} 0 & -\varepsilon \\ +\varepsilon & 0 \end{pmatrix}. \qquad (21)$$

This relation is sometimes used as a definition of the field for the Majorana particle.

3. In cases of both Weyl and massless Majorana neutrinos the limit m$\rightarrow$0 is smooth. From Dirac neutrino we obtain two independent Weyl fields $\Psi_{\rm L}(x)$ and $\Psi_{\rm R}(x)$. In the left-handed chiral field $\Psi_{\rm L}(x)$ there is particle with negative helicity and antiparticle with positive helicity. In the field $\Psi_{\rm R}(x)$ it is just opposite: $A_{\vec{k}}(+)$ and $B_{\vec{k}}(-)$.

For the massless Majorana neutrino two fields $\Psi_{\rm L}(x)$ and $\Psi_{\rm R}(x)$ are still connected $(\Psi_{\rm R}(x) = -\varepsilon\Psi_{\rm L}^*(x))$. In the statical case it was proved [11] that one Weyl neutrino *e.g.* $\Psi_{\rm L}(x)$ (or separately $\Psi_{\rm R}(x)$) is equivalent to massless Majorana neutrino described by two connected fields $\Psi_{\rm L}$ and $\Psi_{\rm R}$. This relation is known as Pauli–Gursey transformation [11] which, for annihilation operators, can be written in the form

$$\begin{aligned} U^{-1}A_{\vec{k}}(-)U &= a_{\vec{k}}(-), \\ U^{-1}B_{\vec{k}}(+)U &= a_{\vec{k}}(+), \end{aligned} \qquad (22)$$

where

$$U = \exp\left[\frac{\pi}{4}\left(B^{\dagger}_{\vec{k}}(-)A_{\vec{k}}(-) - A^{\dagger}_{\vec{k}}(-)B_{\vec{k}}(-) - B^{\dagger}_{\vec{k}}(+)A_{\vec{k}}(+) + A^{\dagger}_{\vec{k}}(+)B_{\vec{k}}(+)\right)\right], \quad (23)$$

and Majorana operators are defined in the following way

$$a_{\vec{k}}(-) = \frac{1}{\sqrt{2}}\left[A_{\vec{k}}(-) + B_{\vec{k}}(-)\right],$$
$$a_{\vec{k}}(+) = \frac{1}{\sqrt{2}}\left[A_{\vec{k}}(+) + B_{\vec{k}}(+)\right]. \quad (24)$$

It must be stressed that this equivalence theorem is valid only for not interacting fields. For interacting fields whether the theorem is valid or not depends on the type of interaction. We will see that the massless Weyl–Majorana particles are still indistinguishable if there is only left-handed V-A (or only right-handed $V+A$) interaction. But for other types of interactions this equivalence theorem is no longer true.

4. Real and hypothetical neutrino interactions

4.1. Neutrinos in the Standard Model

It is only one case in which SM predicts masses of particles. The SM predicts that neutrinos are massless. There are three massless Weyl neutrinos ν_e, ν_μ and ν_τ. As a consequence there is no mixing between generations and 1) leptons have universal interactions, 2) both flavour L_e, L_μ, L_τ and total $L = L_e + L_\mu + L_\tau$ lepton numbers are conserved, and 3) there is not CP violation in the lepton sector.

The massless neutrinos have only the left-handed interactions with the charged and neutral gauge bosons

$$L_{\rm CC} = \frac{g}{2\sqrt{2}}\overline{N}\gamma^{\mu}\left(1-\gamma_5\right) l W^{+}_{\mu} + \text{h.c.}, \quad (25)$$

and

$$L_{\rm NC} = \frac{g}{4\cos\theta_W}\overline{N}\gamma^{\mu}\left(1-\gamma_5\right) N Z_{\mu}. \quad (26)$$

The interaction of fermions with the Higgs particles is proportional to the fermion masses, so massless neutrinos do not interact with scalar particles.

This picture of neutrino interaction is confirmed by all terrestrial experiments (maybe LSND results are the first which contradict the presented picture but they still should be better confirmed (*e.g.* by CARMEN)). In frame of the SM there is not any chance to differentiate between Weyl and massless Majorana fermions.

4.2. The other possible neutrino interactions

If neutrinos are massive particles, the mixing between generations appears in the charged and neutral currents

$$L_{\rm CC} = \frac{g}{2\sqrt{2}}\overline{N_a}\gamma^\mu\left(1-\gamma_5\right)K_{al}l_l W_\mu^+ + {\rm h.c.}, \tag{27}$$

and

$$L_{\rm NC} = \frac{g}{4\cos\theta_W}\overline{N_a}\gamma^\mu\left(1-\gamma_5\right)\Omega_{ab}N_b Z_\mu, \tag{28}$$

where K_{al} and Ω_{ab} are suitable mixing matrices resulting from diagonalization of a neutrino mass matrix. If there is a mixing in the lepton sector then the CP symmetry can be broken. It is the first place which differentiates the Dirac from Majorana neutrinos.

For the Dirac neutrinos situation looks like in the quark sector. Both charged leptons and Dirac neutrino fields have the phase transformation freedom

$$N_a \to N_a^{'} = {\rm e}^{i\alpha_a}N_a \quad \text{and} \quad l_l \to l_l^{'} \to {\rm e}^{i\beta_l}l_l, \tag{29}$$

and substantial number of phases can be eliminated from mixing matrix K (matrix Ω is a function of K).

For the Majorana neutrinos the phase transformation (29) is not allowed. We can see it from the Majorana equation where the field and its complex conjugation are present simultaneously. Fewer number of phases can be eliminated, so greater amount of them break the CP symmetry. For example, if neutrinos are Dirac particles we need at least three families to break CP, for Majorana neutrinos the CP can be broken already for two families. This different number of CP violating phases for the Dirac and Majorana neutrino can have real physical consequences which could be potentially used to distinguish them experimentally. In practical calculation the difference in CP breaking effects is visible by the different number of Feynman diagrams. Let assume that the mass of muon neutrino is larger than the electron neutrino mass $m_{\nu_\mu} > m_{\nu_e}$. Then we can calculate the decay width for the process

$$\nu_\mu \to \nu_e + \gamma. \tag{30}$$

Let us assume that the mixing matrix in Eq. (27) has the form

$$K = \begin{pmatrix} c & s\,{\rm e}^{i\delta} \\ -s\,{\rm e}^{-i\delta} & c \end{pmatrix}. \tag{31}$$

If the neutrinos are Dirac fermions two Feynman diagrams will describe the process (30) at the one loop level.

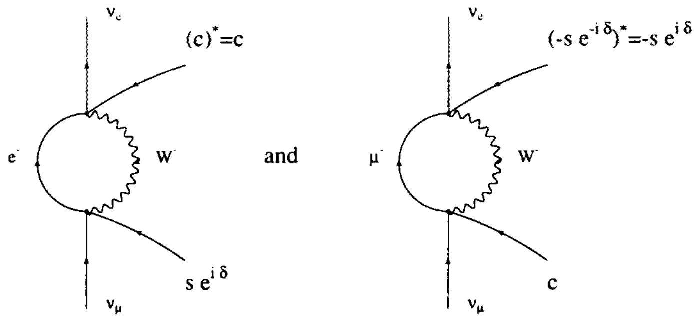


Fig. 1. Two diagrams which describe the radiative neutrino decay $\nu_\mu \to \nu_e + \gamma$ for Dirac neutrinos.

We see from Fig. 1 that the CP violating phase δ multiplies both diagrams in the same way and cancels after taking modulus square of the sum of both of them.

In the case of the Majorana neutrino there are four diagrams instead of two (Fig. 2).

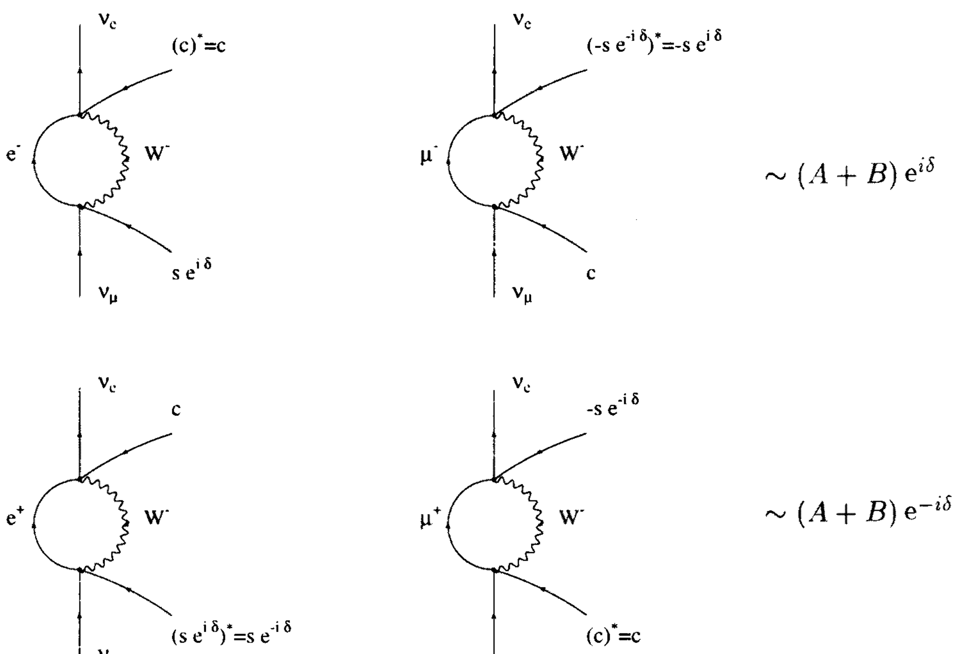


Fig. 2. Four Feynman diagrams which give contributions to the $\nu_\mu \to \nu_e + \gamma$ decay for Majorana neutrino.

We see that calculating the decay width (Fig. 2) the CP violating phase will not disappear and we obtain [21, 22]

$$\Gamma\left(\nu_\mu \to \nu_e + \gamma\right) \sim \left(1 + \left(\frac{m_e}{m_\mu}\right)^2 + 2\left(\frac{m_e}{m_\mu}\right)\cos 2\delta\right). \qquad (32)$$

Even if the CP symmetry is conserved there is an important difference between the Dirac and Majorana case. For Dirac particles the creation operators for particle (A) and antiparticle (B) can be multiplied by different complex phases, and , as a result, any CP phase can be absorbed, so it is not physical [23].

For the Majorana neutrino there is only one operator ($A = B$) and the CP phase cannot be absorbed and it is a physical observable. The CP phase for the Majorana neutrino must be pure imaginary number, and for the helicity state there is [23–25]

$$\mathrm{CP}\,|\vec{p}, \lambda\rangle = \eta_{\mathrm{CP}}\, \mathrm{e}^{-i\frac{\pi}{2}}\,| -\vec{p}, \lambda\rangle, \qquad (33)$$

where $\eta_{\mathrm{CP}} = \pm i$.

This fact can also have real physical consequences. Let us consider, for example, the decay of π^0 into two identical Majorana neutrinos $\pi^0 \to \nu_{\mathrm{M}}\nu_{\mathrm{M}}$ [25]. The initial π^0 state is $J^{\mathrm{PC}} = 0^{-+}$, so the possible final states are

$$J = 0,\ S = L = 0, \quad \text{and} \quad J = 0,\ S = L = 1, \qquad (34)$$

so

$$\mathrm{CP}|\nu_{\mathrm{M}}\nu_{\mathrm{M}}\rangle = (\pm i)^2\,(-)^L\,|\nu_{\mathrm{M}}\nu_{\mathrm{M}}\rangle = -\,|\pi^0\rangle.$$

From this we conclude that $L = 0$ so $S = 0$ and the emitted neutrino's spins are antiparallel.

The next important differences between the Dirac and Majorana neutrinos are their electromagnetic structure. In general any spin 1/2 Dirac fermion can have four independent electromagnetic formfactors.

If we define the one photon interaction diagram with two fermions like in Fig. 3 then the requirements that

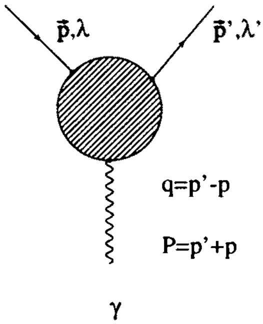
(i) initial and final fermions are on shell,

and

(ii) the current is conserved ($\Gamma^\mu q_\mu = 0$)

give the structure function $\Gamma^\mu\,(P, q)$ [26, 27]

$$\Gamma^\mu_{\mathrm{D}}\,(P, q) = F_{\mathrm{D}}\left(q^2\right)\gamma^\mu + iM_{\mathrm{D}}\left(q^2\right)\sigma^{\mu\nu}q_\nu + E_{\mathrm{D}}\left(q^2\right)\sigma^{\mu\nu}q_\nu\gamma_5 + G_{\mathrm{D}}\left(q^2\right)\left(q^\mu 2m - q^2\gamma^\mu\right)\gamma_5. \qquad (35)$$



$$= \langle \vec{p}', \lambda' \mid J_\mu^{EM}(0) \mid \vec{p}, \lambda \rangle = \bar{u}(\vec{p}', \lambda') \Gamma^\mu(P, q) u(\vec{p}, \lambda)$$

Fig. 3. The one photon interaction with a spin 1/2 fermion which is used to define the electromagnetic structure functions of the Dirac or Majorana neutrinos.

The structure functions for $q^2 \to 0$ correspond to:

$F_{\mathrm{D}}\left(q^2\right) \stackrel{q^2 \to 0}{\longrightarrow} 0$, neutrino charge,

$\frac{1}{2m} F_{\mathrm{D}}\left(q^2\right) + M_{\mathrm{D}}\left(q^2\right) \stackrel{q^2 \to 0}{\longrightarrow} \mu_m$, magnetic moment,

$E_{\mathrm{D}}\left(q^2\right) \stackrel{q^2 \to 0}{\longrightarrow} \mu_e$, electric moment,

and

$G_{\mathrm{D}}\left(q^2\right) \stackrel{q^2 \to 0}{\longrightarrow} T$, anapol moment [28].

For the Majorana particle only one electromagnetic formfactor survives. There are several ways to show it:

— the CPT invariance [22, 29],

— identity of fermions in the final state of the decay, $\gamma \to \nu_{\mathrm{M}} \nu_{\mathrm{M}}$ [25, 30], or

— from the Feynman rules — the effective coupling of Majorana fermion with a neutral vector boson is given by [31]

$$\Gamma_{\mathrm{M}}^\mu = \Gamma_{\mathrm{D}}^\mu + C \Gamma_{\mathrm{D}}^{\mu T} C^{-1}, \tag{36}$$

where

$$C = \begin{pmatrix} -\varepsilon & 0 \\ 0 & \varepsilon \end{pmatrix}.$$

Using any of the above methods we can show that only the anapol formfactor describes the electromagnetic structure of the Majorana neutrino

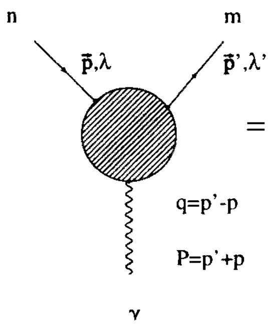
$$\Gamma_{\mathrm{M}}^\mu (P, q) = G_{\mathrm{M}}\left(q^2\right) \gamma^\mu \gamma_5,$$

where

$$G_{\mathrm{M}}\left(q^2\right) = -2 G_{\mathrm{D}}\left(q^2\right) q^2. \tag{37}$$

The electromagnetic structure differentiates the Dirac and Majorana fermions in the obvious way and we can expect to find some visible experimental effect connected with this difference.

Besides the diagonal moments (formfactors) which describe the electromagnetic structure there are also transition moments between different neutrinos (Fig. 4).



$$= \langle m; \vec{p}', \lambda' \mid J_\mu^{EM}(0) \mid n; \vec{p}, \lambda\rangle \equiv \bar{u}_m(\vec{p}', \lambda')\Gamma^\mu_{mn}(P, q)u_n(\vec{p}, \lambda)$$

Fig. 4. The one photon interaction with two different fermions which is used to define the transition electromagnetic moments.

Then the CPT symmetry does not give any restrictions for $\Gamma^\mu_{mn}\ (P, q)$ and all four transition moments exist for the Dirac as well as for the Majorana neutrinos [22, 27].

Up to now we have discussed the V-A interaction of massive neutrinos. Experimentally it is not excluded also that a right-handed current appears in the neutrino interaction with the charged and neutral gauge bosons like in the popular left-right symmetric model [32]. There are also models where scalar particles interact with neutrinos [33]. Generally it is worth remembering that

$$\overline{\Psi}_a \Gamma \Psi_b = \Psi_a^+ \left(\gamma^0 \Gamma\right) \Psi_b, \tag{38}$$

from which we can find that for scalar and tensor interactions $\Gamma = 1, \gamma_5, \sigma^{\mu\nu}, \sigma^{\mu\nu}\gamma_5$ the chirality of Ψ_a and Ψ_b must be opposite but for vector interactions $\Gamma = \gamma^\mu, \gamma^\mu\gamma_5$ the chirality is conserved. It follows very easily from the properties of the projection operators (Eq. (14))

$$P_{\rm L} P_{\rm R} = 0,\ P_{\rm L}^2 = P_{\rm L},\ P_{\rm R}^2 = P_{\rm R}, \tag{39}$$

then *e.g.*

$$\begin{aligned} &P_{\rm L}\gamma^0\gamma^\mu P_{\rm L} \neq 0, \text{ but } P_{\rm R}\gamma^0\gamma^\mu P_{\rm L} = 0, \\ &P_{\rm L}\gamma^0\gamma_5 P_{\rm R} \neq 0, \text{ but } P_{\rm L}\gamma^0\gamma_5 P_{\rm L} = 0. \end{aligned} \tag{40}$$

For relativistic particles, the chirality is almost identical with the helicity, and we can transform the above rule for the helicity of incoming and outgoing particles.

4.3. Differences between the Dirac and Majorana neutrinos for various neutrino interactions

First we consider the situation with vanishing neutrino mass. If there is only a left-handed interaction $\gamma^\mu P_{\rm L}$ (as in the SM), then particles with negative and antiparticles with positive helicities are produced and interact after production.

For Weyl particle: $A(-)$ and $B(+)$.

For Majorana object: $a(-)$ and $a(+)$.

Our interaction will not change $A \longleftrightarrow B$ as well as it will not change $a(-) \longleftrightarrow a(+)$. We know that the A and the B objects interact differently but we have no possibility to check what is the reason for that.

That means that two cases

1. the lepton number is conserved and different helicity in A and B operator has no meaning (so really $A \neq B$ and we have Weyl neutrino), or
2. the interaction depends on the helicity of particles, so $A = B$ and the particles interact differently because they have different helicities (Majorana neutrino)

are physically indistinguishable.

In order to answer the question if $A = B$ or not we must have possibility to compare particle with antiparticle in the same helicity states, so

$$A(-) \text{ with } B(-),$$

or

$$A(+) \text{ with } B(+). \tag{41}$$

But the operators $A(+)$ and $B(-)$ appear in right-handed chiral state $N_{\rm R}$. Such a field will appear if there are right-handed currents or scalar -neutrino interactions. Let us consider a simple example. Beam of massless neutrinos with negative helicity interacts with matter. We assume that there is the left-handed and the right-handed current in the neutrinos interaction with electrons, so

$$L_{\rm CC} = \alpha\bar{l}\gamma^\mu P_{\rm L} N W_\mu^- + \beta\bar{l}\gamma^\mu P_{\rm R} N W_\mu^- + \text{h.c.} \tag{42}$$

Then

• for the Weyl neutrino only electrons will be produced in deep inelastic scattering (Fig. 5).

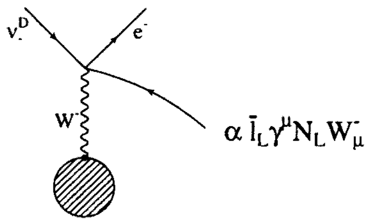


Fig. 5. The deep inelastic scattering of the massless Weyl neutrinos with helicity $h = -1/2$. Even if right-handed current exists, beam of massless neutrinos with negative helicities produces electrons only, contrary to the case of Majorana neutrinos.

• for the Majorana neutrino electrons and positrons will appear (Fig. 6)

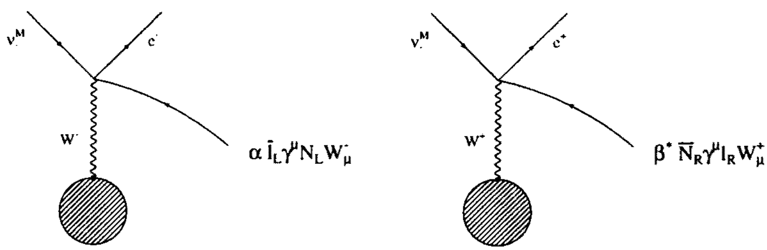


Fig. 6. The interaction of the massless negative helicity Majorana neutrinos with a matter. If right-handed currents exist electrons and positrons are produced. Electrons (positrons) are produced by the left- (right-) handed current.

This simple (but unfortunately unrealistic) example shows us that if the left-handed and the right-handed currents are present we can distinguish the Weyl from the massless Majorana neutrinos (if there are the left and right-handed currents both fields N_L and N_R appear, so we already should talk about Dirac particle). There is also a more realistic example which convinces us that the massless Dirac and Majorana neutrinos are distinguishable (at least in principle) if both left-handed and right-handed currents are present. The magnetic moment calculated in frame of the L-R symmetric model for Dirac neutrino does not vanish even if neutrino mass is equal to zero [22, 27, 34]

$$\mu_{\nu_i} = \left(\frac{\sqrt{2} G_F}{\pi^2} \sin\varphi \cos\varphi m_e \sum_l m_l \mathrm{Re} \left(V_{il}^{+} U_{il} \right) \right) \mu_B \qquad (43)$$

(see references for precise denotation of all the parameters in Eq. (43)). So, with vanishing neutrino mass and only left-handed current interaction, there is no way to distinguish the Dirac from Majorana neutrinos. Such a possibility appears if neutrinos interact also by right-handed currents or if interactions with scalar particle are not proportional to the neutrino mass.

The situation will change if neutrinos have some tiny mass. Then even if there are only left-handed currents, there are ways (at least in principle) to distinguish both types of neutrino. It is so because the left-handed chiral states (Eqs. (18), (19)) are not exactly negative helicity states and, in principle, there is a possibility to compare interaction of particle and antiparticle in the same helicity state:

$$\left(\frac{m}{\sqrt{2E}}\right)|\vec{k},\lambda\ =\ +1/2\rangle_p \quad \text{for the particle}$$

with

$$|\vec{k},\lambda\ =\ +1/2\rangle_a \quad \text{for the antiparticle.} \tag{44}$$

The several sources of neutrinos are known (reactor, accelerator, the sun, supernova) but usually they are produced with relativistic energy $E \sim 0$ (MeV) $\{m_{\nu_e} < 3.5$ eV [35] $m_{\nu_\mu} < 160$ keV [35] $m_{\nu_\tau} < 18.2$ MeV [36] but from Big–Bang Nucleosynthesis $m_{\nu_\tau} < 0.95$ MeV [37] and from matter density of the Universe, $m_{\nu_\tau} < 23$ eV [37]$\}$. Only if we relax the astrophysical and cosmological information the weight factor $(\frac{m_\tau}{2E_\tau})$ for τ neutrino is large enough to have interesting value from the experimental point of view. But, unfortunately, up to now we have not produced the beam of τ neutrinos.

Let us consider the interaction of massive Dirac and Majorana neutrinos with a matter. In both cases the interaction Lagrangian is the same

$$L_{\rm CC} = \frac{g}{\sqrt{2}}\left\{\left(\overline{N}\gamma^\mu P_{\rm L} l\right) W_\mu^+ + \left(\bar{l}\gamma^\mu P_{\rm L} N\right) W_\mu^-\right\}. \tag{45}$$

If the beam of Dirac neutrinos $\nu^{\rm D}$ with helicity h_ν interacts with a matter only electrons with helicities h_e are produced (Fig. 7).

The amplitudes for this process are proportional to the factor $\left[(E_\nu - 2h_\nu k_\nu)\,(E_e - 2h_e k_\nu)\right]^{1/2}$ where $E_\nu\,(E_e)$, $k_\nu\,(k_e)$ are energy and momentum of the neutrinos (electrons), respectively. Positrons will not be produced by the Dirac neutrinos even if neutrinos with positive helicity $\left(\nu_+^{\rm D}\right)$ exist in the beam. Neutrinos in both helicity states will produce electron only, but mainly $\nu_-^{\rm D}$ will do it, $\nu_+^{\rm D}$ produces e$^-$ ($h_e = \pm 1/2$) with the small weight $(m_\nu^2/4E_\nu^2)$.

If the beam of massive Majorana neutrinos interacts with a matter the picture is different. Both electrons and positrons can be produced (Fig. 8).

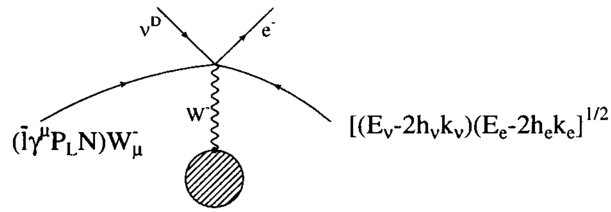


Fig. 7. If only left-handed current exists then the beam of massive Dirac neutrinos will produce electrons. Positrons are not produced even if neutrinos with positive helicity exist in the beam.

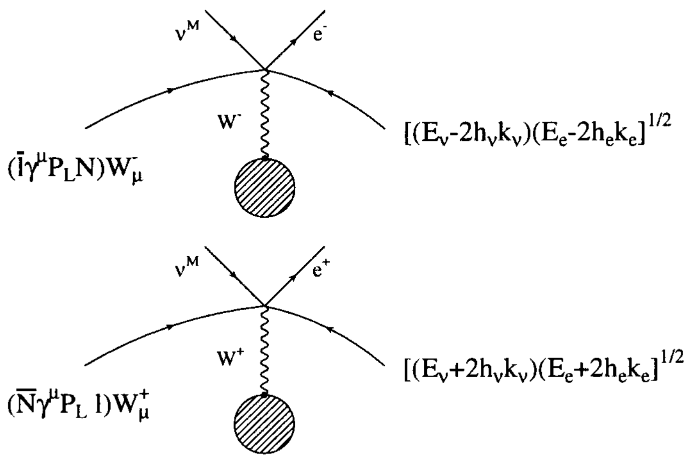


Fig. 8. If a beam of massive Majorana neutrinos interacts with a matter, electrons and positrons can be produced, even if only the left-handed current describes the neutrino interactions. The second diagram which is absent for Dirac neutrinos distinguishes both neutrino characters.

We see that electrons are produced mostly by neutrinos with negative helicity $\nu_-^{\rm M}$, contrary to positrons which are produced mainly by $\nu_+^{\rm M}$. The Dirac and Majorana neutrinos are distinguishable (in principle) if the second diagram for the Majorana neutrino (absent in the Dirac case) gives any contribution to the neutrino interaction with a matter. It happens if the factor $(E_\nu + 2h_\nu k_\nu) \neq 0$, and we can conclude that both neutrinos are distinguishable if $m_\nu \neq 0$ or neutrinos with helicity $h_\nu = +1/2$ exist in the beam (even if $m_\nu = 0$). The second case means that the massless Dirac and Majorana neutrinos are distinguishable in the charged current interaction.

From presented considerations we could get the impression that massive Dirac neutrinos are very easily distinguishable from massive Majorana neutrinos. But it is not the case. In the relativistic beam of neutrinos produced by the chiral left-handed interaction the ν_+ neutrinos occur very seldom.

So, it is very difficult with the tiny neutrino mass to distinguish Dirac from Majorana neutrinos using a process where the charged current dominates the interaction.

What about processes with the neutral currents? There are two things which are worth considering in this context.

(*i*) At first sight the neutral current for the Majorana neutrino looks completely different in comparison with the Dirac neutrino. The Majorana neutrino has no vector current $\overline{\nu}_{\rm M}\gamma^{\mu}\nu_{\rm M} = 0$ [15, 25].

(*ii*) In processes where we usually have Dirac neutrino and antineutrino in Majorana case two identical particles appear. For example in the Z_0 decay we get two identical Majorana neutrinos $Z_0 \to \nu_{\rm M}\nu_{\rm M}$. Here it seems also that it is easy to distinguish both cases, because of symetrization procedure for identical particles.

Let us now consider the first problem. The second one, which was the cause of many mistakes, we will study in the next Chapter.

From the property (21) for Majorana neutrino it follows that

$$\overline{\nu}^{\rm M}(x)\,\gamma^{\mu}\nu^{\rm M}(x) = 0,$$

and we have [25, 38]

$$\left\langle \nu_f^{\rm M}\left|\overline{\nu}^{\rm M}\gamma^{\mu}(1-\gamma_5)\nu^{\rm M}\right|\nu_i^{\rm M}\right\rangle_{|x=0} = -\left\langle \nu_f^{\rm M}\left|\overline{\nu}^{\rm M}\gamma^{\mu}\gamma_5\nu^{\rm M}\right|\nu_i^{\rm M}\right\rangle_{|x=0}. \tag{46}$$

If we decompose the neutrino fields in momentum representation (Table I) we get

$$\begin{aligned}&\langle\nu_f^{\rm M}\left|\overline{\nu}^{\rm M}\gamma^{\mu}(1-\gamma_5)\,\nu^{\rm M}\right|\nu_i^{\rm M}\rangle_{|x=0}\\ &= \overline{u}\left(\vec{k}_f,h_f\right)\gamma^{\mu}\gamma_5\,u\left(\vec{k}_i,h_i\right) + \overline{v}\left(\vec{k}_i,h_i\right)\gamma^{\mu}\gamma_5 v\left(\vec{k}_f,h_f\right),\end{aligned} \tag{47}$$

and from the property

$$v = C\overline{u}^T,\ \overline{v} = -u^TC^{-1}, \tag{48}$$

there is

$$\left\langle \nu_f^{\rm M}\left|\overline{\nu}^{\rm M}\gamma^{\mu}(1-\gamma_5)\,\nu^{\rm M}\right|\nu_i^{\rm M}\right\rangle_{|x=0} = -2\overline{u}_f\gamma^{\mu}\gamma_5 u_i. \tag{49}$$

For Dirac neutrino at first sight the result is different

$$\left\langle \nu_f^{\rm D}\left|\overline{\nu}^{\rm D}\gamma^{\mu}(1-\gamma_5)\,\nu^{\rm D}\right|\nu_i^{\rm D}\right\rangle_{|x=0} = \overline{u}_f\gamma^{\mu}(1-\gamma_5)\,u_i. \tag{50}$$

But we have to our disposal relativistic neutrinos produced by the left-handed current so with precision $\left(\frac{m}{2E}\right)$ the negative helicity state $(h = -1/2)$

is the chiral left-handed state (see Eq. (18)) so for our initial spinors there is

$$P_{\rm L} u_i \simeq u_i + \mathcal{O}\left(\frac{m}{2E}\right), \qquad P_{\rm R} u_i \simeq \mathcal{O}\left(\frac{m}{2E}\right), \tag{51}$$

from which it follows that

$$\gamma^5 u_i = -u_i + \mathcal{O}\left(\frac{m}{2E}\right). \tag{52}$$

If we use this relation in (49) and (50) we see that with precision $\mathcal{O}\left(\frac{m}{2E}\right)$ both matrix elements are equal

$$\left\langle \nu_f^{\rm M} \left| \overline{\nu}^{\rm M} \gamma^\mu \left(1-\gamma_5\right) \nu^{\rm M} \right| \nu_i^{\rm M} \right\rangle = \left\langle \nu_f^{\rm D} \left| \overline{\nu}^{\rm D} \gamma^\mu \left(1-\gamma_5\right) \nu^{\rm D} \right| \nu_i^{\rm D} \right\rangle + \mathcal{O}\left(\frac{m}{2E}\right). \tag{53}$$

For a tiny neutrino mass it was also shown that, if there are only the left-handed weak currents, the electromagnetic structure for the Dirac and Majorana neutrino smoothly becomes indistinguishable [15, 39]. As $\sigma^{\mu\nu}$ and $\sigma^{\mu\nu}\gamma_5$ operators in Eq. (35) change chirality and the spinors in the electromagnetic currents (35) are, with precision $\left(\frac{m}{2E}\right)$, the chiral eigenstates ($h = -1/2 \Longleftrightarrow P_{\rm R} u \cong 0$) the electric and magnetic formfactors must vanish for $m_\nu \to 0$

$$M_{\rm D}\left(q^2\right) \overset{m_\nu \to 0}{\longrightarrow} 0, \qquad E_{\rm D}\left(q^2\right) \overset{m_\nu \to 0}{\longrightarrow} 0. \tag{54}$$

For the other formfactors in (35) we have

$$\begin{aligned} \left\langle \nu_-^{\rm D} \left| J_\mu^{EM} \right| \nu_-^{\rm D} \right\rangle &= \overline{u}_f(-) \left[F_{\rm D}\gamma^\mu - G_{\rm D} q^2 \gamma^\mu \gamma_5\right] u_i(-) \\ &\approx \left(F_{\rm D} + G_{\rm D} q^2\right) \overline{u}_f(-) \gamma^\mu u_i(-). \end{aligned} \tag{55}$$

For the same reasons (only the left-handed current is present) there is no transition for the $\nu_+^{\rm D}$ Dirac states, so

$$0 \cong \left\langle \nu_+^{\rm D} \left| J_\mu^{EM} \right| \nu_+^{\rm D} \right\rangle = \left(F_{\rm D} - G_{\rm D} q^2\right) \overline{u}_f(+) \gamma^\mu u_i(+), \tag{56}$$

and we have

$$F_{\rm D} \approx G_{\rm D} q^2. \tag{57}$$

At the same time for the Majorana neutrinos there is (see Eq. (37))

$$\left\langle \nu_-^{\rm M} \left| J_\mu^{EM} \right| \nu_-^{\rm M} \right\rangle = -2G_{\rm D} q^2 \overline{u}_f(-) \gamma^\mu \gamma^5 u_i(-) = +2G_{\rm D} q^2 \overline{u}_f(-) \gamma^\mu u_i(-). \tag{58}$$

So, if we compare Eqs. (55) and (57) with (58) we see that in the limit $m_\nu \to 0$ both electromagnetic currents go smoothly to the same value

$$\left\langle \nu_-^{\rm D} \left| J_\mu^{EM} \right| \nu_-^{\rm D} \right\rangle \overset{m_\nu \to 0}{\longrightarrow} \left\langle \nu_-^{\rm M} \left| J_\mu^{EM} \right| \nu_-^{\rm M} \right\rangle \overset{m_\nu \to 0}{\longrightarrow} 2G_{\rm D} q^2 \overline{u}_f \gamma^\mu u_i . \tag{59}$$

In the next Chapter we will see that also the fact that Majorana particles are indistinguishable from their antiparticles will not help and, for the left-handed interacting neutrino, differences in all observables for the Dirac and Majorana neutrino smoothly vanish for $m_\nu \to 0$. This statement was formulated in two papers by Kayser and Shrock [15] in 1982 and is known as "Practical Dirac–Majorana Confusion Theorem". Since that time many papers have appeared [40–44] and in the recent time many e-mail texts have become available on the hep-ph list [45–50]. They try to find observables where both neutrinos give the most visible different effects even if their masses are small [40–42, 44]. Some of them try to find effects in frame of extensions of the standard model. Some of them are technically correct. Some of them are not concerned about the practical value of the presented concept [50]. There are also simply wrong concepts [43, 45, 48].

If there are other neutrino interactions (right-handed currents, interaction with scalars) observable differences between the Dirac and Majorana neutrinos could be substantial even for small neutrino mass. But the SM works very well so effects of any of SM extensions must be small at least for presently attainable energies.

5. Review of various processes

5.1. Processes where the differences between Dirac and Majorana neutrinos are not seen

The main neutrino processes which measure their masses do not feel the differences between the Dirac and Majorana neutrinos.

◊ In all processes which give the bounds on the neutrino mass *e.g.* tritium β decay $(H_1^3 \to H_2^3 \to e^- + \bar{\nu}_e)$; pion and tau decays $\pi^+ \to \mu^+ + \nu_\mu$, $\tau^- \to 2\pi^+ 3\pi^- (\pi^0)\nu_\tau$, there is only one neutrino which interacts by the charged current. In these circumstances all differences in measured quantities between the Dirac and Majorana neutrino disappear.

◊ In the case of flavour neutrino oscillations, differences between both types of neutrinos disappear too [51]. It is very easy to see that. Probability for transition $\nu_\alpha \to \nu_\beta$ is given by

$$P\left(\nu_\alpha \to \nu_\beta, t\right) = \left| \sum_{a=1}^{n} U_{\beta a}\, {\rm e}^{-iE_a t}\, U_{\alpha a}^* \right|^2 . \tag{60}$$

The Dirac and Majorana neutrinos give unique signals through the different structures of the mixing matrices U. There are more CP violating phases for the Majorana neutrinos. But the formula (60) does not feel these additional phases; they can be eliminated and remaining number of the CP phases is the same like for the Dirac neutrino.

5.2. Terrestrial experiments

There are many physical observables which feel the difference between Dirac and Majorana neutrinos. The problem is of course with the size of these effects.

$\diamondsuit$ In general, any process which violates the total lepton number (as for example $e^-e^- \to W^-W^-$, $K^- \to \pi^+\mu^-\mu^-$, $K^- \to \pi^+e^-e^-$, $\nu \to \overline{\nu}$ oscillation, ...) will indicate that neutrinos have the Majorana character. Also the neutrinoless double β decay violates the total lepton number. We will comment on it later.

$\diamondsuit$ There are also processes which do not violate the total lepton number and occur for Dirac as well as for Majorana neutrinos. But physical observables (cross sections, angular distributions, energy distributions, decay widths, polarizations) have specific properties which distinguish both types of neutrino.

For example the angular distributions for the processes like $\nu e^- \to \nu e^-$, $\nu N \to \nu N$, $e^+e^- \to \nu\nu$ look different for the Dirac and Majorana neutrinos [15].

If we could observe, for example, the photon polarization in the process of neutrino decay $\nu_i \to \nu_j + \gamma$ then the ratio of the left-handed ($M_{\rm L}$) to right-handed ($M_{\rm R}$) photon polarization distinguishes both types of neutrinos [22]

$$\frac{M_{\rm L}\left(\nu_i \to \nu_j + \gamma\right)}{M_{\rm R}\left(\nu_i \to \nu_j + \gamma\right)} = \begin{cases} \left(\frac{m_{\nu_j}}{m_{\nu_i}}\right)^2 & \text{for Dirac neutrinos,} \\ 1 & \text{for Majorana neutrinos.} \end{cases} \tag{61}$$

There are many other processes where the difference can be written (but only written not observed). Now we consider two examples which were the places of wrong interpretation in the past [45, 48].

$\star$ Let us exam the scattering process

$$\nu_\mu + e^- \to \nu_\mu + e^- \tag{62}$$

which is measured experimentally [52]. We assume that a beam of neutrinos is not a pure negative helicity state ($h = -1/2$), there is a mixture of ($h = +1/2$) and the density matrix in the helicity basis is

$$\rho = \begin{pmatrix} \varepsilon & 0 \\ 0 & 1-\varepsilon \end{pmatrix}, \tag{63}$$

where $0 \leq \varepsilon << 1$.

To be general, we take the coupling of neutrinos and electrons with the neutral boson Z_0 in the form

$$\frac{g}{2\cos\theta}\left[\overline{\nu}\gamma^{\mu}\left(A_{\rm L}^{\nu}P_{\rm L} + A_{\rm R}^{\nu}P_{\rm R}\right)\nu + \overline{e}\gamma^{\mu}\left(A_{\rm L}^{e}P_{\rm L} + A_{\rm R}^{e}P_{\rm R}\right)e\right]Z_{\mu}. \tag{64}$$

Let us define

$$y = \frac{E_e^{\rm LAB}}{E_\nu^{\rm LAB}} = \frac{1}{2}\left(1 - \cos\theta_{\rm CM}\right), \tag{65}$$

where $E_{e(\nu)}^{\rm LAB}$ is energy of outgoing electron (incoming neutrino) in the LAB system, $\theta_{\rm CM}$ is the CM scattering angle. Then we can calculate the electron LAB energy distribution ($m_\nu \approx 0$, $m_e \approx 0$)

$$\begin{aligned} \frac{d\sigma}{dy} &= \frac{G_F^2 \rho s}{4\pi}\left\{(A_{\rm R})^2\left[(A_{\rm R}^e)^2 + (A_{\rm L}^e)^2(1-y)^2\right]\varepsilon \right.\\ &+ \left.(A_{\rm L})^2\left[(A_{\rm L}^e)^2 + (A_{\rm R}^e)^2(1-y)^2\right](1-\varepsilon)\right\}, \end{aligned} \tag{66}$$

where

$$A_{\rm R} = \begin{cases} A_{\rm R}^{\nu} & \text{for Dirac,} \\ A_{\rm R}^{\nu} - A_{\rm L}^{\nu} & \text{for Majorana,} \end{cases}$$

and

$$A_{\rm L} = \begin{cases} A_{\rm L}^{\nu} & \text{for Dirac,} \\ A_{\rm L}^{\nu} - A_{\rm R}^{\nu} & \text{for Majorana.} \end{cases} \tag{67}$$

Such distribution is measured by CHARM collaboration [52] and the result agrees with the SM $\left(A_{\rm L}^{l} = -1 + 2\sin^2\theta_W\ ,\ A_{\rm R}^{l} = -1 + 2\sin^2\theta_W,\ A_{\rm L}^{\nu} = 1,\ A_{\rm R}^{\nu} = 0,\ \varepsilon = 0\right)$. But let us assume that we have better data. Do we have any chance to distinguish (at least in principle) the Dirac from Majorana neutrino from the energy distribution (66)? The answer depends on the polarization of initial neutrino (ε) and the existence of the right-handed currents ($A_{\rm R}^{\nu}$).

- If all initial ν_μ neutrinos are in the pure state ($\varepsilon = 0$) and the interaction is pure left-handed ($A_{\rm R}^{\nu} = 0$) then $\frac{d\sigma^{\rm D}}{dy} = \frac{d\sigma^{\rm M}}{dy}$. This situation we have in the SM.
- If $\varepsilon = 0$ but the right-handed current exists, then there is a difference in normalization of both cross sections

$$\frac{d\sigma^{\rm M}}{dy} = \left(1 - \frac{A^{\nu}_{\rm R}}{A^{\nu}_{\rm L}}\right)^2 \frac{d\sigma^{\rm D}}{dy},$$

and the same is true for the total cross section so in principle the Dirac and Majorana neutrinos are distinguishable.

• If $\varepsilon > 0$ but there are no the right-handed currents ($A^{\nu}_{\rm R} = 0$) both neutrino types are in principle distinguishable. As $(A^{e}_{\rm L})^2 \neq (A^{e}_{\rm R})^2$ the energy distribution is different for the Dirac and Majorana neutrinos.

• The best situation is for $\varepsilon > 0$ and $A^{\nu}_{\rm R} \neq 0$, there are two factors which change the angular distribution.

With present experimental precision there is no chance to measure such details of the energy distribution. The data agree with the SM ($\varepsilon = 0$, $A^{\nu}_{\rm R} = 0$) very well, and only the products of the neutrino and electron couplings are measured

$$(A^{\nu}_{\rm L} A^{e}_{\rm L})^2 \quad \text{and} \quad (A^{\nu}_{\rm L} A^{e}_{\rm R})^2 . \tag{68}$$

Even if we know from other experiments the electron couplings, from neutrino energy distribution (66) we can find only the sum of the vector g^{ν}_{V} and the axial vector g^{ν}_{A} couplings $A^{\nu}_{\rm L} = g^{\nu}_{V} + g^{\nu}_{A}$ .We can say nothing about g^{ν}_{V} and g^{ν}_{A} separately [46, 47]. Particularly we cannot conclude that $g^{\nu}_{V} \neq 0$ and because of this muon neutrino is a Dirac particle as it was wrongly suggested in [45].

⋆ The other interesting problem arises for production of two neutrinos in the e^+e^- collision. If the neutrinos are Dirac particles, neutrino and antineutrino are produced, for Majorana neutrino two indistinguishable particles appear in the final state. At first sight the angular distribution of two final neutrinos in the CM frame should look completely different in both cases. For two identical particles in the final state ($\nu\nu$) the angular distribution must be forward-backward symmetric, for two different particles ($\nu\overline{\nu}$) there are no special reasons to have this symmetry. Two (three) Feynman diagrams give contribution to the process $e^+e^- \to \nu\overline{\nu}$ ($e^+e^- \to \nu\nu$) in the lowest order [53]. Let us specify the momenta and helicities of the particles

$$e^-(p,\sigma) + e^+\left(\overline{p},\overline{\sigma}\right) \to \nu(k,\lambda) + \overline{\nu}\left(\overline{k},\overline{\lambda}\right), \tag{69}$$

and denote $\Delta\sigma = \sigma - \overline{\sigma}$, $\Delta\lambda = \lambda - \overline{\lambda}$.

In what follows we use the same denotation for the Z-leptons couplings as in (Eq. 64) but the charged current interaction we take in the form

$$L_{\rm CC} = \frac{g}{2\sqrt{2}} B_{\rm L} \overline{\nu}\gamma^{\mu}(1-\gamma_5)\, l W^{+}_{\mu} + {\rm h.c.} \tag{70}$$

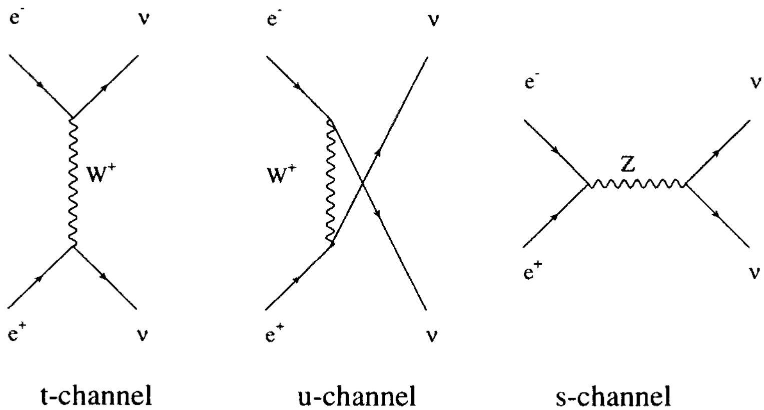


Fig. 9. Three Feynman diagrams which contribute to the $e^-e^+ \to \nu\nu$ process for two identical Majorana neutrinos. Only two diagrams, these from t and s channels, give contribution to the Dirac neutrinos production $e^-e^+ \to \nu\bar{\nu}$ process.

If we neglect the lepton masses then only two (four) helicity amplitudes do not vanish for the Dirac (Majorana) neutrinos [53]. They are (we should remember to divide the Majorana amplitude by $\sqrt{2}$ for two identical particles in the final states; θ is the CM scattering angle)

$$\begin{aligned} M_{\rm M}\left(\Delta\sigma = 1,\, \Delta\lambda = \pm 1\right) &= \pm\tfrac{1}{2} f_{\rm R}\, {\rm e}^{i\varphi}\left(1 \pm \cos\theta\right), \\ M_{\rm M}\left(\Delta\sigma = -1,\, \Delta\lambda = \pm 1\right) &= \pm\tfrac{1}{2} g_{\rm L}\left(\mp\cos\theta\right) {\rm e}^{-i\varphi}\left(1 \mp \cos\theta\right) \end{aligned} \qquad (71)$$

for the Majorana neutrinos, and

$$M_{\rm D}\left(\Delta\sigma = \pm 1,\, \Delta\lambda = -1\right) = \sqrt{2} M_{\rm M}\left(\Delta\sigma = \pm 1,\, \Delta\lambda = -1\right) \qquad (72)$$

for the Dirac neutrinos ($\Delta\lambda = +1$ neutrinos are sterile in this case). All other amplitudes are equal to zero.

The $f_{\rm R}$, $g_{\rm L}$ parameters are defined by

$$f_{\rm R} = \sqrt{2} s \left(\frac{e}{2\sin\theta_W \cos\theta_W}\right)^2 \frac{A_{\rm R}^e A_{\rm L}^\nu}{s - M_W^2 - i\Gamma_W M_W}, \qquad (73)$$

and

$$\begin{aligned} g_{\rm L}\left(\cos\theta\right) &= \sqrt{2} s \left\{ \left(\frac{e}{\sqrt{2}\sin\theta_W}\right)^2 \frac{|B_{\rm L}|^2}{\frac{s}{2} s\cos\theta - \frac{s}{2} + M_W^2} \right. \\ &+ \left. \left(\frac{e}{2\sin\theta_W\cos\theta_W}\right)^2 \frac{A_{\rm L}^e A_{\rm R}^\nu}{s - M_W^2 - i\Gamma_i M_W} \right\}. \end{aligned} \qquad (74)$$

The cross section is calculated from the formula

$$\frac{d\sigma(\Delta\sigma,\ \Delta\lambda)}{d\cos\theta} = \frac{1}{32\pi s}\left|M\,(\Delta\sigma,\ \Delta\lambda)\right|^2, \tag{75}$$

for the polarized particles, and

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{128\pi s}\sum_{\Delta\sigma,\Delta\lambda}\left|M\,(\Delta\sigma,\ \Delta\lambda)\right|^2,$$

in the unpolarized case.

As the t and u amplitudes describe the scattering of Majorana neutrino, the unpolarized angular distribution is symmetric and completely differs from the Dirac neutrino distribution which is asymmetric. Does it mean that we have "very easy" way of distinguishing the Dirac from Majorana neutrino [48]? Of course not [49], even if we could measure the angular distribution for the outgoing neutrinos. The point is that all detectors which we have to our disposal are not able to distinguish the nature of particles from their helicities. Detection of a particle in direction (θ,φ) and an antiparticle in direction $(\pi-\theta,\ \pi+\varphi)$ in the CM frame is technically indistinguishable from the situation that one particle with helicity $h=-1/2$ travels to the solid angle (θ,φ) and the other one to $(\pi-\theta,\ \pi+\varphi)$.

As an example let us calculate the angular distribution if neutrinos are Dirac particles (for simplicity for polarized electron and positron, $\Delta\sigma=-1$). Only one amplitude with $\Delta\lambda=-1$ is necessary, and

$$\begin{aligned} P_{\rm D}\,(\Delta\sigma=-1,\ \Delta\lambda=-1;\theta,\varphi) &= \mid M_{\rm D}\,(\Delta\sigma=-1,\ \Delta\lambda=-1)\mid^2 \\ &= \mid g_{\rm L}(\cos\theta)\mid^2 \frac{(1+\cos\theta)^2}{2}. \end{aligned} \tag{76}$$

In the Majorana case, the final particles are identical, and we have to add incoherently two experimentally indistinguishable situations

$$\begin{aligned} &P_{\rm M}\,(\Delta\sigma=-1,\ \Delta\lambda=-1) = \mid M_{\rm M}\,(\Delta\sigma=-1,\ \Delta\lambda=-1;\theta,\varphi)\mid^2 \\ &+\mid M_{\rm M}\,(\Delta\sigma=-1,\ \Delta\lambda=+1;\pi-\theta,\ \pi+\varphi)\mid^2 \\ &= 2\mid M_{\rm M}\,(\Delta\sigma=-1,\ \Delta\lambda=-1;\theta,\varphi)\mid^2 = \mid g_{\rm L}(\cos\theta)\mid^2\frac{(1+\cos\theta)^2}{2}, \end{aligned} \tag{77}$$

where we use the relation

$$M_{\rm M}\,(\Delta\sigma,\ \Delta\lambda;\theta,\varphi) = M_{\rm M}\,(\Delta\sigma,-\Delta\lambda;\pi-\theta,\ \pi+\varphi), \tag{78}$$

which follows from the identity of neutrinos. We see that really two distributions are identical. The same is true for other electron polarization (it simply follows from the relations (72) and (78)). So the total cross section for both electron polarizations are equal

$$\sigma_{\rm D}\,(\Delta\sigma,\,\Delta\lambda=-1)=\sigma_{\rm M}\,(\Delta\sigma,\,\Delta\lambda=-1)\,,$$

where the angular distributions for the Dirac (76) and the Majorana (77) neutrinos are integrated over full solid angle.

In the Majorana case we can also calculate

$$P_{\rm M}\,(\Delta\sigma,\,\Delta\lambda=+1;\theta,\varphi)=2\,|M_{\rm M}\,(\Delta\sigma,\,\Delta\lambda=+1;\theta,\varphi)|^2 \tag{79}$$

which does not exist for Dirac neutrino. But from (72) and (78) it follows that the distribution (79) is equal

$$2\,|M_{\rm M}\,(\Delta\sigma,\,\Delta\lambda=-1;\pi-\theta,\ \pi+\varphi)|^2=P_{\rm D}\,(\Delta\sigma,\,\Delta\lambda=-1;\pi-\theta,\ \pi+\varphi) \tag{80}$$

and corresponds to the case of the neutrino which is flying in the direction $\pi-\theta$, $\pi+\varphi$ and the antineutrino has opposite momentum. So, for the integrated cross section there is

$$\sigma_{\rm M}\,(\Delta\sigma,\,\Delta\lambda=+1)=\sigma_{\rm D}\,(\Delta\sigma,\,\Delta\lambda=-1)\,. \tag{81}$$

For the unpolarized cross section we have to sum (average) over final (initial) particle polarizations and we get

$$\frac{d\sigma^{\rm D}}{d\cos\theta}=\frac{1}{4}\frac{1}{32\pi s}\sum_{\Delta\sigma}P_{\rm D}\,(\Delta\sigma,\ \Delta\lambda=-1;\theta)\,, \tag{82}$$

and for the Majorana neutrino where also the $\Delta\lambda=+1$ final neutrino polarization exists

$$\frac{d\sigma^{\rm M}}{d\cos\theta}=\frac{d\sigma^{\rm D}}{d\cos\theta}\,(\theta)+\frac{d\sigma^{\rm D}}{d\cos\theta}\,(\pi-\theta)\,, \tag{83}$$

which is also obvious as the Majorana case is equivalent to $\nu\,(h=-1/2)+\bar{\nu}\ (h=+1/2)$ for the Dirac neutrinos. But for the total, unpolarized cross section we once more recover the equivalence between both types of neutrinos. In order not to take into account the same spin configuration two times, we have to integrate the Majorana cross section only over half of the solid angle and we have

$$\sigma_{\rm tot}\,(M)=\int\limits_{-1}^{0}d\cos\theta\frac{d\sigma^{\rm M}}{d\cos\theta}=\int\limits_{-1}^{+1}d\cos\theta\frac{d\sigma^{\rm D}}{d\cos\theta}=\sigma_{\rm tot}\,(D)\,. \tag{84}$$

We see that the Practical Dirac–Majorana Confusion Theorem still works.

The main problem in distinguishing Dirac from Majorana neutrino is the lack of neutrinos with positive helicity. There are two ways, discussed in literature, how to obtain the neutrino with reversed helicity

(1) to overtake it [54]

(2) to reverse the spin of the neutrino in an external magnetic field [22].

Let us assume that we have the beam of π^+ with high energy (*e.g.* 600 GeV from Tevatron) in the laboratory system. In the rest frame of π^+ the decay $\pi^+ \to \mu^+ + \nu_\mu$ looks like in Fig. 10. All neutrinos with momentum in the backward direction (with respect to the π^+ beam) will have forward momentum in the LAB frame. As a neutrino spin will not turn after Lorentz transformation, this means that helicity will change from $h_\nu = -1/2$ in the CM system to $h_\nu = +1/2$ in the LAB system.

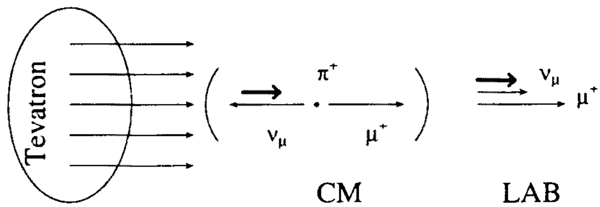


Fig. 10. The CM and the LAB frames for decaying pions produced in the Tevatron. The negative helicity neutrino in the CM frame will change momentum after Lorentz transformation to the LAB frame so neutrinos with positive helicities appear in this frame.

This phenomenon is possible only if neutrino is massive and its efficiency depends on the neutrino mass. It was shown that $m_{\nu_\mu} \approx 10$ keV will not be enough if all practical limitations are taken into account [54]. But there is also some chance that the muon neutrino will oscillate to the tau neutrino which can be much heavier. Now the result depends on the oscillation probability. It was shown that for $U_{\mu\tau} \approx 0.03$ and $m_{\nu_\tau} > 1$ MeV the final results do not look like a completely wild scheme [55].

The other possibility considered in literature is to reverse the neutrino helicity by the influence of an external magnetic field [22]. The problem is that we need large neutrino magnetic moments and large magnetic fields to obtain visible effect. The SM predicts that for the Dirac neutrino with mass m_ν the magnetic moment is equal [56]

$$\mu_\nu = \frac{3eG_F m_\nu}{8\pi^2\sqrt{2}} \approx 3 \times 10^{-19} \left(\frac{m_\nu}{1eV}\right) \mu_{\rm Bohr}. \qquad (85)$$

Then a magnetic field which is needed for feasible experiment is too large even for astronomical scale. But present limits on neutrino magnetic moments are not so small ($\mu_\nu < 1.8 \times 10^{-10}\mu_B$, for reactor antineutrino [57]; $\mu_\nu < 0.3 \times 10^{-11}\mu_B$, from stellar cooling [58]) and stellar magnetic field could have a chance to reverse neutrino helicity.

There is only one terrestrial experimental approach which currently promises to state whether neutrinos are Majorana or Dirac particles — it is the neutrinoless double β decay,$(\beta\beta)_{0\nu}$[4]. The quantity measured in the $(\beta\beta)_{0\nu}$ is the average of neutrino masses [59]

$$\langle m_\nu \rangle = \sum_n U_{en}^2 m_n. \tag{86}$$

If we could find experimentally that $\langle m_\nu \rangle > \kappa$ then we would obtain two important items of information, namely that (i) neutrinos are Majorana type and (ii) at least for one neutrino $m_\nu > \kappa$.

Experiments which try to find the amplitudes for the $(\beta\beta)_{0\nu}$ decay

$$(Z, A) \to (Z+2, A) + 2e^- \tag{87}$$

are presently conducted in several places, using different even-even nuclei. Up to now the best limit on $\langle m_\nu \rangle$ has been obtained by the Heidelberg–Moscow collaboration from observing the half- time of ^{76}Ge [60],

$$T_{1/2}^{0\nu} > 2.0 \times 10^{25} \text{ year (68\% of C.L.)},$$

which gives

$$\langle m_\nu \rangle < (0.44 - 1.1)\,\text{eV (68\% of C.L.) [61]}$$

depending on the method in which the nuclear matrix element is calculated. Future experiments which are planed will move the bounds [61]:

Heidelberg–Moscow coll.

$$^{76}\text{Ge} \to 5 \times 10^{25} \text{ year} \;\Rightarrow\; \langle m_\nu \rangle \sim 0.2 \text{ eV} \tag{88}$$

NEMO coll.

$$^{100}\text{Mo} \to 10^{25} \text{ year} \;\Rightarrow\; \langle m_\nu \rangle \sim 0.16 \text{ eV}. \tag{89}$$

5.3. *"Half-terrestrial" experiments*

Experiments which use neutrinos from non terrestrial sources (the sun, supernova) were also considered. One such a possibility in which the solar neutrinos are used was recently proposed [62] . If electron neutrino magnetic moment is in the range of present astrophysical limit $m_\nu < 3 \times 10^{-12}\mu_B$ there is some chance that strong magnetic fields in the sun will reverse the neutrino helicity. In [62] the energy distribution $\frac{d\sigma}{dT}$ for a final electron in the process

$$\nu + e^- \to \nu + e^- \tag{90}$$

was calculated.As the ν_e (+) state is (is not) sterile for Dirac (Majorana) neutrinos the distribution $\frac{d\sigma}{dT}$ differs very much for both kinds of neutrinos (the size of the effect depends on the neutrino density matrix). The electron energy distribution is different for the Dirac and Majorana neutrino mostly for the low electron energy. So experiments which can measure the low energy of outgoing electrons are welcome. The HELLAZ experiment (with the threshold energy 100 keV) seems to be a good place [62].

It is also possible to find in literature the arguments which compare neutrino emitted from the supernova with the sun neutrinos [63]. The argument is as follows. The observation of neutrinos from the SN 1987 A explosion suggests that the diagonal and the transition moments for Dirac ν_e neutrino should be small $\mu_\nu < 10^{-13}\mu_B$. Such magnetic moments are too small to explain hypothetical anticorrelation in the sun neutrino observation (visible in Davis experiment but not in Kamiokande). For explanation of such anticorrelations we need larger neutrino magnetic moment. So, if neutrinos are of the Majorana type, both observations could agree.

6. Conclusions

The problem whether the neutrino is identical to its own antiparticle is the central problem in neutrino physics and very important in particle physics too.

If the neutrinos have no masses then

• it was proved (Pauli–Gursey transformation) that without interactions each of Weyl neutrinos is equivalent with massless Majorana neutrinos,

• if neutrinos interact with gauge bosons through left-handed currents (like in the SM) then only one Weyl neutrino appears in the theory and again we can never distinguish it from the massless Majorana neutrino,

• if, besides the left-handed also the right-handed currents describe the gauge bosons –neutrino interaction, or massless neutrinos interact with scalar bosons then Weyl and Majorana neutrinos interact differently with the matter. Even if in such case there exist both Weyl neutrinos (equivalent with one massless Dirac neutrino), the beam of any kind of Weyl neutrinos behaves in a different way than the beam of Majorana neutrinos.

If the neutrinos are massive particles, by definition Dirac neutrino ($\nu \neq \bar{\nu}$) differs from Majorana neutrino ($\nu = \bar{\nu}$). They interact with the matter in different way even if only SM governs the neutrino interaction. But unfortunately in such a case (only left-handed current) all differences in physical observables disappear in a smooth way with vanishing neutrino mass (Kayser–Shrock theorem).

As

- the SM works very well and any interaction signals beyond the SM have not been discovered,
- the neutrino interactions described by the SM are checked experimentally and are very weak ($\sigma_{\nu e} \approx 10^{-44}\,\mathrm{cm}^2$),
- the masses of light neutrinos are much smaller than the masses of charged fermions,

it is extremely difficult to find clear experimental evidence which informs us about the nature of existing light neutrinos.

There is only one terrestrial, experimental test that can reveal it, which is the neutrinoless double beta-decay. Several experimental groups, using different even-even nuclei, placed the upper limit on the lifetime of the $(\beta\beta)_{0\nu}$. Up to now, nobody has found the evidence for the $(\beta\beta)_{0\nu}$ decay. However there are plans to improve the upper limit for the $(\beta\beta)_{0\nu}$ by almost one order of magnitude and we can still have hope that the problem if neutrinos are of the Dirac or the Majorana type can be solved in the nearby future.

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