

# CONSTRAINED DIFFERENTIAL RENORMALIZATION\*

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We review the method of differential renormalization, paying special attention to a new constrained version for symmetric theories.

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## 1. Introduction

Popular regularization and renormalization methods work in momentum space. Typically, the divergences which appear in loop integrals at large internal momenta (ultraviolet divergences) are first regulated — *i.e.*, the integrals are modified so that they are finite, but diverge in the limit of no regulator — and then subtracted by adding the necessary counterterms. For example, in dimensional regularization [1] the regulated integrals are defined in  $n$  dimensions by analytical continuation, with  $n$  an arbitrary complex number. Eventually, the poles appearing at  $n = 4$  are cancelled by appropriate counterterms and a finite (renormalized) result is obtained for  $n \rightarrow 4$ . Renormalization without intermediate regularization is also possible, as in the BPHZ method [2], where the first terms of the Taylor expansion in external momenta of the integrand are subtracted off before integrating. Although momentum space is more natural for calculations of scattering amplitudes with fixed external momenta, nothing prevents us from working in coordinate space and, if required, perform a Fourier transform at the end. In coordinate space, ultraviolet divergences correspond to a singular behaviour at short distances.

In Ref. [3] a method of renormalization in coordinate space was proposed: differential renormalization (DR). It is based on the observation that

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primitively divergent Feynman graphs are well defined in coordinate space for non-coincident points, but too singular at coincident points to allow for Fourier transform. In other words, the corresponding expressions are not well-behaved distributions. The idea of DR is to substitute the singular expressions by derivatives of well-behaved distributions, in such a way that the former ('bare') and latter ('renormalized') expressions are equal almost everywhere. These derivatives are prescribed to act formally by parts in integrals such as Fourier transforms. In this way, finite Green functions are obtained, without the need of intermediate regularization. DR acts directly on bare Feynman graphs and does not introduce explicit counterterms. The

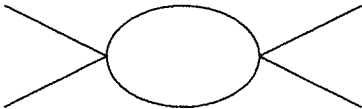


Fig. 1. One-loop diagram contributing to the four-point vertex in  $\Phi^4$ .

procedure is best illustrated in terms of one example: the one-loop four-point bubble graph of massless  $\Phi^4$  (Fig. 1). We work in euclidean space, which leads to simpler functions. The massless propagator in position space is  $\Delta(x - y) = \frac{1}{4\pi^2} \frac{1}{(x-y)^2}$  and the vertex,  $-\lambda\delta(x_1 - x_4)\delta(x_2 - x_4)\delta(x_3 - x_4)$ . The bare expression for the amputated graph is

$$\Gamma(x_1, x_2, x_3, x_4) = \frac{\lambda^2}{2} \frac{1}{16\pi^4} \delta(x_1 - x_2)\delta(x_3 - x_4) \frac{1}{(x_1 - x_3)^4} + 2 \text{ perms.} \tag{1}$$

This involves the singular function  $\frac{1}{x^4}$ , which has a logarithmically divergent Fourier transform. To renormalize it with DR, one must solve a differential equation and find  $f(x^2)$  such that

$$\frac{1}{x^4} = \square f(x^2) \tag{2}$$

for  $x \neq 0$ . Actually, one derivative would be enough in this case, but one uses the D’Alambertian  $\square = \partial_\mu \partial_\mu$  to preserve manifest euclidean invariance. The solution of Eq. (2) is

$$f(x^2) = -\frac{1}{4} \frac{\log x^2 M^2}{x^2}, \tag{3}$$

where  $M$  is an arbitrary constant with dimensions of mass, required for dimensional reasons, and we have omitted a possible but irrelevant additive constant. Although we shall not discuss it here, it is worth mentioning that

the constant  $M$  plays a central role in DR: the renormalized amplitudes satisfy renormalization group equations, with  $M$  the renormalization scale. The renormalized expression of the singular function reads

$$\left[\frac{1}{x^4}\right]^R = -\frac{1}{4}\square\frac{\log x^2 M^2}{x^2}. \quad (4)$$

With the formal integration by parts rule, this is a tempered distribution which admits a finite Fourier transform:

$$\begin{aligned} \int d^4x e^{ip\cdot x} \left[\frac{1}{x^4}\right]^R &= -p^2 \int d^4x e^{ip\cdot x} \left(-\frac{1}{4}\right) \frac{\log x^2 M^2}{x^2} \\ &= -\pi^2 \log \frac{p^2}{M^2}, \end{aligned} \quad (5)$$

where  $\bar{M} = 2M/\gamma_E$ , and  $\gamma_E = 1.781, \dots$  is Euler's constant. Substituting Eq. (4) in Eq. (1), the renormalized vertex graph is obtained:

$$\begin{aligned} \Gamma^R(x_1, x_2, x_3, x_4) &= -\frac{\lambda^2}{128\pi^4} \delta(x_1 - x_2) \delta(x_3 - x_4) \square \frac{\log(x_1 - x_3)^2 M^2}{(x_1 - x_3)^2} \\ &\quad + 2 \text{ perms.} \end{aligned} \quad (6)$$

The renormalized expression in momentum space follows from Eq. (5):

$$\begin{aligned} \hat{\Gamma}^R(p_1, p_2, p_3, p_4) &= -\frac{\lambda^2}{32\pi^2} \log \left[ \frac{(q_1 + q_2)^2 (q_2 + q_3)^2 (q_1 + q_3)^2}{M^6} \right] \\ &\quad \times (2\pi)^4 \delta \left( \sum_i q_i \right). \end{aligned} \quad (7)$$

DR has been successfully applied in different contexts: the Wess–Zumino model [4], lower-dimensional [5] and non-abelian gauge theories [6], two-loop QED [7], a chiral model [8], a non-relativistic anyon model [9], curved space-time and finite temperature [10], the calculation of  $(g - 2)_l$  in supergravity [11], Chern–Simons theories [12] and non-perturbative calculations in supersymmetric gauge theories [13]. Other formal aspects of the method have been developed in Refs [14–17] and different versions of DR can be found in [18, 19].

When symmetries are an issue, it is important that the renormalization program preserves the corresponding Ward identities. In general, even when the regularization procedure breaks some relevant symmetry, one can still recover it by the addition of finite counterterms<sup>1</sup>. In practical calculations

<sup>1</sup> The exception is called an anomaly: the quantum renormalized theory does not have a symmetry of the classical theory.

and formal proofs to all orders, it is nevertheless more convenient to have a method that directly preserves the Ward identities. The great success of dimensional regularization is mainly due to the fact that it automatically respects gauge invariance. It is known, however, that it has problems in dimension-dependent theories like chiral and supersymmetric theories. Its variant dimensional reduction [20] is usually employed in these cases, although inconsistencies may arise at high orders [21]. DR does not change the space-time dimension and it was expected to become a renormalization procedure respecting gauge and chiral symmetry. In its original version, however, this is not automatic. The ambiguities inherent to the manipulation of singular functions are taken care of by introducing arbitrary renormalization scales for different diagrams. Different choices of the renormalization scales give rise to different renormalization schemes and only a subset of these schemes corresponds to a symmetric renormalized theory. Hence, the scales must be fixed to enforce the relevant Ward identities. A change of renormalization scales is equivalent to the addition of finite counterterms, so the situation does not differ much from the one with symmetry-breaking regulators.

In Refs [11, 22] a procedure of DR was proposed which fixes all the manipulations and only introduces the necessary renormalization group scale. This *constrained* DR has been explicitly shown to respect the one-loop Ward identities of abelian gauge symmetry [22]<sup>2</sup> and to preserve supersymmetry in a supergravity calculation [11].

In the following, we first describe the method of constrained DR and then use it to calculate in detail the electron self-energy and the vertex correction in QED, as an illustration. We also derive the corresponding momentum space expressions and check that the corresponding Ward identity is automatically fulfilled.

## 2. Constrained differential renormalization

In this section we briefly describe the constrained procedure of DR at one loop introduced in Ref. [22]. The idea is to find a consistent way of performing the manipulations of singular expressions carried out in the process of renormalization. It turns out that a small set of formal rules is sufficient to completely fix the renormalization scales (except one, associated with the renormalization group invariance). Furthermore, the resulting renormalized amplitudes were explicitly shown in Refs [11, 22] to satisfy the one-loop Ward identities of abelian gauge symmetry and to render a vanishing value for the magnetic moment of a charged lepton in supergravity (which is the required

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<sup>2</sup> In Ref. [23] Smirnov presented an abelian gauge invariant method within his version of DR.

value if supersymmetry is respected [24]).

The set of rules contains the two basic DR rules: the use of DR identities like Eq. (4) (always with the same renormalization scale!) (Rule 1) and the formal integration by parts prescription (Rule 2). In addition, we need another two rules. One is (Rule 3):

$$[F(x, x_1, \dots, x_n) \delta(x - y)]^R = [F(x, x_1, \dots, x_n)]^R \delta(x - y) , \quad (8)$$

where  $F$  is an arbitrary function. The other one requires the general validity of the propagator equation (Rule 4):

$$F(x, x_1, \dots, x_n) \square \Delta(x) = F(x, x_1, \dots, x_n) (-\delta(x)) , \quad (9)$$

where  $\Delta(x) = \frac{1}{4\pi^2} \frac{1}{x^2}$  is the massless propagator (the massive case is analogous). This is a valid mathematical identity between tempered distributions if  $F$  is well-behaved enough. This rule formally extends its range of applicability to an arbitrary function. The main point of the constrained method is to require consistency of renormalization with these rules. Such requirement fixes all the ambiguities in DR at least to one loop. Let us explain how the rules are used in practice with some simple examples. The results will be used in the next section. We first introduce some convenient notation: we define the *bubble* and *triangular* basic functions as

$$B[\mathcal{O}] \equiv \Delta(x) \mathcal{O} \Delta(x) , \quad (10)$$

$$T[\mathcal{O}] \equiv \Delta(x) \Delta(y) \mathcal{O}^x \Delta(x - y) , \quad (11)$$

where  $\mathcal{O}$  is a differential operator. The significance of this kind of functions is that any one-loop bubble or triangular Feynman diagram can be expressed in terms of them (and their derivatives) using only algebraic manipulations and the Leibnitz rule for derivatives. Hence, the problem of renormalization reduces (at this order) to finding the renormalized expressions of the singular basic functions. Note that  $B[\mathcal{O}]$  is singular<sup>3</sup> at  $x = 0$  for any  $\mathcal{O}$ , whereas  $T[\mathcal{O}]$  is only singular (at  $x = y = 0$ ) when  $\mathcal{O}$  contains two or more derivatives. These basic functions are easily renormalized using the set of rules described above. For example,

$$\begin{aligned} B^R[1] &= [\Delta(x) \Delta(x)]^R \\ &= \frac{1}{(4\pi^2)^2} \left[ \frac{1}{x^4} \right]^R \\ &\stackrel{\text{Rules 1,2}}{=} -\frac{1}{4} \frac{1}{(4\pi^2)^2} \square \frac{\log x^2 M^2}{x^2} \end{aligned} \quad (12)$$

<sup>3</sup> In this paper the term ‘singular’ should always be understood as ‘too singular to allow for a Fourier transform’.

and

$$\begin{aligned}
 T^R[\Box] &= [\Delta(x)\Delta(y)\Box^x\Delta(x-y)]^R \\
 &\stackrel{\text{Rule 4}}{=} -\left[(\Delta(x))^2\delta(x-y)\right]^R \\
 &= -[B[1](x)\delta(x-y)]^R \\
 &\stackrel{\text{Rule 3}}{=} -B^R[1](x)\delta(x-y) \\
 &\stackrel{\text{Eq. (12)}}{=} \frac{1}{4} \frac{1}{(4\pi^2)^2} \Box \frac{\log x^2 M^2}{x^2} \delta(x-y) .
 \end{aligned} \tag{13}$$

For basic functions with non-trivial tensor structure the procedure is more involved and can be found in Ref. [22]. In particular, we shall need in the following the identity:

$$\begin{aligned}
 T^R[\partial_\mu\partial_\nu] &= T[\partial_\mu\partial_\nu - \frac{1}{4}\delta_{\mu\nu}\Box] \\
 &+ \left( \frac{1}{16} \frac{1}{(4\pi^2)^2} \Box \frac{\log x^2 M^2}{x^2} \delta(x-y) - \frac{1}{32} \frac{1}{4\pi^2} \delta(x)\delta(y) \right) \delta_{\mu\nu} ,
 \end{aligned} \tag{14}$$

where the first term is finite thanks to its tracelessness. The local term proportional to  $\delta(x)\delta(y)$  was not considered in the earlier literature (before Ref. [22], where Eq. (14) is worked out in detail) and comes from imposing consistency with the propagator equation (rule 4). Notice that

$$\delta_{\mu\nu} T^R[\partial_\mu\partial_\nu] \neq [\delta_{\mu\nu} T[\partial_\mu\partial_\nu]]^R . \tag{15}$$

This might seem strange, but in fact also occurs in other schemes like dimensional or Pauli-Villars regularization. Differentially renormalized expressions of basic functions appearing in one-, two- and three-point one-loop Green functions can be found in the tables of Ref. [22]. The treatment of four-point one-loop Green functions will be presented in Ref. [25].

### 3. Simple applications

We now show how the method works in practice with two detailed examples: the renormalization of the one-loop 1PI electron self-energy and electron-electron-photon vertex in massless QED. The Feynman rules of massless QED in euclidean coordinate space are gathered in Fig. 2, with  $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ . We work in the Feynman gauge. The case of an arbitrary Lorentz gauge was discussed in Ref. [22]. Let us first calculate the electron

$$\begin{array}{ll}
 y \longrightarrow x & - \not{\partial}^x \Delta(x - y) \\
 y \overset{\mu}{\sim} \overset{\nu}{\sim} x & \delta_{\mu\nu} \Delta(x - y) \\
 \begin{array}{c} \mu \\ \text{wavy line} \\ \swarrow \searrow \\ \text{fermion lines} \end{array} & -ie\gamma_\mu
 \end{array}$$

Fig. 2. Feynman rules of massless QED in euclidean coordinate space.

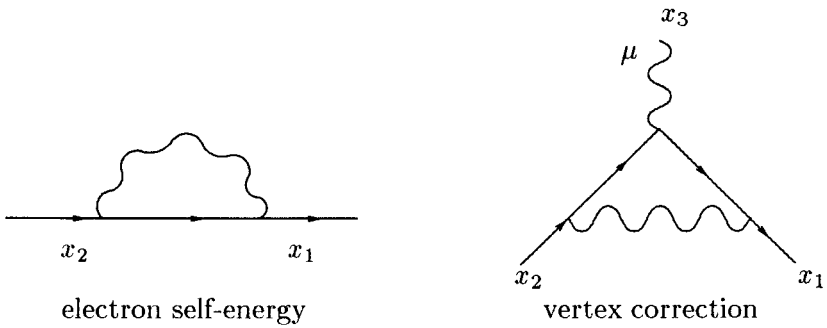


Fig. 3. One loop diagrams contributing to the electron self-energy and the electron-electron-photon vertex in QED

self-energy, given by the first Feynman graph in Fig 3. The bare expression is

$$\Sigma(x) = e^2 \gamma_\alpha \Delta(x) \not{\partial} \Delta(x) \gamma_\alpha, \quad (16)$$

where  $x = x_1 - x_2$ . Due to translation invariance,  $\Sigma$  only depends on  $x$ . Notice that Eq. (16) involves no integration, in contradistinction with the corresponding expression in momentum space<sup>4</sup>. After some straightforward (four-dimensional) diracology and the use of Leibnitz rule to extract the

<sup>4</sup> In general, coordinate space calculations involve one integral less than the momentum space ones. If one is interested in quantities defined at determined momenta (like scattering amplitudes), the missing integral must be done at the end as a Fourier transform (without regulator!). One example when this Fourier transform is not needed is the calculation of beta functions.

derivative, one obtains

$$\begin{aligned}\Sigma(x) &= -e^2 \not{\partial}(\Delta(x))^2 \\ &= -e^2 \not{\partial}B[1] .\end{aligned}\quad (17)$$

The renormalized value is, from Eq. (12),

$$\Sigma^R(x) = \frac{1}{64\pi^4} e^2 \not{\partial} \square \frac{\log x^2 M^2}{x^2} . \quad (18)$$

Let us now deal with the vertex correction (see Fig. 3). Reading directly from the Feynman rules,

$$V_\mu(x, y) = (-ie)^3 \gamma_\alpha \not{\partial}^x \Delta(x) \gamma_\mu (-\not{\partial}^y) \Delta(y) \gamma_\alpha \Delta(x - y) , \quad (19)$$

with  $x = x_1 - x_3$  and  $y = x_2 - x_3$ . Simplifying the Dirac algebra and using systematically the Leibnitz rule to rearrange derivatives,  $V_\mu(x, y)$  can be expressed in terms of triangular basic functions:

$$\begin{aligned}V_\mu(x, y) &= ie^3 \{ -2\gamma_b \gamma_\mu \gamma_a (\partial_a^x \partial_b^y T[1] + \partial_a^x T[\partial_b] - \partial_b^y T[\partial_a]) \\ &\quad - 2\gamma_\mu T[\square] + 4\gamma_a T[\partial_a \partial_\mu] \} .\end{aligned}\quad (20)$$

The renormalized expression is obtained directly from Eqs (13) and (14):

$$\begin{aligned}V_\mu^R(x, y) &= ie^3 \{ -2\gamma_b \gamma_\mu \gamma_a (\partial_a^x \partial_b^y T[1] + \partial_a^x T[\partial_b] \\ &\quad - \partial_b^y T[\partial_a]) + 4\gamma_a T[\partial_a \partial_\mu - \frac{1}{4} \delta_{a\mu} \square] \\ &\quad - \frac{1}{4} \frac{1}{(4\pi^2)^2} \gamma_\mu \square \frac{\log x^2 M^2}{x^2} \delta(x - y) - \frac{1}{8} \frac{1}{4\pi^2} \gamma_\mu \delta(x) \delta(y) \} .\end{aligned}\quad (21)$$

Once the graphs have been renormalized in coordinate space, one can perform a Fourier transform (without any regulator) to obtain the corresponding finite expressions in momentum space. We need the Fourier transforms of the basic functions in Eqs (18) and (21). The latter are more involved, so let us see in some detail how to calculate them. The Fourier transform of a distribution “of two variables”,  $f(x, y)$ , is

$$\hat{f}(p, p') = \int d^4x \, e^{ix \cdot p} e^{iy \cdot p'} f(x, y) . \quad (22)$$

With the integration by parts prescription, total derivatives in  $f(x, y)$  yield

$$\partial_\mu^x \rightarrow -ip_\mu ; \quad \partial_\mu^y \rightarrow -ip'_\mu . \quad (23)$$



For the finite triangular functions we have:

$$\begin{aligned}
 \hat{T}[\mathcal{O}] &= \int d^4x d^4y e^{ix \cdot p} e^{iy \cdot p'} \Delta(x) \Delta(y) \mathcal{O}^x \Delta(x-y) \\
 &= \int d^4x d^4y e^{ix \cdot p} e^{iy \cdot p'} \\
 &\quad \times \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} e^{-ix \cdot k_1} e^{-iy \cdot k_2} e^{-i(x-y) \cdot k} \frac{\hat{\mathcal{O}}(k)}{k_1^2 k_2^2 k^2} \\
 &= \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{\hat{\mathcal{O}}(k)}{k_1^2 k_2^2 k^2} \\
 &\quad \times (2\pi)^4 \delta(p - k_1 - k) (2\pi)^4 \delta(p' - k_2 + k) \\
 &= \int \frac{d^4k}{(2\pi)^4} \frac{\hat{\mathcal{O}}(k)}{(p-k)^2 (p'+k)^2 k^2}, \tag{24}
 \end{aligned}$$

where  $\hat{\mathcal{O}}(k)$  is obtained from  $\mathcal{O}^x$  by the replacement  $\partial^x \rightarrow -ik$ . The integrals in Eq. (24) appear (with a regularization which is not present here) in standard one-loop calculations in momentum space and can be evaluated with standard techniques. We shall do that later on, in the limit  $p' \rightarrow 0$ . On the other hand, the Fourier transform of the renormalized basic function in Eq. (21) reduces to Eq. (5):

$$\begin{aligned}
 \hat{T}^R[\square] &= \int d^4x d^4y e^{ix \cdot p} e^{iy \cdot p'} \frac{1}{64\pi^4} \square \frac{\log x^2 M^2}{x^2} \delta(x-y) \\
 &= \int d^4x e^{ix \cdot (p+p')} \frac{1}{64\pi^4} \square \frac{\log x^2 M^2}{x^2} \\
 &= \frac{1}{16\pi^2} \log\left(\frac{(p+p')^2}{M^2}\right). \tag{25}
 \end{aligned}$$

With all these formulae, we get the renormalized vertex correction in momentum space:

$$\begin{aligned}
 \hat{V}_\mu^R(p, p') &= ie^3 \left\{ -\gamma_b \gamma_\mu \gamma_a (-p_a p'_b \hat{T}[1] - ip_a \hat{T}[\partial_b] + ip'_b \hat{T}[\partial_a]) \right. \\
 &\quad + 4\gamma_a \hat{T}[\partial_a \partial_\mu - \frac{1}{4} \delta_{a\mu} \square] \\
 &\quad \left. - \frac{1}{16\pi^2} \gamma_\mu \left( \log \frac{(p+p')^2}{M^2} + \frac{1}{2} \right) \right\}. \tag{26}
 \end{aligned}$$

The Fourier transform (in one variable) of the self-energy in Eq. (18) is directly given by Eq. (5):

$$\hat{\Sigma}^R(p) = -i \frac{e^2}{16\pi^2} \not{p} \log\left(\frac{p^2}{M^2}\right). \tag{27}$$

#### 4. The vertex Ward identity

Finally, let us verify the Ward identity between the renormalized vertex correction and electron self-energy. For 1PI Green functions it reads

$$(\partial_\mu^x + \partial_\mu^y) V_\mu^R(x, y) = ie \Sigma^R(x - y)(\delta(x) - \delta(y)) , \quad (28)$$

where  $\partial^{x_3} f(x_1 - x_3, x_2 - x_3) = -(\partial^x + \partial^y) f(x, y)$  has been used to express it in the translated variables  $x$  and  $y$ . At points away from the origin this identity must hold, since the bare Green functions have not been modified there. A possible disagreement can only arise from the contact terms at  $x = y = 0$ . In fact, both sides of Eq. (28) are distributions, and to compare them one must either use formal properties of delta functions, *etc.*, or integrate with an arbitrary test function  $\phi(x, y)$ . In particular, one can perform a Fourier transform ( $\phi(x, y) = e^{ix \cdot p} e^{iy \cdot p'}$ ), which retains all the information. In other words, we can check the Ward identity in momentum space,

$$-i(p_\mu + p'_\mu) \hat{V}_\mu^R(p, p') = ie[\hat{\Sigma}^R(-p') - \hat{\Sigma}^R(p)] , \quad (29)$$

using the momentum space renormalized Green functions in Eqs (26) and (27). For simplicity, we consider the limit  $p' \rightarrow 0$  (*i.e.*, the Fourier transform in  $y$  reduces to an integral without any weight). In this limit the relevant integrals reduce to

$$\hat{T}[\partial_\alpha] \xrightarrow{p' \rightarrow 0} \frac{i}{16\pi^2} \frac{p_\alpha}{p^2} , \quad (30)$$

$$\hat{T}[\partial_\alpha \partial_\beta - \frac{1}{4} \delta_{\alpha\beta} \square] \xrightarrow{p' \rightarrow 0} -\frac{1}{32\pi^2} \frac{p_\alpha p_\beta - \frac{1}{4} \delta_{\alpha\beta} p^2}{p^2} , \quad (31)$$

whereas  $\hat{T}[1]$  is logarithmically infrared divergent and  $p'_\alpha \hat{T}[1] \xrightarrow{p' \rightarrow 0} 0$ . With these values, we obtain for both sides of the Ward identity in Eq. (29), in the limit  $p' \rightarrow 0$ , the same result:

$$-\frac{e^3}{16\pi^2} \not{p} \log \frac{p^2}{M^2} .$$

Since both sides are equal, the Ward identity is indeed satisfied: constrained DR has preserved it automatically, *i.e.*, without any *a posteriori* adjustment.

## 5. Conclusions

DR is a renormalization method which works in coordinate space and does not introduce any intermediate regulator [3]. We have shown how it can be easily applied to the calculation and renormalization of one-loop Feynman diagrams. In many cases, worked out calculations at higher orders are also simpler in DR than in other methods [3, 15]. Some nice features of the method are the following:

- It is minimal, in the sense that the Green functions are never modified except at the singular points.
- It does not change the space-time dimension, making the diracology and tensor manipulation, and the treatment of chiral theories [4, 8, 11] easier.
- One integration less has to be performed, unless one is interested in some quantity defined for fixed external momenta. In such case, one has to Fourier transform the renormalized expressions, but without any regularization.
- Some overlapping divergencies disentangle in coordinate space.
- It is better suited for theories which are naturally defined in coordinate space, like theories with conformal invariance [6], in curved space or at finite temperature [10].

In the constrained procedure of DR [22] all the local terms are fixed. This determines a renormalization scheme which turns out to be symmetric in all known examples [11, 22]. Here, we have used constrained DR to calculate two one-loop Green functions in QED and we have verified that the Ward identity relating them is automatically fulfilled after renormalization. We have dealt with massless theories for simplicity. The treatment of massive theories in (constrained) DR is worked out in Ref. [17] (Ref. [22]).

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