THE Z BOSON RESONANCE AND RADIATIVE CORRECTIONS*

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The Z line shape is measured at LEP with an accuracy at the per mill level. Usually it is described in the Standard Model of electroweak interactions with account of quantum corrections. Alternatively, one may attempt different model-independent approaches in order to extract quantities like mass and width of the Z boson. If a fit deviates from that in the standard approach, this may give hints for New Physics contributions. I describe two model-independent approaches and compare their applications to LEP data with the Standard Model approach.

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1. Introduction

From 1989 till 1995 about 16 millions of Z bosons have been produced at LEP1 and several hundreds of thousands at SLC. Due to this, and due to the lack of direct hints for the existence of a Higgs boson, the Z boson and its interactions became for several years the central theme of tests of the Standard Model [1–5], recently accompanied by the discovery of the t quark at the Tevatron [6, 7].

The predictions of the Standard Model depend on the particle masses, fermion mixings [8] and one coupling constant. A central role plays also the weak mixing angle which relates (*i*) the Z boson field and the photon to the symmetry fields, (*ii*) the two coupling constants of the theory, (*iii*) the Z and W mass ratio, (*iv*) the vector and axial vector couplings of the Z to fermions.

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The corresponding electroweak Standard Model relations, modulo radiative corrections, are:

$$Z = \cos\theta_w W^0 - \sin\theta_w B\,, \tag{1}$$

$$\gamma = \sin\theta_w W^0 + \cos\theta_w B\,, \tag{2}$$

$$g\sin\theta_w \equiv e = \sqrt{4\pi\alpha_{em}}\,, \tag{3}$$

$$a_{\text{lept}} = -0.5\,, \tag{4}$$

$$v_{\text{lept}} = -0.5\left(1-4\sin^2\theta_w\right)\,, \tag{5}$$

$$G_\mu/\sqrt{2} = g^2/(8M_W^2)\,, \tag{6}$$

$$M_Z \equiv M_W/\cos\theta_w\,, \tag{7}$$

From (3), (6), and (7) one derives:

$$M_Z > M_W = \sqrt{\frac{\pi\alpha_{em}}{G_\mu\sqrt{2}}}\,\frac{1}{\sin\theta_w} > 37.281 \text{ GeV}\,, \tag{8}$$

as an absolute (though model-dependent) lower limit for the gauge boson masses. From Born unitarity considerations it was expected that the Fermi theory should loose validity above a mass scale of about $\mathcal{O}(100)$ GeV. In 1973 the first observation of neutrino-induced weak neutral current events was reported by the Gargamelle collaboration operating at CERN. One event was observed for the reaction $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ from 360000 $\bar{\nu}_\mu$ and 375000 ν_μ scatterings with 3% background [9]. Since $\sigma \sim s = 2\,E_\nu\, m_{\text{target}}$ at low energies, the process $\nu N \to \nu N$ is about several thousand times ($\sim m_p/m_e$) more frequent. In fact, about one third of νN scattering is NC mediated and was also observed by that time [10]. From the $\bar{\nu}_\mu e$ cross-section, the authors derived with 90% CL: $0.1 < \sin^2\theta_w < 0.6$. This corresponds to:

$$M_Z \sim 75\cdots 125 \text{ GeV}\,, \tag{9}$$

staying well in the limits mentioned above.

In 1983, at the $p\bar{p}$ collider SPS (CERN), both the Z [11, 12] and the W bosons [13, 14] were discovered and their masses could be determined at that time with an accuracy of several GeV; in 1986:

$$M_Z = 92.6 \pm 1.7 \text{ GeV}\,. \tag{10}$$

At the end of 1989 LEP1 and SLC started operation and dominated the precision experiments for tests of the electroweak Standard Model for a decade. This may be exemplified by quoting the following improvements of precision from August 1989 [15] till October 1997 [16]:

$$
\begin{array}{rcll}
 & 1989 & \rightarrow & 1997 & \\
M_Z = & 91.120 \pm 0.160 \text{ GeV} & \rightarrow & 91.1867 \pm 0.0002 \text{ GeV} & (11) \\
\sin^2\theta_w^{\text{eff}} = & 0.23300 \pm 0.00230 & \rightarrow & 0.23152 \pm 0.00023 & (12) \\
m_t^{\text{pred}} = & 130 \pm 50 \text{ GeV} & \rightarrow & m_t^{\text{meas}} = 175.6 \pm 5.5 \text{ GeV} & (13) \\
M_H^{\text{pred}} > & \text{few GeV} & \rightarrow & \geq 77 \text{ GeV} & (14) \\
\alpha_s(M_Z) = & 0.110 \pm 0.010 & \rightarrow & 0.119 \pm 0.003 & (15)
\end{array}
$$

The few remarks on the past may remind you that Z physics in times before the advent of LEP was exciting, and may also indicate that the operation of LEP1 gave a striking experimental support for the simplest realistic version of a renormalizable theory of weak interactions. Maybe it is worth mentioning that in the early seventies the Standard Model was considered by many of us to be an extremely complicated, very artificially looking model. Soon later, when hundreds of alternatives were constructed, minds changed and now many of us wonder that such a simple construct like the Standard Model survives one precision test after the other. But this needs not go on – future is unpredictable. The only thing we know for sure: future will be exciting.

2. The Z line shape

The Z boson may be studied as a resonance at LEP from a measurement of the cross-section

$$e^+e^- \rightarrow (\gamma, Z) \rightarrow \bar{f}f(+n\gamma) \tag{16}$$

as a function of the beam energy. The determinations of mass M_Z and width Γ_Z are dominated by hadron production in a small region around the peak: $|\sqrt{s} - M_Z| < 3$ GeV. The Z is not a pure Breit–Wigner resonance. We want to study a $2 \rightarrow 2$ process with intermediate Z, but we have also virtual photon exchange. In addition, there are huge $2 \rightarrow 3$ contributions due to initial state radiation (ISR) of photons and due to final state radiation (FSR). Further, many virtual corrections are contributing as quantum corrections: vertex insertions, self energy insertions, box diagrams, and all their iterations.

3. Real photonic corrections

The QED corrections may be taken into account by the following convolution formula ([17–19] and references therein):

$$\sigma(s) = \int \frac{ds'}{s}\sigma_0(s')\,\rho\left(\frac{s'}{s}\right) + \int \frac{ds'}{s}\sigma_0^{\rm int}(s,s')\,\rho^{\rm int}\left(\frac{s'}{s}\right) \qquad (17)$$

- $\rho(s'/s)$ — the radiator describes initial and final state radiation, including leading higher order effects and soft photon exponentiation;
- $\sigma_0(s')$ — the basic scattering cross-section, which is the object of investigation.

The $\rho^{\rm int}(s'/s)$ takes into account the initial-final state interference effects which are comparatively small (a few per mill) near the Z resonance but are bigger off the resonance, and $\sigma_0^{\rm int}(s,s')$ is a function similar to $\sigma_0(s')$, but suppressed if $\rho^{\rm int}(s'/s)$ is small.

The dominant part of the QED corrections is ISR:

$$\rho\left(\frac{s'}{s}\right) = \beta\left(1-\frac{s'}{s}\right)^{\beta-1}\delta^{\rm soft+virtual} + \delta^{\rm hard}. \qquad (18)$$

where

$$\beta = 2\frac{\alpha_{em}}{\pi}(L-1), \qquad L = \ln(s/m_e^2)\,, \qquad (19)$$

$$\delta^{\rm soft+virtual} = 1+\frac{\alpha_{em}}{\pi}\left[\frac{3}{2}L+2\zeta(2)-2\right]+\mathcal{O}\left(\frac{\alpha_{em}}{\pi}\right)^2, \qquad (20)$$

$$\delta^{\rm hard} = -\frac{\alpha_{em}}{\pi}\left(1+\frac{s'}{s}\right)(L-1)+\mathcal{O}\left(\frac{\alpha_{em}}{\pi}\right)^2. \qquad (21)$$

It is impossible to perform a reasonable model-independent fit to the Z resonance shape without a dedicated treatment of QED corrections. Their influence on the resonance shape near the peak (at LEP1 energies) will be discussed in the next section. Here I show that the so-called radiative tail is proportional to M_Z/Γ_Z; at higher energies (*e.g.* at LEP2) this is substantial but may be influenced by experimental cuts.

If we apply no cuts on the photon kinematics (*i.e.* observe none of them), the $\mathcal{O}(\alpha_{em})$ corrections to the pure Z contribution to $\sigma_{\rm tot}$ are [20]:

$$\sigma_{\rm tot}^Z = \frac{4}{3}\frac{\pi\alpha_{em}^2}{s}|\chi(s)|^2\left\{(v_e^2+a_e^2)(v_f^2+a_f^2)\left[1+\frac{\alpha_{em}}{\pi}(Q_e^2H_0^T(s,m_Z) + Q_f^2H_2^T(s,m_Z))\right]+4v_ea_ev_fa_f\left[\frac{\alpha_{em}}{\pi}Q_eQ_fH_4^T(s,m_Z)\right]\right\} \qquad (22)$$

with $R_Z = m_Z/s, m_Z = M_Z^2 - iM_Z\Gamma_Z$ and χ defined in (39). The numerically most important contribution is ISR:

$$H_0^T(s, m_Z) = \left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \left(\ln\frac{s}{m_e^2} - 1\right) \Re e \left[2R_Z + \frac{1}{2} - |R_Z|^2 + \frac{i \cdot s}{M_Z\Gamma_Z}(1 - R_Z^*)R_Z(1 + R_Z^2)\ln\frac{R_Z - 1}{R_Z}\right]. \quad (23)$$

The last term describes the radiative tail of the Z resonance. If $R_Z < 1$, *i.e.* if $M_Z < s$, it is:

$$\Re e\, i \cdot \frac{M_Z}{\Gamma_Z}\ln\frac{R_Z - 1}{R_Z} = \mathcal{O}(\pi) \cdot \frac{M_Z}{\Gamma_Z}. \quad (24)$$

Otherwise (*i.e.* $R_Z > 1$, at energies below the peak), this term stays small.

The analytic structure of this QED contribution, as well as that of the others, is completely different from any Born-like expression. This will be of importance if one tries to describe measured cross-sections by simple parameterizations: The QED corrections have to be treated explicitly.

4. Model (I): A model-independent ansatz

QED corrections are treated by the convolution formula introduced in Section 3. For a careful discussion of their influence on height and location of the Z peak see [21].

The following ansatz for $\sigma_0(s')$ is a good choice without explicit reference to the Standard Model [22–25]:

$$\sigma_0(s) = \frac{4}{3}\pi\alpha_{em}^2 \left[\frac{r^\gamma}{s} + \frac{s \cdot R + (s - M_Z^2) \cdot J}{|s - M_Z^2 + is\Gamma_Z/M_Z|^2}\right]. \quad (25)$$

The line shape is described by five parameters:

- $r^\gamma \sim \alpha_{em}^2(M_Z^2)$ — may be assumed to be known
- M_Z, Γ_Z
- R — measure of the Z peak height
- J — measure of the γZ interference

A simpler, also reasonable ansatz would be a pure Breit–Wigner function:

$$\sigma_0^{(Z)}(s) \sim \frac{M_Z^2 \cdot R}{|s - M_Z^2 + iM_Z\Gamma_Z|^2}. \quad (26)$$

The effects of the QED corrections are huge; among others, a shift of the peak position arises:

$$\begin{aligned}\sqrt{s_{\max}} - M_Z &= \delta_{\rm QED} = \frac{\pi}{8}\beta\left(1+\delta^{\rm soft+virtual}\right)\Gamma_Z + \text{small corr's.} \\ &\approx 90\,{\rm MeV}\,.\end{aligned} \tag{27}$$

From the replacements $M_Z^2 R \to M_Z^2 s$, $M_Z\Gamma_Z \to (s/M_Z)\Gamma_Z$ additional shifts arise:

$$\sqrt{s_{\max}} - M_Z = \delta_{\rm QED} \oplus \frac{1}{4}\frac{\Gamma_Z^2}{M_Z} \ominus \frac{1}{2}\frac{\Gamma_Z^2}{M_Z} \sim (90+17-34)\quad {\rm MeV}\,. \tag{28}$$

Finally, adding the effect of the γZ interference J:

$$\begin{aligned}\sqrt{s_{\max}} - M_Z &= \delta_{\rm QED} \oplus \frac{1}{4}\frac{\Gamma_Z^2}{M_Z}\left(1+\frac{J}{R}\right) \ominus \frac{1}{2}\frac{\Gamma_Z^2}{M_Z} \\ &\sim \left[90 + 17\times\left(1+\frac{J}{R}\right) - 34\right]\quad {\rm MeV}\,.\end{aligned} \tag{29}$$

Neglecting this interference (setting J=0) leads to an erroneous systematic shift of the Z mass of 17 MeV$\otimes(J/R)$. If one wants to take into account the J, a model for its prediction is needed. In the Standard Model ([26], table 7): $J = 0.22, R = 2.96$ for hadron production. Thus, $J/R \otimes 17$ MeV ≈ 1.2 MeV.

4.1. Z line shape fit (I)

With the model-independent ansatz, the following nearly uncorrelated observables may be determined from the Z peak data [27, 28]:

$$M_Z = 91.186\,7 \pm 0.002\,0\ {\rm GeV} \quad (\delta = 0.0025\ \%)\,, \tag{30}$$

$$\Gamma_Z = 2.494\,8 \pm 0.002\,5\ {\rm GeV} \quad (\delta = 1.3\ \%)\,, \tag{31}$$

$$\sigma_0^{\rm had} = 41.486 \pm 0.053\ {\rm nb} \quad (\delta = 1.9\ \%)\,, \tag{32}$$

$$R_l = \frac{\sigma_0^{\rm had}}{\sigma_0^{\rm lept}} = 20.775 \pm 0.027 \quad (\delta = 1.5\ \%)\,, \tag{33}$$

$$A_{FB,0}^{\rm lept} = 0.017\,1 \pm 0.001\,0\,. \tag{34}$$

Here, $M_Z, \Gamma_Z, \sigma_0^{\rm had}$ are from $\sigma^{\rm had}(s)$, while R_l and A_{FB} from $\sigma^{\rm lep}(s)$: $\sigma_0^{\rm had(lep)}$ — hadronic (leptonic) peak cross-section, and $A_{FB,0}^{\rm lept}$ — forward-backward asymmetry at the peak. These parameters are considered to be primary parameters in contrast to derived ones, *e.g.* the effective leptonic weak neutral current couplings of leptons or the effective weak mixing angle [27, 28]:

$$v_l = -0.037\,93 \pm 0.000\,58\,, \tag{35}$$

$$a_l = -0.501\,03 \pm 0.000\,31\,, \tag{36}$$

$$\sin^2\theta_w^{\rm eff} \equiv \frac{1}{4}\left(1 - \frac{v_l}{a_l}\right) = 0.231\,52 \pm 0.000\,23\,. \tag{37}$$

5. Model (II): Virtual corrections in the Standard Model

All virtual corrections may be written in some theory, *e.g.* the Standard Model, for massless particle production in the following way (see *e.g.* [29–31] and references therein):

$$\begin{aligned}\mathcal{M}_{\rm net} \sim \frac{\alpha_{em}}{s}\Bigg\{ &\frac{\alpha_{em}(s)}{\alpha_{em}}\,|Q_eQ_f|\,\gamma_\beta\otimes\gamma_\beta + \chi(s)\varrho_{ef}\Big[L_\beta\otimes L_\beta \\ &- 4s_w^2|Q_e|\kappa_e\gamma_\beta\otimes L_\beta - 4s_w^2|Q_f|\kappa_b L_\beta\otimes\gamma_\beta \\ &+ 16s_w^4|Q_eQ_f|\kappa_{eb}\gamma_\beta\otimes\gamma_\beta\Big]\Bigg\}\,.\end{aligned} \tag{38}$$

We use short notations: $L_\beta = \gamma_\beta(1+\gamma_5)$, $A_\beta\otimes B_\beta = [\bar{v}_eA_\beta u_e]\cdot[\bar{u}_bB_\beta v_b]$, and

$$\chi = \chi(s) = \frac{G_\mu}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha_{em}}\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z(s)}, \quad \Gamma_Z(s) = \frac{s}{M_Z^2}\Gamma_Z\,. \tag{39}$$

The effective Born cross-section now is uniquely determined once the net matrix element $\mathcal{M}_{\rm net}$ is known:

$$\begin{aligned}\sigma_0(s) =\; & N_c^f\sqrt{1-4m_f^2/s}\;\frac{4\pi\alpha_{em}^2}{3s} \\ &\times\Bigg\{\left(1+\frac{2m_f^2}{s}\right)\Big[|Q_eQ_f|^2\frac{|\alpha_{em}(s)|^2}{\alpha_{em}^2} + 2|Q_eQ_f|\Re e\Big(\chi\frac{\alpha_{em}^*(s)}{\alpha_{em}}\varrho_{ef}v_{ef}\Big) \\ &+|\chi\varrho_{ef}|^2(1+|v_e|^2+|v_f|^2+|v_{ef}|^2)\Big] - \frac{6m_f^2}{s}|\chi\varrho_{ef}|^2(1+|v_e|^2)\Bigg\}\end{aligned} \tag{40}$$

with

$$v_i = 1 - 4s_w^2|Q_i|\kappa_i, \qquad i = e, f \tag{41}$$

$$v_{ef} = 1 - 4s_w^2|Q_e|\kappa_e - 4s_w^2|Q_f|\kappa_f + 16s_w^4|Q_eQ_f|\kappa_{ef}\,. \tag{42}$$

Further, $N_c^f = 1, 3$ is the colour factor and QCD corrections also have to be taken into account.

The virtual corrections with higher order parts are (see for details [29, 31, 32] and references therein):

$$\varrho_{ef} = \frac{(1+\tau_f)^2}{1 - \Delta\rho - \Delta\varrho_{ef}^{\rm 1loop,\alpha} + \Delta\bar{\rho}^{\alpha} + 2\Delta\bar{\rho}_f - \Delta\rho^{\rm 2loop,\alpha\alpha_s} + \Delta\bar{\rho}^{\alpha\alpha_s}}, \quad (43)$$

$$\begin{aligned} \kappa_f &= \left[1 + \left(\Delta\kappa_f^{\rm 1loop,\alpha} - \frac{c_w^2}{s_w^2}(\Delta\bar{\rho}^{\alpha} + \bar{X}) + \Delta\bar{\rho}_f\right)\right. \\ &\quad \left. + \left(\Delta\kappa^{\rm 2loop,\alpha\alpha_s} - \frac{c_w^2}{s_w^2}\Delta\bar{\rho}^{\alpha\alpha_s}\right)\right] \frac{1 + \frac{c_w^2}{s_w^2}(\Delta\rho + X)}{1+\tau_f}, \quad (44) \\ \kappa_{ef} &= \left[1 + \left(\Delta\kappa_{ef}^{\rm 1loop,\alpha} - 2\frac{c_w^2}{s_w^2}(\Delta\bar{\rho}^{\alpha} + \bar{X}) + 2\Delta\bar{\rho}_f\right)\right. \\ &\quad \left. + 2\left(\Delta\kappa_f^{\rm 2loop,\alpha\alpha_s} - \frac{c_w^2}{s_w^2}\Delta\bar{\rho}^{\alpha\alpha_s}\right)\right] \frac{\left(1 + \frac{c_w^2}{s_w^2}(\Delta\rho + X)\right)^2}{(1+\tau_f)^2}. \quad (45) \end{aligned}$$

The corrections $\Delta\bar{\rho}_f$ and τ_f contribute only to b quark pair production.

The one loop form factors are, due to the ZZ and WW box contributions, dependent on the scattering angle ϑ:

$$\begin{aligned} \rho_{ef}^{\rm 1loop,\alpha}(s,\cos\vartheta) &= 1 + \Delta\rho_{ef}^{\rm non-box}(s) + \rho_{ef}^{\rm box}(s,\cos\vartheta) \\ &\quad + \delta_{fb}\left[\delta\rho_{eb}^{t}(s) + \delta\rho_{WW}^{{\rm box},t}(s,\cos\vartheta)\right], \quad (46) \\ \kappa_e^{\rm 1loop,\alpha}(s,\cos\vartheta) &= \kappa(e,f) + \delta_{fb}\left[\delta\kappa_e^{t}(s) - \delta\rho_{WW}^{{\rm box},t}(s,\cos\vartheta)\right], \quad (47) \\ \kappa_f^{\rm 1loop,\alpha}(s,\cos\vartheta) &= \kappa(f,e) - \delta_{fb}\left[\delta\rho_{eb}^{t}(s) + \delta\rho_{WW}^{{\rm box},t}(s,\cos\vartheta)\right] \quad (48) \end{aligned}$$

with

$$\kappa(e,f) = 1 + \Delta\kappa^{\rm non-box}(e,f;s) + \kappa_e^{\rm box}(s,\cos\vartheta) \quad (49)$$

and finally

$$\begin{aligned} \kappa_{ef}^{\rm 1loop,\alpha}(s,\cos\vartheta) &= 1 + \Delta\kappa_{ef}^{\rm non-box}(s) + \kappa_{ef}^{\rm box}(s,\cos\vartheta) \\ &\quad - \delta_{fb}\left[\delta\rho_{eb}^{t}(s) + \delta\rho_{WW}^{{\rm box},t}(s,\cos\vartheta)\right]. \quad (50) \end{aligned}$$

The form factors with super index t are for b quark pair production only. If the total cross-section is written in terms of four form factors with a dependence on only s, it is implicitly assumed that the dependence of the weak box terms on $\cos\vartheta$ is negligible. This is a very good approximation near the Z resonance but not at much higher energies.

Further, we need expressions for M_W, Γ_Z, α_{em} and a reasonable treatment of QCD corrections and explicit expressions for the structures shown above. The W boson mass is:

$$M_W = M_Z \sqrt{1 - \sqrt{1 - \frac{4\pi\alpha_{em}}{\sqrt{2}G_\mu M_Z^2[1-\Delta r]}}} \tag{51}$$

with

$$\frac{1}{1-\Delta r} = \frac{1}{(1-\Delta\alpha_{em})\left(1 + \frac{c_w^2}{s_w^2}(\Delta\rho + X)\right) - \Delta r_{\text{rem}}} \tag{52}$$

and

$$\Delta r_{\text{rem}} = \Delta r^{1\text{loop},\alpha} + \Delta r^{2\text{loop},\alpha\alpha_s} + \frac{c_w^2}{s_w^2}\left(\Delta\bar{\rho}^{\alpha} + \Delta\bar{\rho}^{\alpha\alpha_s} + \bar{X}\right) - \Delta\alpha_{em}\,. \tag{53}$$

For the Z width [33–36], α_{em} [37, 38], as well as QCD corrections [39], and all the other expressions left out here I have to refer to literature quoted above and to references therein.

5.1. Z line shape fit (II)

I quoted all the above formulae in order to demonstrate explicitly how involved a Standard Model fit ansatz is. The input quantities are: α_{em}, G_μ (for M_W), M_Z, m_f, M_H, α_s. Some of them are precisely known (*e.g.* G_μ), others are subject of determination at LEP (*e.g.* M_Z), others are completely unknown (M_H). The t quark mass may be determined from weak loop corrections at LEP or directly from t quark production at Fermilab.

Quantities like the Z width or the weak mixing angle are not a subject of fits since they are considered to be secondary quantities. In this respect, there is a basic difference to the approach of the foregoing section.

The most recent Standard Model fit is [27, 28]. The t quark mass from Fermilab is:

$$m_t = 175.6 \pm 5.5 \;\; \text{GeV}\,. \tag{54}$$

The global fit to all data yields:

$$M_Z = 91.1867 \pm 0.0020 \ \text{GeV}, \quad (55)$$
$$m_t = 173.1 \pm 5.4 \ \text{GeV}, \quad (56)$$
$$M_H = 115^{+116}_{-66} \ \text{GeV}, \quad (57)$$
$$\alpha_s(M_Z) = 0.120 \pm 0.003. \quad (58)$$

In a next step, one may calculate the other quantities like the Z width and relate to values from model-independent fits. Whatever one does, there is no unique hint to New Physics. For a detailed discussion of this see [16].

6. Model (III): The S-matrix approach

In view of the extremely high precision in the measurement of mass and width of the Z boson, one may question the ansatz used. A potential bias in the Standard Model (if it is not correct) would not show up in the experimental errors. For this reason, one is interested in a model-independent Z peak description with a minimum of assumptions. We saw in Sections 3 and 4 that we must take into account real photonic corrections; but we saw also that this is possible with the use of an effective Born cross-section to be folded with a function depending on the kinematics, M_Z, and Γ_Z.

When the Z boson is treated as a resonance, the S-matrix approach[1] may be used for its description.

This was proposed in the context of LEP physics in [24], where the perturbation expansion in the Standard Model was studied. In [25] it was proposed to use this approach for a direct fit to LEP data and the first S-matrix fit was performed therein. The first fit by a LEP collaboration was due to L3 [42,43]. The treatment of asymmetries near the peak was discussed in [25]. For the role of QED corrections to asymmetries see [44].

A recent survey on the definition of Z mass and width and their treatment in fermion pair production is [45]. Here, I give a short introduction to the technical essentials.

Consider four independent helicity amplitudes in the case of massless fermions f:

$$\mathcal{M}^{fi}(s) = \frac{\mathrm{R}^{\mathrm{f}}_{\gamma}}{s} + \frac{\mathrm{R_Z}^{\mathrm{fi}}}{s - s_Z} + \sum_{n=0}^{\infty} \frac{\mathrm{F}^{\mathrm{fi}}_{\mathrm{n}}}{\overline{m}_Z^2} \left(\frac{s - s_Z}{\overline{m}_Z} \right)^n, \quad i = 1, \ldots, 4. \quad (59)$$

The position of the Z pole in the complex s plane is given by s_Z:

$$s_Z = \overline{m}_Z^2 - i \overline{m}_Z \overline{\Gamma}_Z. \quad (60)$$

[1] For an introduction to S-matrix theory, see [40, 41].

The R^f_γ and $R_Z{}^{fi}$ are complex residua of the photon and the Z boson, respectively. One may approximate (59) by setting $F^{fi}_n \to 0$. There are four residua $R_Z{}^{fi}$ for $e^-_L e^+_R \to f^-_L f^+_R$, $e^-_L e^+_R \to f^-_R f^+_L$, $e^-_R e^+_L \to f^-_R f^+_L$, $e^-_R e^+_L \to f^-_L f^+_R$. The amplitudes $\mathcal{M}^{fi}(s)$ give rise to four cross-sections σ_i:

$$\begin{aligned} & & \sigma^0_T(s) &= +\sigma_0+\sigma_1+\sigma_2+\sigma_3\,, \\ \sigma^0_{lr-pol}(s) &= & \sigma^0_{FB}(s) &= +\sigma_0-\sigma_1+\sigma_2-\sigma_3\,, \\ \sigma^0_{FB-lr}(s) &= & \sigma^0_{pol}(s) &= -\sigma_0+\sigma_1+\sigma_2-\sigma_3\,, \\ \sigma^0_{lr}(s) &= & \sigma^0_{FB-pol}(s) &= -\sigma_0-\sigma_1+\sigma_2+\sigma_3\,. \end{aligned} \tag{61}$$

Here, the σ^0_T — the total cross-section, σ^0_{FB} — numerator of the forward-backward asymmetry, σ^0_{pol} — that of the final state polarization *etc.* All these cross-sections may be parameterized by the following master formula ($A = T, FB, \ldots$):

$$\sigma^0_A(s) = \frac{4}{3}\pi\alpha^2_{em}\left[\frac{r^{\gamma f}_A}{s} + \frac{s r^f_A + (s-\overline{m}^2_Z) j^f_A}{(s-\overline{m}^2_Z)^2 + \overline{m}^2_Z \overline{\Gamma}^2_Z}\right] + \ldots\,. \tag{62}$$

The parameters are related to the residua of the pole terms. The $r^{\gamma f}_A$ is the photon exchange term and it is assumed to be known. It vanishes for all asymmetric cross-sections. The Z exchange residuum r^f_A and the γZ-interference j^f_A are, together with Z mass and width, subject of a fit.

Without QED corrections, asymmetries are:

$$\mathcal{A}^0_A(s) = \frac{\sigma^0_A(s)}{\sigma^0_T(s)} = A^A_0 + A^A_1\left(\frac{s}{\overline{m}^2_Z} - 1\right) + \ldots, \quad A \neq T\,. \tag{63}$$

They take the above extremely simple approximate form around the Z resonance. At LEP1, the higher order terms in the Taylor expansion may be neglected since $(s/\overline{m}^2_Z - 1)^2 < 2\times 10^{-4}$. The coefficients have a quite simple form:

$$A^A_0 = \frac{r^f_A}{r^f_T}\,, \qquad A^A_1 = \left[\frac{j^f_A}{r^f_A} - \frac{j^f_T}{r^f_T}\right] A^A_0\,. \tag{64}$$

The variation of asymmetries with s near the peak is due to the γZ interference (A_1). QED corrections modify the coefficient A_1, but in a model-independent form. It is important to note that they may not be neglected in a fit due to resonance tail effects similarly as was discussed above for the cross-section.

Another comment is necessary concerning the definition of mass and width of the Z boson. The so-called pole definition with a constant width

(60) as a natural consequence of the S-matrix ansatz leads to different numerical values compared to the usual Standard Model approach (39) [46–48]. A very precise approximation is:

$$\overline{m}_Z = [1 + (\Gamma_Z/M_Z)^2]^{-1/2} M_Z \approx M_Z - \tfrac{1}{2}\Gamma_Z^2/M_Z = M_Z - 34 \text{ MeV} \,. \quad (65)$$

Similar observations were made for hadron resonances in 1968 [49].

6.1. Z line shape fit (III)

The interest of the community in an S-matrix based fit to the LEP data has several origins. One is the wish for a model-independent description of the resonance. Closely related is the question about the number of independent parameters needed to describe the peak: four (per channel) suffice to describe a cross-section: M_Z, Γ_Z, r_T, j_T, provided we assume QED interactions to be understood. Among these parameters, Z mass and width are universal for all channels. Any asymmetry introduces two additional degrees of freedom (per channel): r_A, j_A.

There are practical aspects of all this. If the number of different energy points needed for a scan of the Z peak is asked for, the answer is at least five (four plus one) for cross-sections, at least three (two plus one) for asymmetries. Further, the γZ interferences j_A are separate degrees of freedom. The j_T and M_Z are highly correlated. This became more important recently when the highest statistics became available and also when the data taking at energies farther away from the peak collected more statistics. There the interference gets more influential.

TABLE I

Results from a combined LEP1 line shape fit

Parameter	S-matrix fit	SM Prediction
$\overline{m}_Z$[GeV]	91.153 4±0.003 3	—
Γ_Z [GeV]	2.492 4±0.002 6	2.493 2
$r_T^{\rm had}$	2.962 3±0.006 7	2.960 3
$j_T^{\rm had}$	0.15±0.15	0.22
$r_T^{\rm lept}$	0.142 39±0.000 34	0.142 53
$j_T^{\rm lept}$	0.009±0.012	0.004
$r_{FB}^{\rm lept}$	0.003 04±0.000 18	0.002 66
$j_{FB}^{\rm lept}$	0.789±0.013	0.799

Recent experimental studies are summarized in [26]. Data of Table I are obtained from the LEP Z line shape scans which were performed mainly in 1993 and 1995 (from Table 7 of [26][2]). The biggest error correlations are shown in Table II (from Table 8 of [26]). Including into the analysis cross-sections measured at TRISTAN energies does not improve substantially *e.g.* the resolution of M_Z and j_T [26].

TABLE II

Biggest correlations in the S-matrix fit

Correlation	Value
M_Z-$j_{\mathrm{T}}^{\mathrm{had}}$	-0.77
M_Z-$j_{\mathrm{T}}^{\mathrm{lept}}$	-0.47
Γ_Z-$r_{\mathrm{T}}^{\mathrm{had}}$	0.80
Γ_Z-$r_{\mathrm{T}}^{\mathrm{lept}}$	0.62
$r_{\mathrm{T}}^{\mathrm{had}}$-$r_{\mathrm{T}}^{\mathrm{lept}}$	0.78
$j_{\mathrm{T}}^{\mathrm{had}}$-$j_{\mathrm{T}}^{\mathrm{lept}}$	0.49

7. Summary

We have presented three different approaches to a numerical analysis of the Z boson line shape – a Breit-Wigner ansatz, the Standard Model of electroweak interactions, and the S-matrix approach; QED corrections must be applied. They all agree basically in the determination of the Z mass. The S-matrix approach treats the γZ interference as an independent quantity, which enlarges the error for M_Z. Two different mass definitions are used but may be related with a high accuracy. In the Standard Model, Γ_Z is a derived quantity; the other two approaches allow direct fits. The approaches agree numerically for the Z width.

We see an essential difference of the S-matrix ansatz to the Standard Model. The latter assumes *fixed* relations among many of the parameters. They rely thus on stronger theoretical assumptions.

From the strong experimental correlations in the S-matrix fit together with the excellent agreement of the central values of fitted parameters in all fit scenarios one may conclude that the different scenarios are highly compatible with each other.

[2] Note that the table shows values of the complex pole mass $\overline{m}_Z$. The Standard Model fits use the on shell mass M_Z; the relation of both is given in (65).

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