PHENOMENOLOGY OF SUPERGRAVITY UNIFIED MODELS *

CHUNG KAO

Department of Physics, University of Wisconsin
Madison, WI 53706, USA

(Received November 4, 1997)

Recent developments in the phenomenology of supergravity grand unified models are reviewed. Two special topics are discussed: *(i)* the search for neutral Higgs bosons in the minimal supergravity model via their decays into muon pairs at the CERN LHC, and *(ii)* constraints from theoretical requirements and the relic density of neutralino dark matter on the supergravity parameter space.

PACS numbers: 14.80. Ly, 12.60. Jv, 13.85. Qk

1. Introduction

The supersymmetry (SUSY) between fermions and bosons solves the problem of quadratic divergence in the Higgs sector of the Standard Model (SM) and provides a natural explanation of the Higgs mechanism for electroweak symmetry breaking (EWSB) in the framework of a grand unified theory (GUT). The evolution of renormalization group equations (RGEs) [1] with the particle content of the minimal supersymmetric standard model (MSSM) [2] is consistent with a grand unified scale ($M_{\rm GUT}$) at about 2×10^{16} GeV and an effective SUSY mass scale in the range $M_Z < M_{\rm SUSY} \lesssim 1$ TeV. Radiative corrections with a large top quark Yukawa coupling (Y_t) to Higgs bosons at the GUT scale, can drive the corresponding Higgs boson mass squared parameter negative, spontaneously breaking the electroweak symmetry and naturally explaining the origin of the electroweak scale. In a supersymmetric GUT with a large (Y_t), there is an infrared fixed point (IRFP) [3, 4] at low $\tan\beta$, and the top quark mass is predicted to be $m_t = 200\ {\rm GeV} \sin\beta$ [3]. There exits another IRFP solution ($dY_t/dt \simeq 0$) at

* Presented at the XXI School of Theoretical Physics "Recent Progress in Theory and Phenomenology of Fundamental Interactions", Ustroń, Poland, September 19–24, 1997.

$\tan\beta \sim 60$ [3, 4]. For $m_t = 175$ GeV, the two IRFPs of top quark Yukawa coupling appear at $\tan\beta \simeq 1.8$ and $\tan\beta \simeq 56$ [3].

The minimal supersymmtric standard model requires at least two Higgs doublets ϕ_1 and ϕ_2 [2, 5] that couple to the $t_3 = -1/2$ and $t_3 = +1/2$ fermions, respectively. After spontaneous symmetry breaking, there remain five physical Higgs bosons: a pair of singly charged Higgs bosons $H^\pm$, two neutral CP-even scalars H^0 (heavier) and h^0 (lighter), and a neutral CP-odd pseudoscalar A^0. At the tree level, all Higgs boson masses and couplings are determined by just two independent parameters, which are commonly chosen to be the mass of the pseudoscalar (m_A) and the ratio of vacuum expectation values (VEVs) of Higgs fields $\tan\beta \equiv v_2/v_1$.

In supergravity (SUGRA) models [6], it is assumed that SUSY is broken in a hidden sector with SUSY breaking communicated to the observable sector through gravitational interactions, leading naturally to a common scalar mass (m_0), a common gaugino mass ($m_{1/2}$), a common trilinear coupling (A_0) and a common bilinear coupling (B_0) at the GUT scale. Through minimization of the Higgs potential, the B parameter and magnitude of the superpotential Higgs mixing parameter μ are related to $\tan\beta$ and M_Z. In SUSY theories with a conserved R-parity[1], the lightest supersymmetric particle (LSP) cannot decay into normal particles and the LSP is an attractive candidate for cosmological dark matter [7, 8]. In most of the supergravity parameter space, the lightest neutralino (χ_1^0) is the LSP [3, 9].

The SUSY particle masses and couplings at the weak scale can be predicted by the evolution of RGEs from the unification scale [3, 9]. We evaluate SUSY mass spectra and couplings in the minimal supergravity model (MSUGRA) with four parameters: m_0, $m_{1/2}$, A_0 and $\tan\beta$; and the sign of the Higgs mixing parameter μ. Since A_0 mainly affects the masses of third generation sfermions, it is taken to be zero in most of our analysis. We calculate masses and couplings in the Higgs sector with one loop corrections from the top and the bottom Yukawa interactions in the RGE-improved one-loop effective potential [10] at the scale $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$ [11, 12]. At this high scale, the RGE improved one-loop corrections approximately reproduce the dominant two loop corrections [13] to the mass of the lighter CP-even scalar (m_h). The mass matrix of the charginos in the weak eigenstates ($\tilde{W}^\pm$, $\tilde{H}^\pm$) has the following form [3]

$$M_C = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & -\mu \end{pmatrix} . \quad (1)$$

This mass matrix is not symmetric and must be diagonalized by two matrices

[1] $R = +1$ for particles in the standard model and Higgs bosons and $R = -1$ for their superpartners.

[2]. The form of Eq. (1) establishes our sign convention for μ.

In Figure 1, we present masses, in the case of $\mu < 0$, for the lightest neutralino (χ_1^0), the lighter top squark ($\tilde{t}_1$), the lighter tau slepton ($\tilde{\tau}_1$), and two neutral Higgs bosons: the lighter CP-even (h^0) and the CP-odd (A^0). For $m_0 \sim 100$ GeV and $m_{1/2} \gtrsim 400$ GeV, the mass of $\tilde{\tau}_1$ can become smaller than $m_{\chi_1^0}$ so such regions are theoretically excluded. Also shown are the regions that do not satisfy the following theoretical requirements: electroweak symmetry breaking (EWSB), tachyon free, and the lightest neutralino as the LSP. The region excluded by the $m_{\chi_1^+} > 85$ GeV limit from the chargino search [14] at LEP 2 is indicated.

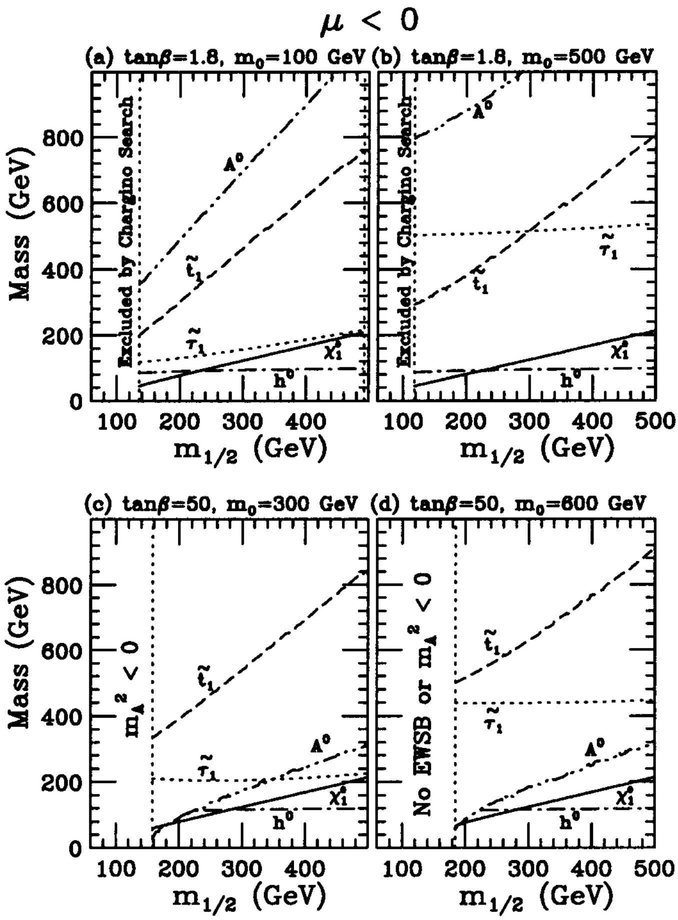

Fig. 1. Masses of χ_1^0, $\tilde{t}_1$, $\tilde{\tau}_1$ at the mass scale of M_Z and masses of h^0, and A^0 at the mass scale $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, versus $m_{1/2}$.

In this article, we briefly review recent developments in the search of neutral Higgs bosons at the LHC [15] and the neutralino relic density [12] over the full range of $\tan\beta$ and SUGRA parameter space with universal GUT scale boundary conditions. Recent measurements of the $b \to s\gamma$ decay rate by the CLEO [16] and the LEP collaborations [17] place strong constraints on the parameter space of the minimal supergravity model [18]. It was found that $b \to s\gamma$ excludes most of the MSUGRA parameter space for $\mu > 0$ with a large $\tan\beta$. Although we choose $\mu < 0$ in our analysis, our results and conclusions are almost independent of the sign of μ.

2. Discovering the Higgs bosons with muons

In our analysis, the cross section of $pp \to \phi \to \mu\bar{\mu} + X$ ($\phi = A^0, H^0$, or h^0) is evaluated from the Higgs boson cross section $\sigma(pp \to \phi + X)$ multiplied with the branching fraction of the Higgs decay into muon pairs $B(\phi \to \mu\bar{\mu})$ with two dominant subprocesses: $gg \to \phi$ and $gg \to \phi b\bar{b}$. The parton distribution functions of CTEQ3L [19] are chosen to evaluate the cross section of $pp \to \phi + X$ with $\Lambda_4 = 0.177$ GeV and $Q = M_{gg}$, where M_{gg} is invariant mass of the gluons. We take $M_Z = 91.19$ GeV, $\sin^2\theta_W = 0.2319$, $M_W = M_Z\cos\theta_W$, $m_b = 4.8$ GeV, and $m_t = 175$ GeV. The Higgs masses and couplings are evaluated with one loop corrections from the top and the bottom Yukawa interactions in the one-loop effective potential [10].

At the LHC energy, the SM Higgs boson ($h^0_{\rm SM}$) is produced dominantly from gluon fusion, and from vector boson fusion if the Higgs boson is heavy. In the MSSM, gluon fusion ($gg \to \phi$) is the major source of neutral Higgs bosons for $\tan\beta$ less than about 4. If $\tan\beta$ is larger than about 10, neutral Higgs bosons in the MSSM are dominantly produced from b-quark fusion ($b\bar{b} \to \phi$) [20]. We have evaluated the cross section of Higgs bosons in pp collisions $\sigma(pp \to \phi + X)$, with two dominant subprocesses: $gg \to \phi$ and $gg \to \phi b\bar{b}$. The cross section of $gg \to \phi b\bar{b}$ is a good approximation to the 'exact' cross section [20] of $b\bar{b} \to \phi$ for M_ϕ less than about 500 GeV. In addition, the subprocesses $gg \to \phi b\bar{b}$ and $gg \to g\phi$ are complementary to each other for producing large P_T Higgs bosons at future hadron colliders [21, 22]. Since the Yukawa couplings of $\phi b\bar{b}$ are enhanced by $1/\cos\beta$, the production rate of neutral Higgs bosons is usually enhanced with large $\tan\beta$. For m_A larger than about 150 GeV, the couplings of the lighter scalar h^0 to gauge bosons and fermions become close to those of the SM Higgs boson, therefore, gluon fusion is the major source of the h^0 even if $\tan\beta$ is large.

The QCD radiative corrections to the subprocess $gg \to \phi$ was found to be large [23], the same corrections to $gg \to \phi b\bar{b}$ are still to be evaluated. To be conservative, we take a K-factor of 1.5 for the contribution from $gg \to \phi$ and a K-factor of 1.0 for the contribution from $gg \to \phi b\bar{b}$ to evaluate the cross section of $pp \to \phi + X$. For the dominant Drell–Yan background [24,26,27], we have adopted the well known K-factor from referee [28].

If the $b\bar{b}$ mode dominates Higgs decays, the branching fraction of $\phi \to \mu\bar{\mu}$ is about $m_\mu^2/3m_b^2$, where 3 is the color factor of the quarks. It has been found that QCD radiative corrections greatly reduce the decay width of $\phi \to b\bar{b}$. Therefore, the branching fraction of $\phi \to \mu\bar{\mu}$ is about 2×10^{-4} when the $b\bar{b}$ mode dominates. For $\tan\beta \gtrsim 10$, the branching fraction of $B(\phi \to \mu\bar{\mu})$ ($\phi = A^0, H^0$, or h^0) in most of the parameter space is about 2×10^{-4}, even when the A^0 and the H^0 can decay into SUSY particles. For m_A less than about 80 GeV, the H^0 decays dominantly into h^0h^0, A^0A^0 and ZA^0.

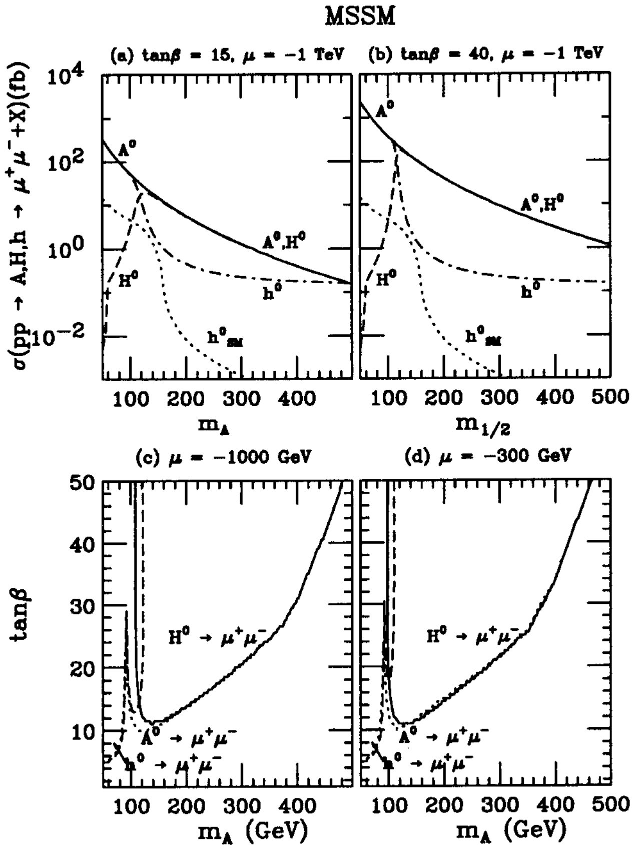

Fig. 2. The cross section of $pp \to \phi \to \mu\bar{\mu} + X$ versus m_A and 5σ contours in the $(m_A, \tan\beta)$ plane at the LHC with $L = 300\ \text{fb}^{-1}$.

In Figs 2(a) and 2(b), we present the cross section of the MSSM Higgs bosons at the LHC, $pp \to \phi \to \mu\bar{\mu} + X$, as a function of m_A for $\tan\beta = 15$ and $\tan\beta = 40$. As $\tan\beta$ increase, the production cross section is enhanced because *(i)* m_A is reduced, and *(ii)* for $\tan\beta \gtrsim 10$, the production cross section is dominated by $gg \to \phi b\bar{b}$ and enhanced by the $\phi b\bar{b}$ Yukawa coupling. Also shown is the same cross section for the SM Higgs boson h^0_{SM} with $m_{h_{SM}} = m_A$. For $m_{h_{SM}}) > 140$ GeV, the SM h^0_{SM} mainly decays into gauge bosons, therefore, the branching fraction $B(h^0_{SM} \to \mu\bar{\mu})$ drops sharply.

We define the signal to be observable if the 99% confidence level upper limit on the background is smaller than the corresponding lower limit on the signal plus background [29, 30], namely,

$$L(\sigma_s + \sigma_b) - N\sqrt{L(\sigma_s + \sigma_b)} > L\sigma_b + N\sqrt{L\sigma_b}\,,$$
$$\sigma_s > \frac{N^2}{L}[1 + 2\sqrt{L\sigma_b}/N]\,, \qquad (2)$$

where L is the integrated luminosity, and σ_b is the background cross section within a bin of width $\pm\Delta M_{\mu\bar{\mu}}$ centered at M_ϕ; $N = 2.32$ corresponds to a 99% confidence level and $N = 2.5$ corresponds to a 5σ signal.

To study the observability for the muon pair decay mode, the dominant background from the Drell–Yan (DY) process, $q\bar{q} \to Z, \gamma \to \mu\bar{\mu}$ is considered. We take $\Delta M_{\mu\bar{\mu}}$ to be the larger of the ATLAS muon mass resolution (about 2% of the Higgs bosons mass) [27, 31] or the Higgs boson width.[2] The minimal cuts applied are (1) $p_T(\mu) > 20$ GeV and (2) $|\eta(\mu)| < 2.5$ for both the signal and background. For $m_A \gtrsim 130$ GeV, m_A and m_H are almost degenerate while for $m_A \lesssim 100$ GeV m_A and m_{h^0} are very close to each other [24, 26]. Therefore, We sum up the cross sections of the A^0 and the h^0 for $m_A \leq 100$ GeV and those of the A^0 and the H^0 for $m_A > 100$ GeV.

The 5σ discovery contours at $\sqrt{s} = 14$ TeV and an integrated luminosity $L = 300$ fb^{-1} are shown in Figs 2(c) and 2(d) for $m_{\tilde{q}} = m_{\tilde{g}} = -\mu = 1$ TeV and $m_{\tilde{q}} = m_{\tilde{g}} = -\mu = 300$ GeV. The discovery region of $H^0 \to \mu\bar{\mu}$ is slightly enlarged for a smaller m_A, but the observable region of $h^0 \to \mu\bar{\mu}$ is slightly reduced because the lighter top squarks make the H^0 and the h^0 lighter and enhance the $H^0 b\bar{b}$ coupling while reducing the $h^0 b\bar{b}$ coupling.

The total cross section of $pp \to A, H \to \mu\bar{\mu} + X$ in fb, as a function of $m_{1/2}$, with $\sqrt{s} = 14$ TeV, for $\mu > 0$, $m_0 = 300$ GeV, and (a) $\tan\beta = 1.8$, (b) $\tan\beta = 10$, (c) $\tan\beta = 35$, and (d) $\tan\beta = 50$.

Also shown are the m_A versus $m_{1/2}$ and the invariant mass distribution $(d\sigma/dM_{\mu\bar{\mu}})$ of the background from the Drell–Yan process (dash) at $M_{\mu\mu} = m_A$. For $\tan\beta \sim 1.8$, m_A is very large and the cross section of the signal is much smaller than the background. For $\tan\beta \gtrsim 35$, and the cross section of the signal is greatly enhanced and can become slight larger than the background. For $\tan\beta \sim 50$ and $m_{1/2} < 200$ GeV, m_A can become close to

[2] The CMS mass resolution will be better than 2% of m_ϕ for $m_\phi \lesssim 500$ GeV [24, 26].

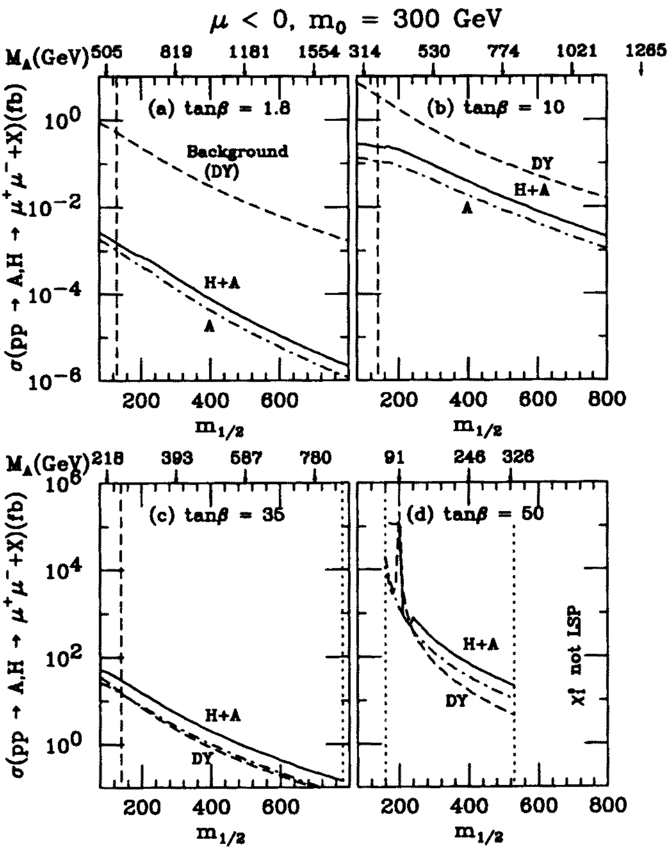

Fig. 3. The total cross section of $pp \to A, H \to \mu\bar{\mu} + X$ in fb, as a function of $m_{1/2}$.

m_Z and a peak appears in the invariant mass distribution of the background from the Drell–Yan process.

In Figure 4, we present the LHC discovery contours in the minimal supergravity model, for (a) the $m_{1/2}$ versus $\tan\beta$ plane with $m_0 = 150$ GeV, (b) the $m_{1/2}$ versus $\tan\beta$ plane with $m_0 = 500$ GeV, (c) the $m_{1/2}$ versus m_0 plane with $\tan\beta = 15$, and (d) the $m_{1/2}$ versus m_0 plane with $\tan\beta = 40$. The discovery region is the part of the parameter space between the curve of square symbol and the dash line. An increase in m_0 raises the mass of Higgs bosons.

There are a couple of interesting aspects to note: *(i)* an increase in $\tan\beta$ leads to a larger m_h but a reduction in m_A and m_H; *(ii)* increasing m_0 raises m_A, m_H and masses of the other scalars significantly.

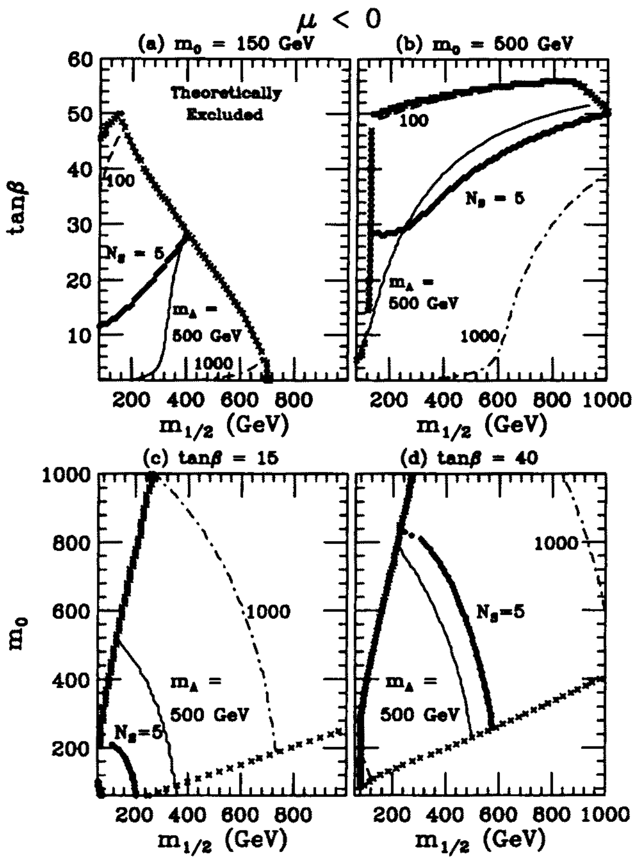

Fig. 4. The 5σ contours at the LHC with $L = 300$ fb^{-1} for muon pair discovery channel of neutral Higgs bosons in the minimal supergravity model.

3. Relic density of neutralino dark matter

The matter density of the Universe ρ is described in terms of a relative density $\Omega = \rho/\rho_c$ with $\rho_c = 3H_0^2/8\pi G_N \simeq 1.88 \times 10^{-29}h^2$ g/cm^3 the critical density to close the Universe. Here, H_0 is the Hubble constant, $h = H_0/(100$ km sec^{-1} Mpc$^{-1})$, and G_N is Newton's gravitational constant.

Studies on clusters of galaxies and large scale structure suggest that the matter density in the Universe should be at least 20% of the critical density ($\Omega_M \gtrsim 0.2$) [32], but the big-bang nucleosynthesis and the measured primordial abundance of helium, deuterium and lithium constrain the baryonic density to $0.01 \lesssim \Omega_b h^2 \lesssim 0.03$ [33]. The anisotropy in the cosmic microwave background radiation measured by the Cosmic Background Explorer (COBE) suggests that at least 60% of the dark matter should be cold (nonrelativistic) [34]. Inflationary models usually require $\Omega = 1$ for the Universe [7]. Recent measurements on the Hubble constant are converging to $h \simeq 0.6 - 0.7$ [35]. Therefore, we conservatively consider the cosmologically interesting region for $\Omega_{\chi_1^0}$ to be

$$0.1 \lesssim \Omega_{\chi_1^0} h^2 \lesssim 0.5. \tag{3}$$

Following the classic papers of Zel'dovich, Chiu, and Lee and Weinberg [36], many studies of the neutralino relic density in supergravity models have been made [37–39]. The time evolution of the number density $n(t)$ of weakly interacting mass particles is described by the Boltzmann equation [36]

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle \left[n^2 - n_E^2 \right], \tag{4}$$

where $H = 1.66 g_*^{1/2} T^2 / M_{Pl}$ is the Hubble expansion rate with $g_* \simeq 81$ the effective number of relativistic degrees of freedom, $M_{Pl} = 1.22 \times 10^{19}$ is the Planck mass, $\langle \sigma v \rangle$ is the thermally averaged cross section times velocity, v is the relative velocity and σ is the annihilation cross section, and n_E is the number density at thermal equilibrium.

In the early Universe, when the temperature $T \gg m_{\chi_1^0}$, the LSP existed abundantly in thermal equilibrium. Deviation from the thermal equilibrium began when the temperature reached the freeze-out temperature ($T_f \simeq m_{\chi_1^0}/20$). After the temperature dropped well below T_f, the annihilation rate became equal to the expansion rate and $n_{\chi_1^0} = H / \langle \sigma v \rangle$. The resulting relic density can be expressed in terms of the critical density by [7]

$$\Omega_{\chi_1^0} h^2 = m_{\chi_1^0} n_{\chi_1^0} / \rho_c \,, \tag{5}$$

where $n_{\chi_1^0}$ is the number density of χ_1^0.

The relic density at the present temperature (T_0) is then obtained from

$$\Omega h^2 = \frac{\rho(T_0)}{8.0992 \times 10^{-47} \mathrm{GeV}^4}, \tag{6}$$

where

$$\rho(T_0) \simeq 1.66 \times \frac{1}{M_{Pl}} \left(\frac{T_{\chi_1^0}}{T_\gamma} \right)^3 T_\gamma^3 \sqrt{g_*} \frac{1}{\int_0^{x_F} \langle \sigma v \rangle \, dx}. \tag{7}$$

The ratio $T_{\chi_1^0}/T_\gamma$ is the reheating factor [40] of the neutralino temperature ($T_{\chi_1^0}$) compared to the microwave background temperature (T_γ) [12,38]. The scaled freeze-out temperature ($x_F \equiv m_{\chi_1^0}/T_f$) is determined by iteratively solving the freeze out relation

$$x_F^{-1} = \log\left[\frac{m_{\chi_1^0}}{2\pi^3}\sqrt{\frac{45}{2g_* G_N}}\langle\sigma v\rangle_{x_F} x_F^{1/2}\right], \tag{8}$$

starting with $x_F = 1/20$.

The relic density of the neutralino dark matter is presented in Fig. 5.

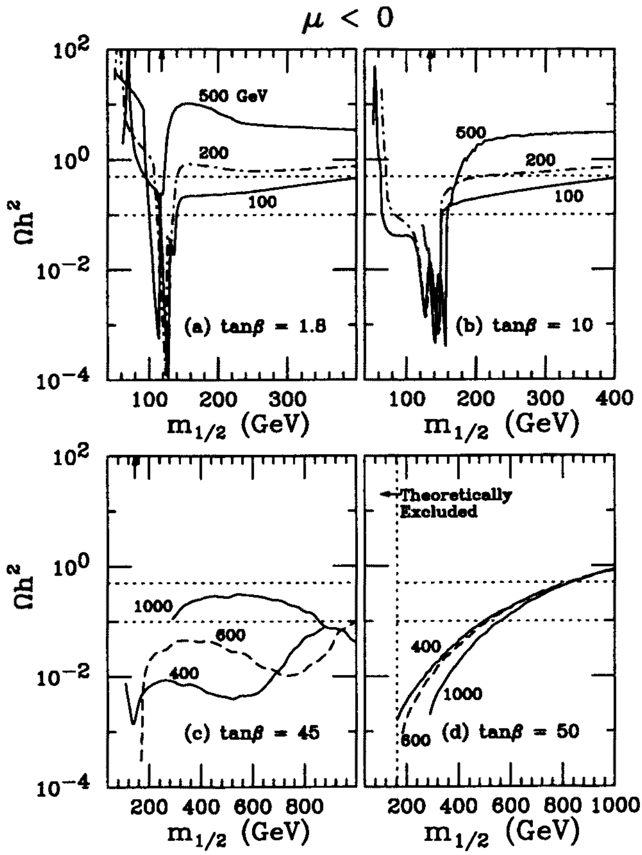

Fig. 5. The relic density of the neutralino dark matter ($\Omega_{\chi_1^0}h^2$) versus $m_{1/2}$.

When $2m_{\chi_1^0}$ is close to the mass of Z, h^0, H^0, or A^0, the cross section is significantly enhanced by the Breit–Wigner resonance pole, and a corresponding dip appears in the relic density distribution. An increase in m_0 raises the mass of Higgs bosons but does not affect the mass of χ_1^0. Increasing $\tan\beta$ slightly raises m_h, slightly lowers $m_{\chi_1^0}$, and greatly reduces

m_A and m_H. For $\tan\beta \sim 45$, extra dips appear in Fig. 5(c). These resonance dips are associated with the A^0 and H^0 masses. For $\tan\beta \leq 10$ and $m_0 > 200$ GeV, the annihilation cross section is too small for a cosmologically interesting $\Omega_{\chi_1^0} h^2$, while for $\tan\beta \sim 45$, m_0 must be close to 1000 GeV for $m_{1/2} \lesssim 750$ GeV and be larger than 400 GeV for $m_{1/2} \gtrsim 750$ GeV to obtain an interesting relic density. For $\tan\beta \gtrsim 50$, $m_A \lesssim 2m_{\chi_1^0}$, and there are no A^0 and H^0 resonant contributions; an acceptable Ωh^2 is found for $m_0 \gtrsim 400$ GeV and 400 GeV $\lesssim m_{1/2} \lesssim$ 800 GeV.

To show the constraints of Eq. (3) on the SUGRA parameter space, we present contours of $\Omega_{\chi_1^0} h^2 = 0.1$ and 0.5 in the $(m_{1/2}, \tan\beta)$ plane in Fig. 6. Also shown are the regions that do not satisfy the theoretical requirements (EWSB, tachyon free, and lightest neutralino as the LSP) and the region excluded by the chargino search ($m_{\chi_1^+} < 85$ GeV) at LEP 2 [14].

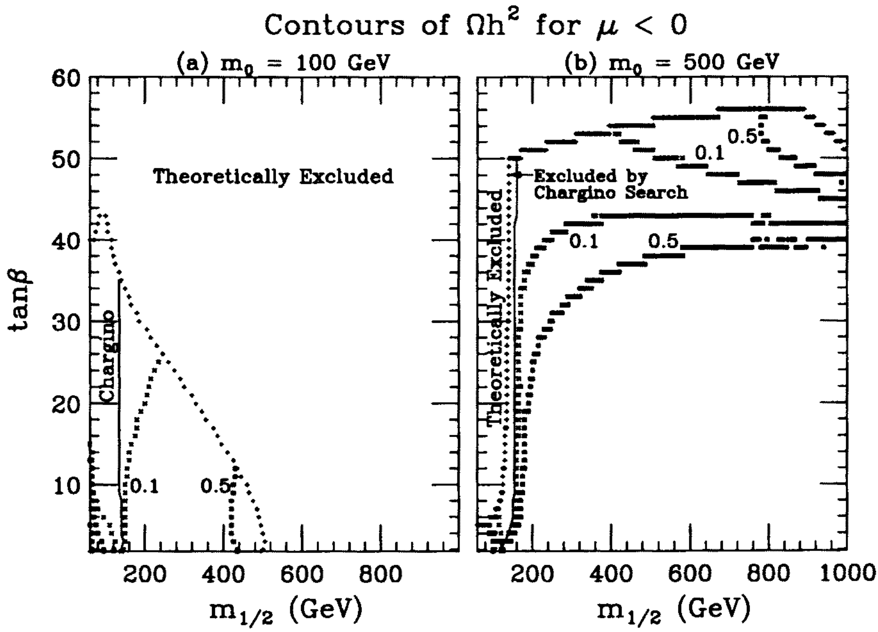

Fig. 6. Contours of $\Omega_{\chi_1^0} h^2 = 0.1$ and 0.5 in the $(m_{1/2}, \tan\beta)$ plane.

We summarize below the central features of these results. If m_0 is close to 100 GeV, (a) Most of the $(m_{1/2}, \tan\beta)$ plane with $\tan\beta \gtrsim 40$ is excluded by the above mentioned theoretical requirements. (b) The chargino search at LEP 2 excludes the region where $m_{1/2} \lesssim 100$ GeV for $\mu > 0$ and $m_{1/2} \lesssim$ 120 GeV for $\mu < 0$. (c) The cosmologically interesting region is 100 GeV $\lesssim m_{1/2} \lesssim$ 400 GeV and $\tan\beta \lesssim 25$ for either sign of μ.

If m_0 is close to 500 GeV, (a) Most of the $(m_{1/2}, \tan\beta)$ plane is theoretically acceptable for $\tan\beta \lesssim 50$ and $m_{1/2} \gtrsim 120$ GeV. (b) The LEP 2 chargino search excludes *(i)* $m_{1/2} \lesssim 140$ GeV for $\tan\beta \gtrsim 10$, and *(ii)* $m_{1/2} \lesssim 80$ GeV ($\mu > 0$) or $m_{1/2} \lesssim 120$ GeV ($\mu < 0$) for $\tan\beta \sim 1.8$. (c) The cosmologically interesting regions lie in two narrow bands.

Contours of $\Omega_{\chi_1^0} h^2 = 0.1$ and 0.5 in the $(m_{1/2}, m_0)$ plane are presented in Fig. 7 for $\mu > 0$ with $\tan\beta = 1.8$ and 50. Also shown are the theoretically excluded regions.

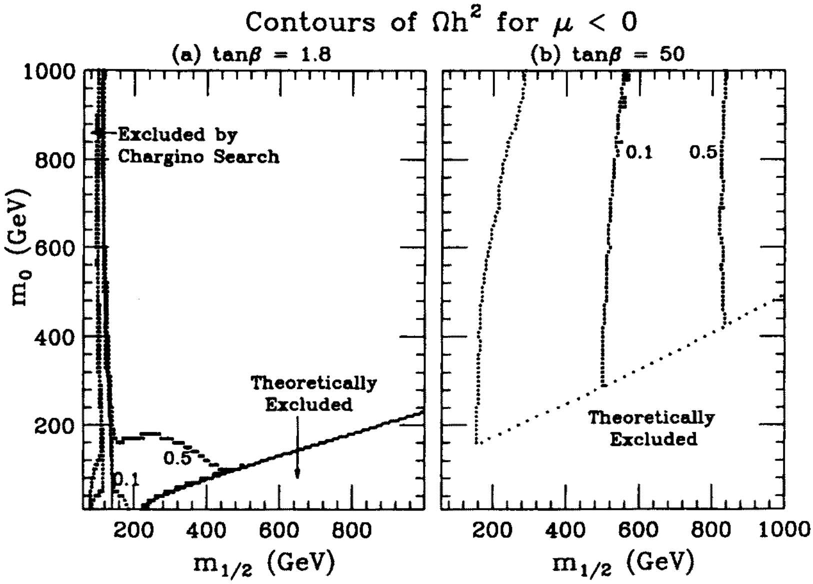

Fig. 7. Contours of $\Omega_{\chi_1^0} h^2 = 0.1$ and 0.5 in the $(m_{1/2}, m_0)$ plane.

If $\tan\beta$ is close to 1.8, *(i)* Most of the $(m_{1/2}, m_0)$ parameter space is theoretically acceptable. *(ii)* The chargino search at LEP 2 excludes the region where $m_{1/2} \lesssim 80$ GeV for $\mu > 0$ and $m_{1/2} \lesssim 110$ GeV for $\mu < 0$. *(iii)* Most of the cosmologically interesting region is 80 GeV $\lesssim m_{1/2} \lesssim 450$ GeV and $m_0 \lesssim 200$ GeV.

If $\tan\beta$ is close to 50, (a) The theoretically acceptable region in the $(m_{1/2}, m_0)$ plane is constrained to have $m_0 \gtrsim 160$ GeV and $m_{1/2} \gtrsim 150$ GeV for $\tan\beta \sim 50$. (b) The LEP 2 chargino search excludes *(i)* the region with $m_{1/2} \lesssim 125$ GeV for $\mu > 0$ or *(ii)* the region with $m_{1/2} \lesssim 135$ GeV for $\mu < 0$, which is already inside the theoretically excluded region. (c) The cosmologically interesting region lies in a band with *(i)* 475 GeV $\lesssim m_{1/2} \lesssim$ 800 GeV for $\mu > 0$, or *(ii)* 500 GeV $\lesssim m_{1/2} \lesssim$ 840 GeV for $\mu < 0$, and $m_0 \gtrsim 300$ GeV.

4. Conclusions

The muon pair decay mode can be a very promising channel to discover the neutral Higgs bosons of minimal supersymmetry and minimal supergravity. The discovery region of the $\mu\bar{\mu}$ might be slightly smaller than the $\tau\tau$ channel but it will allow precise reconstruction for the Higgs boson masses. The A^0 and H^0 might be observable in a large region of parameter space with $\tan\beta \gtrsim 10$. The h^0 might be observable in a region with $m_A < 120$ GeV and $\tan\beta \gtrsim 5$. For $m_A \gtrsim 200$ GeV and $\tan\beta > 25$, $L = 10\ \mathrm{fb}^{-1}$ would be enough to obtain Higgs boson signals with a statistical significance larger than 7. For $M_{\mu\bar{\mu}}$ close to the M_Z, the signal is marginal because it appears on the shoulder of the huge Z peak. Adequate subtraction procedures are required to extract the signal in this region.

Requiring that the neutralino relic density should be in the cosmologically interesting region, we were able to place tight constraints on the SUGRA parameter space, especially in the plane of $m_{1/2}$ versus $\tan\beta$, since the mass of the lightest neutralino depends mainly on these two parameters. The cosmologically interesting regions of the parameter space with $\tan\beta$ close to the top Yukawa infrared fixed points are found to be

$$\begin{aligned} \tan\beta = 1.8: &\quad 80\ \text{GeV} \lesssim m_{1/2} \lesssim 450\ \text{GeV and } m_0 \lesssim 200\ \text{GeV}, \\ \tan\beta = 50: &\quad 500\ \text{GeV} \lesssim m_{1/2} \lesssim 800\ \text{GeV and } m_0 \gtrsim 300\ \text{GeV}, \end{aligned}$$

where the high $\tan\beta$ result is based on $A_0 = 0$. Both regions are nearly independent of the sign of μ. The results presented in this article were based on the GUT scale trilinear coupling choice $A_0 = 0$. A recent study [12] found out that for $\tan\beta \lesssim 10$, the neutralino relic density $\Omega_{\chi_1^0} h^2$ is almost independent of the trilinear couplings A_0. For $\tan\beta \sim 40$, the relic density is reduced by a positive A_0 while enhanced by a negative A_0. The value of A_0 significantly affects $\Omega_{\chi_1^0} h^2$ only when $\tan\beta$ is large and both m_0 and $m_{1/2}$ are small.

I am grateful to my collaborator Vernon Barger for advice and comments in the preparation for this article. In addition, I would like to thank Karol Kolodziej, Jan Sladkowski and Marek Zralek for organizing a very good school of Ustron 97. This research was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-95ER40896, and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

REFERENCES

[1] K. Inoue, A. Kakuto, H. Komatsu, H. Takeshita, *Prog. Theor. Phys.* **68**, 927 (1982); **71**, 413 (1984).

[2] Reviews for the MSSM can be found in: H.P. Nilles, *Phys. Rep.* **110**, 1 (1984); P. Nath, R. Arnowitt, A. Chamseddine, *Applied* $N = 1$ *Supergravity*, ICTP Series in Theoretical Physics, Vol.I, World Scientific, Singapore 1984; H. Haber, G. Kane, *Phys. Rep.* **117**, 75 (1985); X. Tata, in *The Standard Model and Beyond*, ed. J.E. Kim, World Scientific, Singapore 1991; TASI-95 lectures, University of Hawaii Report No. UH-511-833-95 (1995).

[3] V. Barger, M.S. Berger, P. Ohmann, *Phys. Rev.* **D47**, 1093 (1993); *Phys. Rev.* **D49**, 4908 (1994); V. Barger, M.S. Berger, P. Ohmann, R.J.N. Phillips, *Phys. Lett.* **B314**, 351 (1993).

[4] B. Pendleton, G.G. Ross, *Phys. Lett.* **B98**, 291 (1981); C.T. Hill, *Phys. Rev.* **D24**, 691 (1981); H. Arason *et al., Phys. Rev. Lett.* **67**, 2933 (1991); *Phys. Rev.* **D46**, 3945 (1992); C.D. Froggatt, R.G. Moorhouse, I.G. Knowles, *Phys. Lett.* **B298**, 356 (1993); W.A. Bardeen, M. Carena, S. Pokorski, C.E.M. Wagner, *Phys. Lett.* **B320**, 110 (1994); M. Carena, M. Olechowsk, S. Pokorski, C.E.M. Wagner *Nucl. Phys.* **B419**, 213 (1994); B. Schremp, *Phys. Lett.* **B344**, 193 (1995).

[5] J. Gunion, H. Haber, G. Kane, S. Dawson, *The Higgs Hunter's Guide*, Addison-Wesley Publishing Company, Redwood City, CA 1990.

[6] A.H. Chamseddine, R. Arnowitt, P. Nath, *Phys. Rev. Lett.* **49**, 970 (1982); L. Ibañez, G. Ross, *Phys. Lett.* **B110**, 215 (1982); L. Ibañez, *Phys. Lett.* **B118**, 73 (1982); J. Ellis, D. Nanopoulos, K. Tamvakis, *Phys. Lett.* **B121**, 123 (1983); L. Alvarez-Gaumé, J. Polchinski, M. Wise, *Nucl. Phys.* **B121**, 495 (1983); L. Hall, J. Lykken, S. Weinberg, *Phys. Rev.* **D27**, 2359 (1983).

[7] E.W. Kolb, M.S. Turner, *The Early Universe*, Addison-Wesley, Redwood City, CA 1989.

[8] G. Jungman, M. Kamionkowski, K. Griest, *Phys. Rep.* **267**, 195 (1996).

[9] J. Ellis, F. Zwirner, *Nucl. Phys.* **B338**, 317 (1990); G. Ross, R.G. Roberts, *Nucl. Phys.* **B377**, 571 (1992); R. Arnowitt, P. Nath, *Phys. Rev. Lett.* **69**, 725 (1992); M. Drees, M.M. Nojiri, *Nucl. Phys.* **B369**, 54 (1993); S. Kelley *et. al.*, *Nucl. Phys.* **B398**, 3 (1993); M. Olechowski, S. Pokorski, *Nucl. Phys.* **B404**, 590 (1993); G. Kane, C. Kolda, L. Roszkowski, J. Wells, *Phys. Rev.* **D49**, 6173 (1994); D.J. Castaño, E. Piard, P. Ramond, *Phys. Rev.* **D49**, 4882 (1994); W. de Boer, R. Ehret, D. Kazakov, *Z. Phys.* **67**, 647 (1995); H. Baer, M. Drees, C. Kao, M. Nojiri, X. Tata, *Phys. Rev.* **D50**, 2148 (1994); H. Baer, C.-H. Chen, R. Munroe, F. Paige, X. Tata, *Phys. Rev.* **D51**, 1046 (1995).

[10] H. Haber, R. Hempfling, *Phys. Rev. Lett.* **66**, 1815 (1991); J. Ellis, G. Ridolfi, F. Zwirner, *Phys. Lett.* **B257**, 83 (1991); T. Okada, H. Yamaguchi, T. Tanagida, *Prog. Theor. Phys. Lett.* **85**, 1 (1991); We use the calculations of M. Bisset, Ph.D. thesis, University of Hawaii (1994).

[11] H. Baer, C.-H. Chen, M. Drees, F. Paige, X. Tata, *Phys. Rev. Lett.* **79**, 986 (1997).

[12] V. Barger, C. Kao, University of Wisconsin report MADPH-97-992 (1997), e-Print Archive: hep-ph/9704403.

[13] M. Carena, J.R. Espinosa, M. Quiros,, C.E.M. Wagner, *Phys. Lett.* **B355**, 209 (1995); M. Carena, M. Quiros, C.E.M. Wagner, *Nucl. Phys.* **B461**, 407 (1996); H. Haber, R. Hempfling, A. Hoang, CERN-TH/95-216 (1996), hep-ph/9609331.

[14] ALEPH collaboration, talk presented at CERN by G. Cowan, February (1997).

[15] V. Barger, C. Kao, University of Wisconsin report MADPH-97-1020 (1997).

[16] M.S. Alam *et al.*, (CLEO Collaboration), *Phys. Rev. Lett.* **74**, 2885 (1995).

[17] P.G. Colrain, M.I. Williams, talk presented at the International Europhysics Conference on High Energy Physics, Jerusalem, Israel, August 1997.

[18] P. Nath, R. Arnowitt, *Phys. Lett.* **B336**, 395 (1994); *Phys. Rev. Lett.* **74**, 4592 (1995); *Phys. Rev.* **D54**, 2374 (1996); F. Borzumati, M. Drees, M. Nojiri, *Phys. Rev.* **D51**, 341 (1995); H. Baer, M. Brhlik, *Phys. Rev.* **D55**, 3201 (1997); and references therein.

[19] H.L. Lai *et al.*, *Phys. Rev.* **D51**, 4763 (1995).

[20] D. Dicus, S. Willenbrock, *Phys. Rev.* **D39**, 751 (1989).

[21] R. Ellis, I. Hinchliffe, M. Soldate, J. van der Bij, *Nucl. Phys.* **B297**, 221 (1988); U. Baur, E.W.N. Glover, *Nucl. Phys.* **B339**, 38 (1990).

[22] C. Kao, *Phys. Lett.* **B328**, 420 (1994).

[23] S. Dawson, *Nucl. Phys.* **B359**, 283 (1991); A. Djouadi, M. Spira, P.M. Zerwas, *Phys. Lett.* **B264**, 440 (1991); D. Graudenz, M. Spira, P.M. Zerwas, *Phys. Rev. Lett.* **70**, 1372 (1993); M. Spira, A. Djouadi, D. Graudenz, P.M. Zerwas, *Nucl. Phys.* **B453**, 17 (1995).

[24] C. Kao, N. Stepanov, *Phys. Rev.* **D52**, 5025 (1995).

[25] E. Braaten, J.P. Leveille, *Phys. Rev.* **D22**, 715 (1980); M. Drees, K. Hikasa, *Phys. Lett.* **B240**, 455 (1990); *Phys. Lett.* **B262** (1991) 497.

[26] CMS Technical Proposal, CERN/LHCC 94-38 (1994).

[27] E. Richter-Was, D. Froidevaux, F. Gianotti, L. Poggioli, D. Cavalli, S. Resconi, CERN report CERN-TH-96-111, (1996).

[28] V. Barger, R. Phillips, *Collider Physics, updated edition*, Addison-Wesley Publishing Company, Redwood City, CA 1997.

[29] H. Baer, M. Bisset, C. Kao, X. Tata, *Phys. Rev.* **D46**, 1067 (1992).

[30] N. Brown, *Z. Phys.* **C49**, 657 (1991).

[31] Atlas Technical Proposal, CERN/LHCC 94-43 (1994).

[32] A. Dekel, M.J. Rees, *Astrophys. J.* **422**, L1 (1994); A. Dekel, *Annu. Rev. Astron. Astrophys.* **32**, 371 (1994); M. Strauss, J. Willick, *Phys. Rep.* **261**, 271 (1995); N. Bahcall, L.M. Lubin, V. Dorman, *Astrophys. J.* **447**, L81 (1995); R.G. Carlberg, H.K.C. Yee, E. Ellingson, astro-ph/9512087.

[33] T.P. Walker, G. Steigman, D.N. Schramm, K.A. Olive, H-S. Kang, *Astrophys. J* **376**, 51 (1991); M.S. Smith, L.H. Kawano, R.A. Malaney, *Astrophys. J Suppl. Ser.* **85**, 219 (1993); C.J. Copi, D.N. Schramm, M.S. Turner, *Science* **267**, 192 (1995).

[34] M. White, D. Scott, J. Silk, *Annu. Rev. Astron. Astrophys.* **32**, 319 (1994).

[35] A.G. Riess, R.P. Krishner, W.H. Press, *Astrophys. J.* **438**, L17 (1995); *Astrophys. J.* **473**, 88 (1996); D. Branch, A. Fisher, E. Baron, P. Nugent,*Astrophys. J.* **470**, L7 (1996).

[36] Ya.B. Zel'dovich, *Adv. Astron. Astrophys.* **3**, 241 (1965); H.-Y. Chiu, *Phys. Rev. Lett.* **17**, 712 (1966); B. Lee, S. Weinberg, *Phys. Rev. Lett.* **39**, 165 (1977).

[37] R. Barbieri, M. Frigeni, G.F. Giudice, *Nucl. Phys.* **B313**, 725 (1989); K. Griest, D. Seckel, *Phys. Rev.* **D43**, 3191 (1991); J.L. Lopez, D.V. Nanopoulos, K.-J. Yuan, *Phys. Lett.* **B267** (1991) 219; *Nucl. Phys.* **B370**, 445 (1992); *Phys. Rev.* **D48**, 2766 (1993); J. Ellis, L. Roszkowski, *Phys. Lett.* **B283**, 252 (1992); L. Roszkowski, R. Roberts, *Phys. Lett.* **B309**, 329 (1993); M. Drees, M.M. Nojiri, *Phys. Rev.* **D 47**, 376 (1993). G.L. Kane, C. Kolda, L. Roszkowski, J.D. Wells, *Phys. Rev.* **D49**, 6173 (1994); E. Diehl, G.L. Kane, C. Kolda, J.D. Wells, *Phys. Rev.* **D52**, 4223 (1995). R. Arnowitt, P. Nath, *Phys. Lett.* **B299**, 58 (1993); **B307**, 403(E) (1993); *Phys. Rev. Lett.* **70**, 3696 (1993); *Phys. Rev.* **D54**, 2374 (1996); J. Ellis, T. Falk, K.A. Olive, M. Schmitt, *Phys. Lett.* **B388**, 97 (1996); CERN Report No. CERN-TH-97-105, hep-ph/9705444.

[38] H. Baer, M. Brhlik, *Phys. Rev.* **D53**, 597 (1996).

[39] D. Matalliotakis, H.P. Nilles, *Nucl. Phys.* **B435**, 115 (1995); M. Olechowski, S. Pokorski, *Phys. Lett.* **B344**, 201 (1995); V. Berezinskii *et al.*, *Astropart. Phys.* **5**, 1 (1996); P. Nath, R. Arnowitt, *Phys. Rev.* **D56**, 2820 (1997).

[40] J. Ellis *et al.*, *Nucl. Phys.* **B238**, 453 (1984).