

# NEW ANALYSIS OF THE $\Delta I = 1/2$ RULE IN THE $1/N_c$ EXPANSION FOR $K \rightarrow \pi\pi$ DECAYS \*

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We analyze long-distance contributions to the  $K \rightarrow \pi\pi$  amplitudes relevant for the  $\Delta I = 1/2$  selection rule in the framework of the  $1/N_c$  expansion. We use a modified prescription for the identification of meson momenta in the chiral loop corrections to gain a consistent matching with the short-distance part. Our approach involves a separation of non-factorizable and factorizable  $1/N_c$  corrections. Along these lines we calculate the one-loop contributions from the lowest order Lagrangian. Our main result is an additional enhancement of the  $\Delta I = 1/2$  channel amplitude which we find in good agreement with experiment.

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## 1. Introduction

Since the first observation of the  $\Delta I = 1/2$  enhancement more than 40 years ago [1], there have been many attempts to find dynamical mechanisms responsible for it, in particular within the standard model. This  $\Delta I = 1/2$  rule was particularly enigmatic before the birth of QCD when only the current-current operator  $Q_2$  arising from the W exchange was considered and, consequently, the ratio  $\text{Re}A(K \rightarrow (\pi\pi)_{I=0})/\text{Re}A(K \rightarrow (\pi\pi)_{I=2}) \equiv \text{Re}A_0/\text{Re}A_2 \equiv R$  was expected to be about one order of magnitude smaller than the experimentally observed value  $R = 22.2$ . Now, since the establishment of QCD, our understanding of this rule has improved considerably. Within QCD, the  $K \rightarrow \pi\pi$  amplitudes are obtained from the effective hamiltonian for  $\Delta S = 1$  transitions [2-4],

$$\mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^8 c_i(\mu) Q_i(\mu), \quad (1)$$

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involving the Wilson coefficients  $c_i(\mu)$  which can be calculated for a scale  $\mu \gtrsim 1$  GeV using perturbative renormalization group techniques, as well as, the local four-quark operators  $Q_i(\mu)$ . The hadronic matrix elements of these operators are difficult to calculate but can be estimated using non-perturbative techniques generally for  $\mu$  around a scale of 1 GeV. For the  $\Delta I = 1/2$  rule, only the  $z_i$  part of the Wilson coefficient  $c_i$  is numerically relevant, with  $c_i(\mu) = z_i(\mu) - y_i(\mu) V_{ts}^* V_{td} / (V_{us}^* V_{ud})$ . The dominant operators are

$$Q_1 = 4\bar{s}_L\gamma^\mu d_L \bar{u}_L\gamma_\mu u_L, \quad Q_2 = 4\bar{s}_L\gamma^\mu u_L \bar{u}_L\gamma_\mu d_L, \quad Q_6 = -8 \sum_{q=u,d,s} \bar{s}_L q_R \bar{q}_R d_L, \quad (2)$$

with  $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$ . Major improvements were obtained when it was observed that the QCD (and electroweakly) induced effective hamiltonian of Eq. (1) can explain various large enhancements. These can be of short-distance (SD) nature, like the first identified *octet enhancement* [2] in the  $Q_1$ - $Q_2$  sector dominated by the increase of  $z_2$  when  $\mu$  evolves from  $M_W$  down to  $\mu \simeq 1$  GeV. Another important SD enhancement was found to arise in the sector of the QCD penguin operators, in particular for  $z_6$ , through the proper inclusion of the threshold effects (and the associated incomplete GIM cancellation) [5]. Enhancements are also of long-distance (LD) nature like the first identified LD enhancement of the matrix elements of the QCD penguin operators over the matrix elements of  $Q_1$  and  $Q_2$ . The latter was first conjectured and estimated in Ref. [3] using the vacuum insertion method. Due to the non-perturbative character of the LD contribution, a large variety of techniques has been proposed to estimate it (for some recent publications see Ref. [6]). Among the analytical methods, the  $1/N_c$  approach [5] associated with the chiral effective Lagrangian is particularly interesting. In this approach, an additional LD enhancement is obtained from inclusion of chiral loop effects in the  $Q_1$ - $Q_2$  sector. The net result of all enhancements mentioned above is a value of  $R$  in the range of 70-75% [5] of the measured value  $R = 22.2$ , suggesting that the bulk of the  $\Delta I = 1/2$  rule in  $K \rightarrow \pi\pi$  decays is now understood.<sup>1</sup> One might note that the agreement with experiment is not improved by inclusion of the NLO renormalization group equations for the  $z_i$ 's [7].

In this proceeding, we reconsider the calculation of  $K \rightarrow \pi\pi$  amplitudes relevant for the  $\Delta I = 1/2$  rule in the  $1/N_c$  approach of Ref. [5]. Our main improvement consists in the fact that we use a modified matching procedure in order to remove previous ambiguities. This will be done treating the

<sup>1</sup> In fact, the  $\Delta I = 3/2$  channel is usually obtained "sufficiently suppressed" whereas the  $\Delta I = 1/2$  enhancement is obtained only partially. In Ref. [5], for  $m_s(1 \text{ GeV}) = 150$  MeV and  $\Lambda_{\text{QCD}} \equiv \Lambda^{(4)} = 300$  MeV, the latter channel was reproduced to  $\simeq 70\%$ .

factorizable (F) and non-factorizable (NF) contributions differently. Only the NF diagrams are matched with the SD contributions to cancel the scale dependence of the SD Wilson coefficients (after having, as we will see, factorized out the scale dependence of the coefficient  $1/m_s^2$  in the matrix elements of the operator  $Q_6$ ). Factorizable loop contributions which refer uniquely to the strong sector of the theory can be calculated in full chiral perturbation theory ( $\chi$ PT), the corresponding scale dependence being absorbed in the renormalization of the various parameters in the Lagrangian. As a result of this procedure, performing a matching with the SD Wilson coefficients, we obtain an additional enhancement of the  $\Delta I = 1/2$  channel which we find to be in good agreement with experiment. The  $\Delta I = 3/2$  channel, however, exhibits a large dependence on the matching scale, resulting from the difference of two large numbers, but is found to be equal or smaller than the experimental value in such a way that the ratio  $R = 22.2$  is reproduced or even passed beyond.

## 2. General framework

To calculate the matrix elements, we start from the chiral effective Lagrangian for pseudoscalar mesons which involves an expansion in momenta where terms up to  $\mathcal{O}(p^4)$  are included [8],

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{f^2}{4} \left( \langle \partial_\mu U^\dagger \partial^\mu U \rangle + \frac{\alpha}{4N_c} \langle \ln U^\dagger - \ln U \rangle^2 + r \langle \mathcal{M} U^\dagger + U \mathcal{M}^\dagger \rangle \right) \\ & + r L_5 \langle \partial_\mu U^\dagger \partial^\mu U (\mathcal{M}^\dagger U + U^\dagger \mathcal{M}) \rangle, \end{aligned} \quad (3)$$

with  $\langle A \rangle$  denoting the trace of  $A$  and  $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ .  $f$  and  $r$  are free parameters related to the pion decay constant  $F_\pi$  ( $= 93$  MeV) and to the quark condensate, respectively, with  $r = -2\langle \bar{q}q \rangle / f^2$ . Up to terms of  $\mathcal{O}(p^4)$  and to leading order in the  $1/N_c$  expansion, the Lagrangian of Eq. (3) has the most general structure relevant for the operators  $Q_1$ ,  $Q_2$  and  $Q_6$ . The degrees of freedom of the complex matrix  $U$  are identified with the pseudoscalar meson nonet given in a non-linear representation:

$$U = \exp \frac{i}{f} \Pi, \quad \Pi = \pi^a \lambda_a, \quad \langle \lambda_a \lambda_b \rangle = 2\delta_{ab}, \quad (4)$$

where, in terms of the physical states,

$$\Pi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}a\eta + \sqrt{\frac{2}{3}}b\eta' & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}a\eta + \sqrt{\frac{2}{3}}b\eta' & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}b\eta + \sqrt{\frac{2}{3}}a\eta' \end{pmatrix}, \quad (5)$$

with  $a = \cos \theta - \sqrt{2} \sin \theta$  and  $\sqrt{2}b = \sin \theta + \sqrt{2} \cos \theta$ .  $\theta$  is the  $\eta - \eta'$  mixing angle satisfying the relation [9]

$$\tan 2\theta = \frac{2m_{80}^2}{m_{00}^2 - m_{88}^2} = 2\sqrt{2} \left[ 1 - \frac{3\alpha}{2(m_K^2 - m_\pi^2)} \right]^{-1}, \quad (6)$$

which yields  $\theta \simeq -19^\circ$ . Note that we treat the singlet as a dynamical degree of freedom. Consequently, in Eq. (3), we include the strong anomaly term, with the instanton parameter  $\alpha$  ( $\simeq 0.72 \text{ GeV}^2$ ), which gives a non-vanishing mass of the  $\eta_0$  in the chiral limit ( $m_q = 0$ ) reflecting the explicit breaking of the axial  $U(1)$  symmetry. The Lagrangian of Eq. (3) is equivalent to the one of Ref. [5] (provided we identify the coefficient  $1/\Lambda_\chi^2$  of Ref. [5] with  $4L_5/f^2$ ), except for the fact that we explicitly include the  $\eta_0$ .

A straightforward bosonization yields the chiral representation of the quark currents and densities

$$(J_L^\mu)_{ij} = \bar{q}_{iL} \gamma^\mu q_{jL} = -i \frac{f^2}{2} (U^\dagger \partial^\mu U)_{ji} + ir L_5 \left( \partial^\mu U^\dagger \mathcal{M} - \mathcal{M}^\dagger \partial^\mu U + \partial^\mu U^\dagger U \mathcal{M}^\dagger U - U^\dagger \mathcal{M} U^\dagger \partial^\mu U \right)_{ji}, \quad (7)$$

$$(D_L)_{ij} = (D_R^\dagger)_{ij} = \bar{q}_{iR} q_{jL} = -r \left( \frac{f^2}{4} U^\dagger + L_5 \partial_\mu U^\dagger \partial^\mu U U^\dagger \right)_{ji}. \quad (8)$$

Using Eqs (7)–(8), the operators  $Q_1$ ,  $Q_2$  and  $Q_6$  can be expressed in terms of the meson fields.

The  $1/N_c$  corrections to the matrix elements  $\langle Q_i \rangle_I$  are calculated by chiral loop diagrams. The loop expansion involves a series in  $1/f^2 \sim 1/N_c$  which is in direct correspondence with the short-distance expansion in terms of  $\alpha_s/\pi \sim 1/N_c$ . In these diagrams, we encounter integrals which are regularized by a finite cutoff as it was introduced in Ref. [5]. Due to the truncation to pseudoscalar mesons, the cutoff has to be taken at or, preferably, below the  $\mathcal{O}(1\text{GeV})$ . This restriction is a common feature of the phenomenological approaches at hand in which higher resonances are not included.

To retain the physical amplitudes  $A_I$ , which as a matter of principle are scale-independent, the long- and the short-distance contributions are evaluated at the cutoff scale, *i.e.*, the LD ultraviolet scale is identified with the SD infrared one. Performing this identification we must take into account that, within the cutoff regularization, there is a dependence on the way we define the momentum variable inside the loop.

In the standard approach of Refs. [5, 10, 11], the cutoff is associated to the virtual meson, *i.e.*, the integration variable is identified with the

meson momentum. Consequently, as there is no corresponding quantity in the short-distance part, a rigorous matching of long- and short-distance contributions is not possible.

The ambiguity is removed by associating the cutoff to the effective color singlet gauge boson as introduced within a study of the  $K_L$ - $K_S$  mass difference [12]. This is done by identifying in the LD, as well as in the SD part, the momentum variable in the loop integration with the momentum flow between the two currents or densities. With respect to the standard approach, the momentum of the virtual meson is shifted by the external momentum, the former being no longer identical with the integration variable, which affects both the ultraviolet, as well as the infrared structure of the  $1/N_c$  corrections. Note that the matching prescription advocated in Ref. [12] was also used for current-current operators in the chiral limit in Ref. [13]. The authors showed that the coefficient of the quadratic term in the cutoff is increased by a factor  $3/2$  relative to the one obtained from the standard matching prescription [5, 10, 11]. This provides an additional *octet enhancement* which, however, has to be confirmed by a full calculation of the amplitudes relevant for the  $\Delta I = 1/2$  rule. As we will argue, from this calculation we performed, we confirm this enhancement which is even largely increased by the effects beyond the chiral limit.

Obviously, the modified procedure described above is applicable to the non-factorizable part of the interaction and not to the factorizable ones. The factorizable part, however, refers to the strong sector of the theory, and has not to be matched with any SD contribution. Consequently, it can be calculated in pure  $\chi$ PT. In this case dimensional regularization can (and will) be used. Therefore, no momentum prescription ambiguities appear. This separation of F and NF contributions was already applied in a similar way to investigate the  $B_K$  parameter [14].

### 3. Calculation of the amplitudes and results

Expanding the Lagrangian of Eq. (3), as well as, the currents and densities of Eqs (7)–(8) in terms of pseudoscalars fields, the various F and NF one-loop diagrams to be calculated are given in Fig. 1 and Fig. 2, respectively.

In addition to these diagrams, the effect of the wave function renormalization must be included, and the values of the various parameters in the Lagrangian must be expressed in terms of physical quantities. This we did in pure  $\chi$ PT following our general procedure for the factorizable contributions. We checked that the scale dependence coming from the factorizable diagrams is completely cancelled by the scale dependence of the various parameters in the tree level expression of the matrix elements. Explicitly, we obtain the

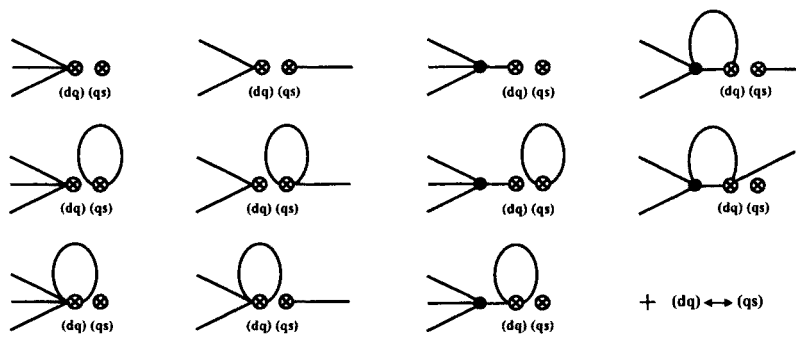


Fig. 1. Factorizable diagrams contributing to the matrix elements of the operators  $Q_i$ . Crossed circles represent the currents or densities (with indices (dq) (qs) here specified for  $Q_6$ ); black circles denote strong vertices. The lines represent the pseudoscalar mesons.

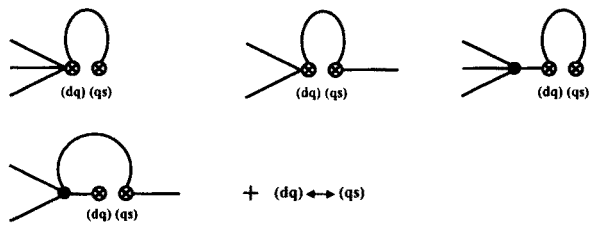


Fig. 2. Non-factorizable diagrams with indices here specified for  $Q_6$ .

following expressions:

$$i\langle\pi^+\pi^-|Q_2|K^0\rangle^F = -i\langle\pi^0\pi^0|Q_1|K^0\rangle^F = X\left(1 + \frac{4L_5^r}{F_\pi^2}m_\pi^2\right), \tag{9}$$

$$i\langle\pi^+\pi^-|Q_1|K^0\rangle^F = i\langle\pi^0\pi^0|Q_2|K^0\rangle^F = 0, \tag{10}$$

$$i\langle\pi^+\pi^-|Q_6|K^0\rangle^F = i\langle\pi^0\pi^0|Q_6|K^0\rangle^F = -4\frac{X}{F_\pi^2}\left(\frac{2m_K^2}{\hat{m} + m_s}\right)^2 L_5^r, \tag{11}$$

with  $X = \sqrt{2}F_\pi(m_K^2 - m_\pi^2)$  and  $L_5^r$  defined by the relation

$$\frac{F_K}{F_\pi} = 1 + \frac{4L_5^r}{F_\pi^2}(m_K^2 - m_\pi^2). \tag{12}$$

For the reason of brevity, in Eqs (9)–(11) we omit scale-independent (finite) terms resulting from the one-loop corrections which, nevertheless, are taken into account within the numerical analysis.

Note that in the definition of  $L_5^r$ , in the denominator on the left-hand side of Eq. (12), we used  $F_\pi$  rather than  $f$ . Formally, the difference represents higher order effects. Nevertheless, the appearance of  $f$  in Eq. (12) would induce residual scale dependence in Eqs (9)–(11) which has no counterpart at the SD level. Therefore, the choice of  $F_\pi$  is more adequate as it ensures that no scale dependence appears in the complete factorizable LD contribution of the matrix elements except the scale dependence of  $m_s$ . To be explicit, in accordance with current conservation, there is no scale dependence in Eqs (9)–(10); as to the density-density operator of Eq. (11), we are just left with the scale dependence of  $1/m_s^2$ . This is to be expected since the evolution of  $m_s$  is just inverse to the evolution of a single density operator. Consequently, the scale dependence of Eq. (11) (which is of leading order in  $N_c$ ) exactly cancels the corresponding diagonal evolution of  $z_6$  at SD. This characteristic has already been observed in Ref. [5].

The non-factorizable contributions are calculated from the diagrams of Fig. 2, by means of introducing a cutoff regulator  $\Lambda_{NF}$ . Using the improved matching prescription as explained above, we obtain

$$i\langle \pi^0 \pi^0 | Q_1 | K^0 \rangle^{NF} = 0, \quad (13)$$

$$i\langle \pi^+ \pi^- | Q_1 | K^0 \rangle^{NF} = \frac{X}{16\pi^2 F_\pi^2} \left( -3\Lambda_{NF}^2 + \frac{1}{4}(m_K^2 + 12m_\pi^2) \ln \Lambda_{NF}^2 \right), \quad (14)$$

$$i\langle \pi^0 \pi^0 | Q_2 | K^0 \rangle^{NF} = \frac{X}{16\pi^2 F_\pi^2} \left( \frac{9}{2}\Lambda_{NF}^2 + \frac{3}{4}(m_K^2 - 6m_\pi^2) \ln \Lambda_{NF}^2 \right), \quad (15)$$

$$i\langle \pi^+ \pi^- | Q_2 | K^0 \rangle^{NF} = \frac{X}{16\pi^2 F_\pi^2} \left( \frac{3}{2}\Lambda_{NF}^2 + (m_K^2 - \frac{3}{2}m_\pi^2) \ln \Lambda_{NF}^2 \right), \quad (16)$$

$$i\langle \pi^0 \pi^0 | Q_6 | K^0 \rangle^{NF} = -\frac{X}{16\pi^2 F_\pi^2} \frac{3}{2} r^2 \ln \Lambda_{NF}^2, \quad (17)$$

$$i\langle \pi^+ \pi^- | Q_6 | K^0 \rangle^{NF} = -\frac{X}{16\pi^2 F_\pi^2} \frac{3}{2} r^2 \ln \Lambda_{NF}^2. \quad (18)$$

Here again we omit finite terms which, however, are taken into account in the numerical analysis. The scale-dependent terms in Eqs (13)–(18) have to be matched with the Wilson coefficients. To this end, we use the numerical values presented in the leading logarithm analyze of Ref. [15].

Note that in the denominators of Eqs (13)–(18) we took the physical value  $F_\pi$  rather than  $f$ , in the same way as for the factorizable diagrams. Again, the difference represents higher order effects. However, the scale dependence of  $f$  in Eqs (13)–(18) has no counterpart in the SD and will be absorbed at the next order of the chiral expansion. Note also that in Ref. [5] only the tree level contribution was taken into account in the matrix

elements of  $Q_6$ . The latter are proportional to the  $L_5$  coefficient, as the tree level contribution of the  $\mathcal{O}(p^0)$  coming from  $\mathcal{L}(p^2)$  vanishes due to the unitarity of  $U$ . From the point of the  $N_c$  counting, it is justified to consider only the tree level contribution for  $Q_6$ , since the  $z_6$  coefficient is  $\mathcal{O}(1/N_c)$ . In Ref. [5], only the one-loop diagrams induced by the operators  $Q_1$  and  $Q_2$  were considered. Loops over  $Q_6$  from  $\mathcal{L}(p^2)$ , on the other hand, were assumed to be zero as the corresponding tree level contribution from  $\mathcal{L}(p^2)$  is zero. However, an explicit calculation of the NF one-loop contributions to the matrix elements of  $Q_6$  coming from  $\mathcal{L}(p^2)$  yields a non-trivial cutoff dependence, as shown in Eqs (17) and (18), which has to be matched with the SD part. This effect cannot be neglected since it corresponds to the leading non-vanishing order of the twofold series expansion in  $1/N_c$  and  $p^2$ ; *i.e.*, it is of the same order as the tree level term proportional to  $L_5$  (the former, being of  $\mathcal{O}(p^0/N_c)$ , is the leading term in the  $p^2$  expansion, while the latter, being of order  $\mathcal{O}(p^2)$ , is leading in the  $1/N_c$  expansion). The resulting scale dependence, however, is only of minor importance within the analysis of the  $\Delta I = 1/2$  rule.<sup>2</sup>

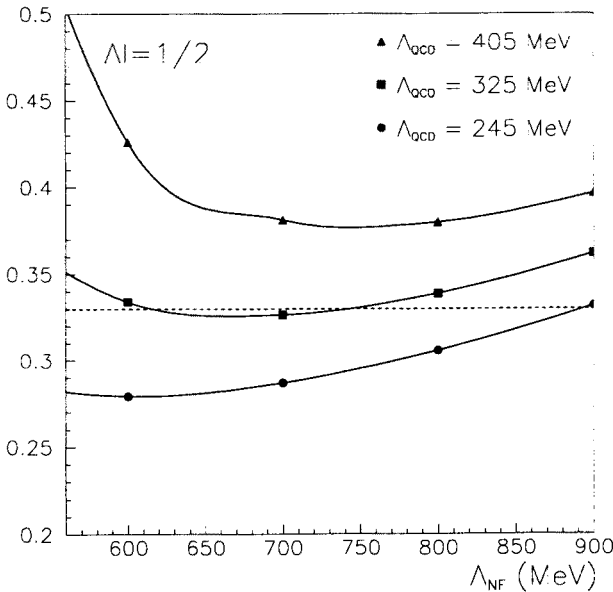


Fig. 3.  $\text{Re}A_0$  in units of  $10^{-3}$  MeV for  $m_s(1 \text{ GeV}) = 150$  MeV as a function of the matching scale  $\Lambda_{NF}$ . The experimental value is represented by the dashed line.

<sup>2</sup> It is actually important within the analysis of the ratio  $\epsilon'/\epsilon$ . Further details on this new calculation of the matrix elements of  $Q_6$  will be presented elsewhere [16].



Our numerical result for the amplitude  $\text{Re}A_0$  is shown in Fig. 3. It shows an additional enhancement which renders the amplitude in good agreement with the observed value. The new contribution arises from the  $Q_1$  and  $Q_2$  operators. It is due to the modified matching prescription in the NF sector (except for approximately one fifth of it which is due to the choice of the physical value  $F_\pi$  in Eqs (13)–(18) as explained above). Our result is remarkably stable with respect to the matching scale. The main uncertainty displayed in Fig. 3 originates from the dependence of the Wilson coefficients on  $\Lambda_{\text{QCD}} (\equiv \Lambda^{(4)})$ . On the other hand, the isospin 3/2 amplitude shown in Fig. 4 is highly unstable. The large uncertainty can be understood from the fact that it is obtained from the difference of two large amplitudes of approximately the same size [namely  $A(K \rightarrow \pi^+\pi^-)$  and  $A(K \rightarrow \pi^0\pi^0)$ ]. Consequently, the 3/2 amplitude is not well reproduced except that, and this is a crucial point, it comes out to be sufficiently suppressed whatever the particular chosen scale is between 500 MeV and 1 GeV.

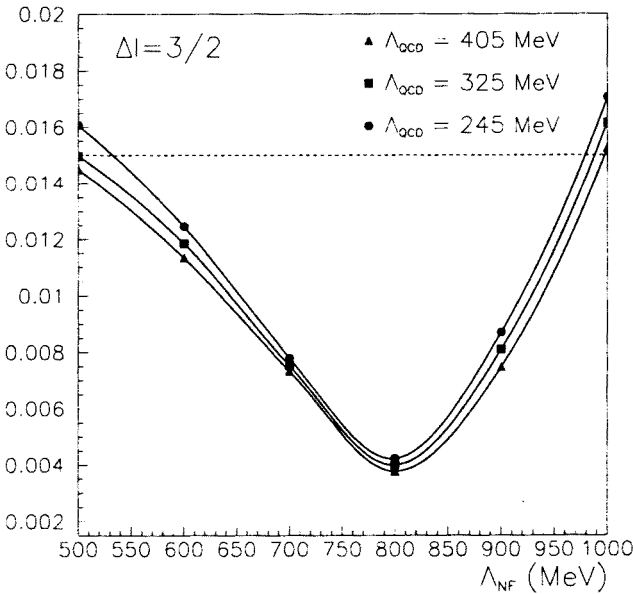


Fig. 4.  $\text{Re}A_2$  in units of  $10^{-3}$  MeV for  $m_s(1 \text{ GeV}) = 150$  MeV. The experimental value is represented by the dashed line.

In conclusion, it is certainly premature to say that the  $\Delta I = 1/2$  rule for  $K \rightarrow \pi\pi$  decays is now understood completely since the  $1/N_c$  approach we use is only an approximate method. In particular, vector mesons and higher resonances should be included in order to take a higher and more secure value for the matching scale. (It is probable that the vector mesons play

an important role for  $A_2$ ). Nevertheless, we believe that the enhancement reported here is a further important indication that the  $1/N_c$  approach can account for the bulk of the  $\Delta I = 1/2$  rule for  $K \rightarrow \pi\pi$  decays.

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