FORM FACTORS AND RADIATIVE CORRECTIONS IN Z' PHYSICS*

A. Leike

Ludwigs–Maximilians-Universität, Sektion Physik
Theresienstr. 37, D-80333 München, Germany
E-mail: leike@graviton.hep.physik.uni-muenchen.de

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This lecture contains a pedagogical approach to the description of Z' physics in the formalism of form factors. Usually, only electroweak corrections are described by form factors, which modify the Weinberg angle and the overall normalization. It is demonstrated how this formalism can be extended to include different Born contributions. In the second part of the lecture, QCD and QED corrections are considered. The development and consequences of the radiative tail for a Z' search are discussed in detail.

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1. Introduction

The existence of an extra neutral gauge boson (Z') is predicted in many theories [1, 2], which go beyond the Standard Model (SM). Although the mass of the Z' is generally not predicted, there are theories where its mass is naturally of the order of the electroweak breaking scale [3]. If a Z' is found at present or future colliders, it will provide us with information on the large gauge group and on its symmetry breaking. A Z' search is therefore foreseen at every present and future collider.

In this lecture, we focus on two topics. The first topic is the description of Z' physics in terms of form factors of the Z [4]. This formalism has the advantage that it appears at the level of the amplitude. It is therefore applicable equally well to all four fermion interactions. These processes would give the main information on a Z' at e^+e^-, $e^\pm p$, $pp(p\bar{p})$ colliders. Form factors allow an easy extension of existing SM programs to Z' physics.

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The second topic are QCD and QED corrections. They are known to be very process dependent. The radiative tail arising due to initial state radiation is discussed in detail for the process $e^+e^- \to f\bar{f}$.

Specific Z' models [1, 2] and present experimental bounds [3, 5] on the Z' mass and the ZZ' mixing angle are not discussed in this lecture. See [6] for simple estimates of these bounds.

The form factors as known from electroweak corrections are introduced in Section 2. They are applied to different Born contributions relevant in Z' processes in Section 3. In Section 4, radiative corrections are discussed. Much room is devoted to the discussion of the radiative tail. We conclude in Section 5.

2. Form factors in four fermion interactions

2.1. Assumptions and definitions

We will consider only the case of massless final fermions. Observables, which are sensitive to transversal polarizations will not be discussed. Then, one can investigate the unpolarized case. Different helicity states can be obtained by a simple substitution of the couplings. Consider the four fermion interaction $e^+e^- \to f\bar{f}$ $(f \neq e)$ as an example.

The SM Z boson couples to all known fermions f with non-zero vector and axial vector couplings v_f, a_f. Denote the amplitude mediated by the SM Z boson by $\mathcal{M}^Z$,

$$\mathcal{M}^Z = \frac{g_Z^2}{s-m_Z^2}\bar{v}(e)\gamma_\beta\left[v_e - \gamma_5 a_e\right]u(e)\cdot\bar{u}(f)\gamma^\beta\left[v_f - \gamma_5 a_f\right]v(f). \quad (1)$$

m_Z stands for the complex mass of the Z boson, $m_Z^2 = M_Z^2 - i\Gamma_Z M_Z$, g_Z is the coupling strength of the fermion current to the Z boson, and s is the c.m. energy.

In general, it is not sufficient to describe four fermion interactions by the Z amplitude alone. Suppose that an extra amplitude $\mathcal{M}$ must be added to $\mathcal{M}^Z$ to reach a given precision,

$$\begin{aligned}
\mathcal{M}^\Sigma &= \mathcal{M}^Z + \mathcal{M} \\
&\equiv \frac{g_Z^2}{s-m_Z^2}\bar{v}(e)\gamma_\beta u(e)\cdot\bar{u}(f)\gamma^\beta v(f)\cdot v_e(1)v_f(1)(1+\varepsilon_{vv}) \\
&\qquad -\bar{v}(e)\gamma_\beta u(e)\cdot\bar{u}(f)\gamma^\beta\gamma_5 v(f)\cdot v_e(1)a_f(1)(1+\varepsilon_{va}) \\
&\qquad -\bar{v}(e)\gamma_\beta\gamma_5 u(e)\cdot\bar{u}(f)\gamma^\beta v(f)\cdot a_e(1)v_f(1)(1+\varepsilon_{av}) \\
&\qquad +\bar{v}(e)\gamma_\beta\gamma_5 u(e)\cdot\bar{u}(f)\gamma^\beta\gamma_5 v(f)\cdot a_e(1)a_f(1)(1+\varepsilon_{aa})\,. \quad (2)
\end{aligned}$$

The extra amplitude can be large but it must have the same structure as $\mathcal{M}^Z$. Then, the contribution of $\mathcal{M}$ can be absorbed into $\mathcal{M}^Z$ by a redefinition of the Z couplings to fermions,

$$\tilde{x}_e \tilde{y}_f = x_e y_f (1 + \varepsilon_{xy}), \quad x, y = a, v. \tag{3}$$

The coefficients ε_{xy} in equation (2) contain all information on the amplitude $\mathcal{M}$. Several examples of ε_{xy} will be discussed in the next sections.

Following the tradition of electroweak corrections [7], the contributions ε_{xy} are parametrized by (complex) form factors $\rho_{\rm ef}$, κ_e, κ_f, $\kappa_{\rm ef}$, which are introduced by replacements of the couplings [8],

$$\begin{aligned}
v_e v_f &\to a_e a_f \left[1 - 4|Q^e| s_W^2 \kappa_e - 4|Q^f| s_W^2 \kappa_f + 16 |Q^e Q^f| s_W^4 \kappa_{\rm ef} \right], \\
v_e &\to a_e \left[1 - 4|Q^e| s_W^2 \kappa_e \right], \\
v_f &\to a_f \left[1 - 4|Q^f| s_W^2 \kappa_f \right], \\
a_e, a_f &\to \text{unchanged}, \\
g_Z^2 &\to g_Z^2 \rho_{\rm ef}.
\end{aligned} \tag{4}$$

$s_W^2 = \sin^2\theta_W$ and θ_W is the Weinberg angle. Comparing with equation (2), the form factors can be expressed through ε_{xy},

$$\begin{aligned}
\rho_{\rm ef} &= 1 + \varepsilon_{aa} \\
\kappa_f &= \frac{1}{\rho_{\rm ef}} \left[1 + \frac{\varepsilon_{av} v_f - \varepsilon_{aa} a_f}{v_f - a_f} \right], \\
\kappa_e &= \frac{1}{\rho_{\rm ef}} \left[1 + \frac{\varepsilon_{av} v_e - \varepsilon_{aa} a_e}{v_e - a_e} \right] \\
\kappa_{\rm ef} &= \frac{1}{\rho_{\rm ef}} \left[1 + \frac{\varepsilon_{vv} v_e v_f + \varepsilon_{aa} a_e a_f - \varepsilon_{av} a_e v_f - \varepsilon_{va} v_e a_f}{[v_e - a_e][v_f - a_f]} \right].
\end{aligned} \tag{5}$$

If the relation

$$(1 + \varepsilon_{va})(1 + \varepsilon_{av}) = (1 + \varepsilon_{aa})(1 + \varepsilon_{vv}) \tag{6}$$

is fulfilled, we have $\kappa_{\rm ef} = \kappa_e \kappa_f$, *i.e.* the product $v_e v_f$ needs no special replacement rule in (4).

2.2. Sum rule for form factors

Consider the case where two additional amplitudes are added to $\mathcal{M}^Z$,

$$\mathcal{M}^\Sigma = \mathcal{M}^Z + \mathcal{M}^1 + \mathcal{M}^2. \tag{7}$$

Suppose that the form factors $\rho^i_{\rm ef}, \kappa^i_e, \kappa^i_f, \kappa^i_{\rm ef}$, $i = 1,2$ are known for both additional amplitudes $\mathcal{M}^1$ and $\mathcal{M}^2$. Then, the combined form factors can be calculated taking into account that $\varepsilon^\Sigma_{xy} = \varepsilon^1_{xy} + \varepsilon^2_{xy}$,

$$\begin{aligned} \rho^\Sigma_{\rm ef} &= \rho^1_{\rm ef} + \rho^2_{\rm ef} - 1, \\ \kappa^\Sigma_f &= \frac{\kappa^1_f\rho^1_{\rm ef} + \kappa^2_f\rho^2_{\rm ef} - 1}{\rho^1_{\rm ef} + \rho^1_{\rm ef} - 1}, \\ \kappa^\Sigma_{\rm ef} &= \frac{\kappa^1_{\rm ef}\rho^1_{\rm ef} + \kappa^2_{\rm ef}\rho^2_{\rm ef} - 1}{\rho^1_{\rm ef} + \rho^1_{\rm ef} - 1}. \end{aligned} \tag{8}$$

The sum rules (8) are exact. In many applications, the form factors are not very different from one. Then, the approximate sum rules

$$\rho^\Sigma_{\rm ef} = \rho^1_{\rm ef}\rho^2_{\rm ef}, \quad \kappa^\Sigma_f = \kappa^1_f\kappa^2_f, \quad \kappa^\Sigma_{\rm ef} = \kappa^1_{\rm ef}\kappa^2_{\rm ef} \tag{9}$$

can be used. In their derivation, contributions proportional to $\varepsilon_{xy}\varepsilon_{x'y'}$ are neglected.

3. Born contributions in terms of form factors

3.1. Photon exchange

If we are not dealing with pure neutrino processes, the complete Born amplitude in the SM consists of contributions of Z and photon exchange. The photon exchange can be treated as the additional matrix element in equation (2)

$$\mathcal{M} = \mathcal{M}^\gamma = \frac{e^2}{s}\bar{v}(e)\gamma_\beta Q^e u(e)\cdot\bar{u}(f)\gamma^\beta Q^f v(f). \tag{10}$$

Comparing equations (10) and (2), we get

$$\varepsilon^\gamma_{av} = \varepsilon^\gamma_{va} = \varepsilon^\gamma_{aa} = 0\,, \quad \varepsilon^\gamma_{vv} = \frac{e^2(s-m_Z^2)}{g_Z^2 s}\frac{Q^eQ^f}{v_ev_f}. \tag{11}$$

The form factors follow from equation (5),

$$\rho^\gamma_{\rm ef} = \kappa^\gamma_e = \kappa^\gamma_f = 1, \quad \kappa^\gamma_{\rm ef} = 1 + \frac{e^2(s-m_Z^2)}{g_Z^2 s}\frac{Q^eQ^f}{[v_e-a_e][v_f-a_f]}. \tag{12}$$

3.2. Z' exchange

The description of four fermion interactions in theories with extra neutral gauge bosons demands a third matrix element at the tree level,

$$\mathcal{M} = \mathcal{M}^{Z'} = \frac{g_{Z'}^2}{s - m_{Z'}^2} \bar{v}(e)\gamma_\beta \left[v_e' - \gamma_5 a_e'\right] u(e) \cdot \bar{u}(f)\gamma^\beta \left[v_f' - \gamma_5 a_f'\right] v(f). \tag{13}$$

$v_f'(a_f')$ stands for the vector (axial vector) coupling of the Z' to the fermion f, $m_{Z'}$ is the complex mass of the Z' and $g_{Z'}$ denotes the coupling strength of the fermion current to the Z'. The functions $\varepsilon_{xy}^{Z'}$ follow immediately,

$$\varepsilon_{xy}^{Z'} = \chi_{Z/Z'} \frac{x_e' y_f'}{x_e y_f}, \quad x, y = a, v, \quad \chi_{Z'/Z} = \frac{g_{Z'}^2 (s - m_Z^2)}{g_Z^2 (s - m_{Z'}^2)}. \tag{14}$$

Again, the resulting form factors can be calculated using equation (5),

$$\begin{aligned}
\rho_{\text{ef}}^{Z'} &= 1 + \chi_{Z'/Z} \frac{a_e' a_f'}{a_e a_f}, \\
\kappa_e^{Z'} &= \frac{1}{\rho_{\text{ef}}^{Z'}} \left[1 + \chi_{Z'/Z} \frac{[v_e' - a_e'] a_f'}{[v_e - a_e] a_f}\right], \\
\kappa_f^{Z'} &= \frac{1}{\rho_{\text{ef}}^{Z'}} \left[1 + \chi_{Z'/Z} \frac{[v_f' - a_f'] a_e'}{[v_f - a_f] a_e}\right], \\
\kappa_{\text{ef}}^{Z'} &= \frac{1}{\rho_{\text{ef}}^{Z'}} \left[1 + \chi_{Z'/Z} \frac{[v_e' - a_e'][v_f' - a_f']}{[v_e - a_e][v_f - a_f]}\right].
\end{aligned} \tag{15}$$

See reference [9] for an earlier derivation of these formulae. We emphasize that the form factors (12) and (15) include the photon and Z' contributions without any approximation. Of course, they are s-dependent, in general not small and even singular (resonating) for $s \to 0$ ($s \approx M_{Z'}^2$).

3.3. ZZ' mixing

There are no quantum numbers, which forbid a mixing between the Z and the Z'.

Indeed, the mass matrix of the Z and Z' receives in the general case non-diagonal entries δM^2 related to the vacuum expectation values of the Higgs fields,

$$\mathcal{L}_M = \frac{1}{2}(Z,\ Z') \begin{pmatrix} M_Z^2 & \delta M^2 \\ \delta M^2 & M_{Z'}^2 \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}. \tag{16}$$

The mass eigenstates Z_1 and Z_2 are connected with the symmetry eigenstates Z and Z' by a mixing matrix,

$$\left(\begin{array}{c} Z_1 \\ Z_2 \end{array}\right) = \left(\begin{array}{cc} c_M & s_M \\ -s_M & c_M \end{array}\right)\left(\begin{array}{c} Z \\ Z' \end{array}\right), \tag{17}$$

which depends on the mixing angle θ_M with $c_M = \cos\theta_M$, $s_M = \sin\theta_M$.

The masses M_1 and M_2 of the mass eigenstates Z_1 and Z_2 are

$$M_{1,2}^2 = \frac{1}{2}\left[M_Z^2 + M_{Z'}^2 \pm \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4(\delta M^2)^2}\right]. \tag{18}$$

It follows $M_1 < M_Z < M_2$. Per definition, Z_1 is the light mass eigenstate investigated at LEP 1.

The mixing predicts the couplings $a_f(n), v_f(n)$, $n = 1, 2$ of the mass eigenstates,

$$a_f(1) = c_M a_f + \frac{g_{Z'}}{g_Z} s_M a_f', \quad v_f(1) = c_M v_f + \frac{g_{Z'}}{g_Z} s_M v_f', \tag{19}$$

$$a_f(2) = c_M a_f' - \frac{g_Z}{g_{Z'}} s_M a_f, \quad v_f(2) = c_M v_f' - \frac{g_Z}{g_{Z'}} s_M v_f. \tag{20}$$

The considered process is now given by amplitudes with a photon, Z_1 and Z_2 exchange. The photon and Z_2 exchange can be treated as explained in the previous sections.

Here, we deal with the Z_1 amplitude in presence of ZZ' mixing,

$$\mathcal{M} = \mathcal{M}^M \tag{21}$$

$$= \frac{g_Z^2}{s - m_1^2}\bar{v}(e)\gamma_\beta\left[v_e(1) - \gamma_5 a_e(1)\right]u(e)\cdot\bar{u}(f)\gamma^\beta\left[v_f(1) - \gamma_5 a_f(1)\right]v(f). \tag{22}$$

m_1 is the complex mass of the Z_1. The functions ε_{xy}^M are

$$\varepsilon_{xy}^M = \chi_{Z_1/Z}\left(c_M + \frac{g_{Z'}}{g_Z}s_M\frac{x_e'}{x_e}\right)\left(c_M + \frac{g_{Z'}}{g_Z}s_M\frac{y_f'}{y_f}\right) - 1, \quad x, y = a, v,$$

$$\chi_{Z_1/Z} = \frac{s - m_Z^2}{s - m_1^2}. \tag{23}$$

Again, the resulting form factors can be calculated using equation (5),

$$\rho_{\mathrm{ef}}^M = \chi_{Z_1/Z}\left(c_M + \frac{g_{Z'}}{g_Z}s_M\frac{a_e'}{a_e}\right)\left(c_M + \frac{g_{Z'}}{g_Z}s_M\frac{a_f'}{a_f}\right),$$

$$\kappa_f^M = \frac{1 + \frac{s_M g_{Z'}}{c_M g_Z}\frac{v'_f - a'_f}{v_f - a_f}}{1 + \frac{s_M g_{Z'}}{c_M g_Z}\frac{a'_f}{a_f}},$$
$$\kappa_{\mathrm{ef}}^M = \kappa_e^M \kappa_f^M . \tag{24}$$

In the expression for κ_{ef}^M, relation (6) is used.

In contrast to the previous sections, the ZZ' mixing introduces a shift of the Z mass taken into account by the factor $\chi_{Z_1/Z}$. As it should be, this normalization factor drops out in the formulae for the κ's. The mass shift could be observed by charged current processes verifying the relation $M_W^2/M_1^2 > M_W^2/M_Z^2 = 1 - s_W^2$. At the Born level, the choice of m_Z^2 or m_1^2 in the Z–amplitude (1) is a question of convention.

In reference [10], the amplitude $\mathcal{M}^Z$ in (1) is redefined replacing m_Z^2 by m_1^2. Then, the coefficients ε_{xy}^M have no factor $\chi_{Z_1/Z}$. The results obtained in reference [10] are reproduced by the form factors (24) in that case.

4. Radiative corrections

4.1. Electroweak corrections

Radiative corrections are necessary to meet the precise experimental data by accurate theoretical predictions.

In the calculation of SM electroweak corrections [7], a huge number of Feynman diagrams must be taken into account. The result can always be expressed in terms of the four form factors [8]

$$\rho_{\mathrm{ef}}^{\mathrm{ew}},\ \kappa_e^{\mathrm{ew}},\ \kappa_f^{\mathrm{ew}},\ \kappa_{\mathrm{ef}}^{\mathrm{ew}} . \tag{25}$$

They are complex functions depending on all parameters of the theory and on kinematical variables. Without box contributions, the κ's would factorize. In many applications, the box contributions are negligible leading to the approximate factorization $\kappa_{\mathrm{ef}}^{\mathrm{ew}} \approx \kappa_e^{\mathrm{ew}}\kappa_f^{\mathrm{ew}}$. See [9] for a review on the weak form factors and further references.

In a Z' search on the Z_1 resonance , weak corrections and ZZ' mixing must be taken into account simultaneously. Weak corrections apply to the symmetry eigenstate Z, which is different from the mass eigenstate Z_1 in the case of a non–zero mixing. The factor $\chi_{Z_1/Z}$ in equation (23) can be expanded in the difference $M_Z - M_1$,

$$\chi_{Z_1/Z} \approx 1 + \frac{2M_Z(M_1 - M_Z)}{s - m_1^2}. \tag{26}$$

The experimental upper bound on this difference is better than 150 MeV, compare [11]. This is much smaller than the Z width. Approximating

$\chi_{Z_1/Z}$ by 1, one makes a normalization error of the order "e.w. corrections $\cdot(M_1 - M_Z)^2/\Gamma_Z^2$", which is completely negligible.

Using the approximate sum rules (9) to combine the form factors of weak corrections (25) and ZZ' mixing (24), one makes an error of the order "e.w. corrections$\cdot\theta_M$", which is still much smaller than the e.w. corrections and therefore negligible.

The form factor $\rho_{\rm ef}^{\rm ew}$ arises through the replacement of the coupling constant g_Z^2 by the muon decay constant,

$$g_Z^2 = \frac{\pi\alpha}{s_W^2 c_W^2} \to \sqrt{2} G_\mu M_Z^2 \rho_{\rm ef}^{\rm ew} \equiv \sqrt{2} G_\mu M_1^2 \rho_{\rm ef}^{\rm ew} \rho_{\rm mix} \tag{27}$$

with $\rho_{\rm mix} = M_Z^2/M_1^2 \approx 1 + 2(M_Z - M_1)/M_1$. $\rho_{\rm mix}$ takes into account the tree level mass shift. Historically, this effect was numerically important [10]. It is marginal with the present experimental constraint on $M_Z - M_1$.

Theories predicting extra neutral gauge bosons always contain many new particles. These particles contribute to the "weak corrections" of the whole theory. All these contributions are neglected in present analyses. As far as there are no hints of physics beyond the SM, the error due to this ignorance is expected to be small.

4.2. QCD corrections

QCD corrections are present in fermion interactions involving quarks. Even in the case of massless fermions at the tree level, the QCD (and QED) corrections depend on fermion masses because they regularize the infrared and collinear singularities. QCD corrections are different in processes where two or four quarks are involved. We concentrate, from now on, on the process $e^+e^- \to f\bar{f}$. $O(\alpha_s)$ corrections are present for $f = q$.

The corrections depend on the parameters of the theory and on kinematic variables. If one integrates over the whole phase space, the QCD corrections reduce simply to a factor, which multiplies the total Born cross section σ_T^0. To order $O(\alpha_s)$, we have

$$\sigma_T^{\rm QCD} = \left(1 + \frac{\alpha_s}{\pi}\right) \sigma_T^0. \tag{28}$$

The $O(\alpha_s)$ QCD corrections can be obtained from the $O(\alpha)$ QED corrections by an implementation of color factors. QCD and QED corrections are very process dependent. They can not be expressed in terms of form factors.

4.3. QED corrections

QED corrections are present in processes involving charged particles.

In the reaction $e^+e^- \to f\bar{f},\ f \neq \nu$, QED corrections can be separated into initial state radiation, final state radiation and the interference between them. All these corrections give different contributions to CP even and CP odd parts of the cross section. The corrections are rather involved functions of the kinematical parameters [12].

For a Z' search, the initial state correction is numerically most important. For later reference, we give the initial state correction to the total cross section for the case where all kinematical variables except the photon energy are integrated out,

$$\sigma_T^{\mathrm{ISR}}(s) = [1 + S(\varepsilon)]\sigma_T^0(s) + \int_\varepsilon^\Delta dv\ \sigma_T^0\left(s(1-v)\right)\ H_T(v). \tag{29}$$

The flux function $H_T(v)$ describes the probability of the emission of a photon with the energy $v\sqrt{s}$, where $\sqrt{s}$ the c.m. energy. To order $O(\alpha)$, it is [13]

$$\begin{aligned} H_T(v) &= \bar{H}_T(v) + \frac{\beta_e}{v} = \beta_e\ \frac{1+(1-v)^2}{2v}, \\ \beta_e &= \frac{2\alpha}{\pi}\ (Q^e)^2\ (L_e - 1), \quad L_e = \ln\frac{s}{m_e^2}, \quad Q^e = -1. \end{aligned} \tag{30}$$

The quantity $[1 + S(\varepsilon)]$ in equation (29) describes the Born term plus corrections due to soft and virtual photons. To order $O(\alpha)$, we have

$$S(\varepsilon) = \bar{S} + \beta_e \ln\varepsilon = \beta_e\ \left(\ln\varepsilon + \frac{3}{4}\right) + \frac{\alpha}{\pi}\ (Q^e)^2\ \left(\frac{\pi^2}{3} - \frac{1}{2}\right). \tag{31}$$

See reference [14] for analytical results of the integral (29) in presence of a Z'.

4.4. The radiative tail

Starting from the convolution (29), the origin of the radiative tail and its magnitude can be estimated. We do this ignoring details of the radiator functions $S(\varepsilon)$ and $H_T^e(v)$. The s' dependence of the Born cross section is

$$\begin{aligned} \sigma_T^0(s') &\approx \frac{1}{s'}\frac{s'}{s'-m_m^2}\frac{s'}{s'-m_n^{*2}} \\ &= \frac{s}{m_n^{*2}-m_m^2}\frac{1}{s}\left[\frac{s'}{s'-m_n^{*2}}-\frac{s'}{s'-m_m^2}\right]. \end{aligned} \tag{32}$$

For $m = n$, the first factor of the last expression becomes

$$\frac{s}{m_n^{*2}-m_n^2} = -\frac{i}{2}\frac{M_n}{\Gamma_n}\frac{s}{M_n^2}. \tag{33}$$

This imaginary quantity will give contributions to the cross section only, if it is met by another imaginary multiplier. It arises from the v–integration (29) over the remaining factors in equation (32). Keeping only the relevant term after partial fraction decomposition, one gets

$$\begin{aligned} \sigma_T^{\mathrm{ISR}}(s) - \sigma_T^0(s) &\approx \frac{i}{2}\frac{M_n}{\Gamma_n}\frac{s}{M_n^2}\frac{M_n^2}{s}\int_0^{\Delta} dv\,\frac{1}{1-v-m_n^{*2}/s} \\ &\approx \frac{i}{2}\frac{M_n}{\Gamma_n}\ln\frac{m_n^{*2}/s-1+\Delta}{m_n^{*2}/s-1}. \end{aligned} \tag{34}$$

The real part of the argument of the logarithm is negative for $s > M_n^2$ and $\Delta > 1 - M_n^2/s$. The first condition demands that the c.m. energy must be larger than the mass of the resonance. The second condition allows the radiation of photons with an energy $\sqrt{s}\Delta = \sqrt{s}(1 - M_n^2/s)$. If both conditions are fulfilled, the whole expression in equation (34) becomes real and the radiative tail develops.

For events of the radiative tail, the energy of the remaining e^+e^- pair equals M_n, *i.e.* it annihilates on top of the Z_n resonance. Fermion pair production *on* resonance is enhanced by a factor M_n^2/Γ_n^2 relative to *off* resonance production. The convolution (29) tells us that we have to integrate over all possible photon energies. Resonance enhancement is only obtained for photons, which shift the energy of the remaining e^+e^- pair in the narrow interval $M_n \pm \Gamma_n$. Therefore, the enhancement factor M_n^2/Γ_n^2 is effectively multiplied by Γ_n/M_n leading to a combined enhancement M_n/Γ_n of the radiative tail. This is exactly what we have in equations (33) and (34).

The magnitude of the radiative tail can now be estimated restoring the missing factor in equation (34). We find from equation (30) a factor $\beta_e/2$ in the limit of large energies, and a multiplier 2 due to the second contribution in the difference in equation (32),

$$\text{rad. tail } \approx \sigma_T^{\text{ISR}}(s) - \sigma_T^0(s) \approx \sigma_T^0(s;n,n) \cdot \beta_e \frac{\pi}{2} \frac{M_n}{\Gamma_n}. \tag{35}$$

The factor β_e contains α because the radiation of an additional photon is a process of higher order, and $L_e = \ln \frac{s}{m_e^2}$ giving an enhancement for photons radiated collinear to the beam.

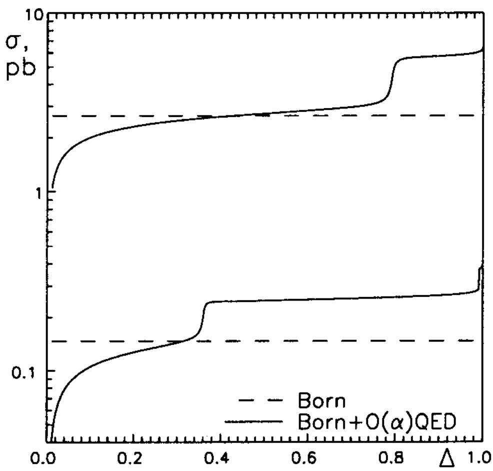

Fig. 1. The total cross section σ_T^μ as a function of the cut on the photon energy $\Delta\sqrt{s}$ for $M_{Z'} = M_\eta = 800\,\text{GeV}$. The upper (lower) set of curves corresponds to $\sqrt{s} = 200(1000)\,\text{GeV}$.

Putting $s_W^2 = \frac{1}{4}$ and considering the Z_1 peak, $M_n/\Gamma_n = M_1/\Gamma_1$, one gets $\sigma_T^l(s;\gamma,Z_1) = 0$ and $\sigma_T^l(s;Z_1,Z_1)/\sigma_T^l(s;\gamma,\gamma) = 1/9$ in the limit $s \gg M_Z^2$. It follows

$$\text{rad. tail } \approx 7 \cdot \sigma_T^{l0}(s;Z_1,Z_1) \approx 7/10 \cdot \sigma_T^{l0}, \tag{36}$$

which is in reasonable agreement with the exact calculation and with figure 1. For b quark production where the Z boson exchange dominates over the photon exchange, the effect of the radiative tail is much more pronounced; $\sigma_T^{b0}(s;\gamma,Z_1) \approx 0$, $\sigma_T^{b0}(s;Z_1,Z_1)/\sigma_T^{b0}(s;\gamma,\gamma) = 4$ in the limit $s \gg M_Z^2$, and hence

$$\text{rad. tail } \approx 7 \cdot \sigma_T^{b0}(s;Z_1,Z_1) \approx 28/5 \cdot \sigma_T^{b0}, \tag{37}$$

which is again in reasonable agreement with the exact calculation.

Only the cross section of the exchange of the vector boson n, $\sigma_T^0(s;n,n)$, appears in equation (35). All other contributions to σ_T^0 are not enhanced by the radiative tail. For $M_1 < \sqrt{s} < M_2$, the contribution $\sigma_T^0(s;Z_1,Z_1)$ is enhanced, while the Z' signal is not enhanced. Therefore, the radiative tail must be removed for a Z' search. This can be done demanding $\Delta < 1-M_1^2/s$.

The dependence of the cross section on Δ is shown in figure 1 for two different energies. The upper curves correspond to an energy above the Z peak but below the Z' peak, the lower curves to an energy above the Z and Z' peaks. One recognizes the step-like behaviour for photon energies where the radiative tail(s) are "switched on". We see that the radiatively corrected cross section is numerically similar to the Born prediction only for a certain cut, which rejects all radiative tails. This property is independent of the observable considered. It is the reason why theoretical Z' analyses with Born cross sections agree very well with those involving SM corrections. Of course, the whole SM corrections are needed for an analysis of real data.

5. Conclusions

In this lecture, much attention was devoted to the implementation of effects connected with Z' physics by form factors. This procedure works at the level of amplitudes. Therefore, it can be applied to any four fermion interaction. Examples are $e^+e^- \to f\bar{f}$, $e^+e^- \to e^+e^-$, $e^-e^- \to e^-e^-$, $e^\pm q \to e^\pm q$, $q\bar{q} \to f\bar{f}$.

QCD and QED corrections depend on the process and on kinematical cuts. Therefore, they have to be calculated for every reaction separately. In the case of a Z' search, the results known from the Standard Model can be applied.

The radiative tail develops only in processes with resonances in the s-channel. The mechanisms of its development and its magnitude are extensively discussed. It is suppressed at hadron colliders mainly due to properties of the structure functions. At e^+e^- colliders, it is most pronounced in fermion pair production and therefore important at LEP 2 and at a future linear collider. For a Z' search, it has to be removed by kinematical cuts.

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