

# HAS THE GENERAL TWO-HIGGS-DOUBLET MODEL UNNATURAL FCNC SUPPRESSION? AN RGE ANALYSIS\*

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There is a widespread belief that the general two-Higgs-doublet model (G2HDM) behaves unnaturally with respect to evolution of the flavor-changing neutral Yukawa coupling parameters (FCNYCP's) — *i.e.*, that the latter, although being suppressed at low energies of probes, in general increase by a large factor as the energy of probes increases. We investigate this, by evolving Yukawa parameters by one-loop renormalization group equations and neglecting contributions of the first quark generation. For patterns of FCNYCP suppression at low energies suggested by existing quark mass hierarchies, FCNYCP's remain remarkably stable (suppressed) up to energies very close to the Landau pole. This indicates that G2HDM preserves FCNYCP suppression, for reasonably chosen patterns of that suppression at low energies.

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Lee [1] proposed, already in 1973, the general model with two Higgs doublets [G2HDM, referred often also as 2HDM(III)]. This is the model with the most general Yukawa couplings of the two Higgs doublets (to quarks). He investigated the model with emphasis on CP-violating phenomena, and was followed by Sikivie *et al.* [2].

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Glashow and Weinberg [3] pointed out in 1977 that only those models with two Higgs doublets which possess specific discrete [or  $U(1)$ -type] family symmetries in the Yukawa sector [2HDM(II) and 2HDM(I)] have the property of complete suppression of all flavor-changing neutral Yukawa coupling parameters (FCNYCP's), and this is then true at any evolution energy<sup>1</sup>. Since then, a large part of physics community has apparently regarded such models as being the only ones with a natural suppression of FCNYCP's. It appears that G2HDM's were then not investigated until late eighties.

During the last twelve years, there has been a moderate resurgence of works on G2HDM's [4–7]<sup>2</sup>. These works investigate *low* energy phenomena of G2HDM's, allowing nonzero but reasonably suppressed FCNYCP's at low (electroweak) energies. Cheng, Sher and Yuan (CSY) [5] and Antaramian, Hall and Rašin (AHR) [6] proposed specific ansätze for these parameters at low (electroweak) energies, motivated largely by the existing mass hierarchies of quarks. They pointed out that neutral scalars can then still be reasonably light ( $M \sim 10^2$  GeV) without violating available data on flavor-changing neutral processes. Thus, these two groups showed that FCNC suppression in G2HDM's is not intrinsically unnatural (*i.e.*, ad hoc), since it can be motivated by existing quark mass hierarchies. Certainly, these arguments regard only the structure of the theory at low (electroweak) energies of probes. We note that most of the phenomenological investigations in G2HDM's by other authors used essentially ansätze of these two groups.

We are not aware of any work on the second aspect of G2HDM's, *i.e.*, the question whether these models really behave unnaturally with respect to *evolution* of FCNYCP's. The present work attempts to address this question, by investigating one-loop evolution of these parameters, for the cases when the patterns of their values at low energies are described by CSY-type of ansätze and modifications thereof.

Yukawa interactions of quarks in G2HDM's in any  $SU(2)_L$ -basis are

$$\begin{aligned} \mathcal{L}_{\text{G2HDM}}^{(E)} = & - \sum_{i,j=1}^3 \left\{ \tilde{D}_{ij}^{(1)} (\bar{q}_L^{(i)} \Phi^{(1)}) \tilde{d}_R^{(j)} + \tilde{D}_{ij}^{(2)} (\bar{q}_L^{(i)} \Phi^{(2)}) \tilde{d}_R^{(j)} \right. \\ & \left. + \tilde{U}_{ij}^{(1)} (\bar{q}_L^{(i)} \tilde{\Phi}^{(1)}) \tilde{u}_R^{(j)} + \tilde{U}_{ij}^{(2)} (\bar{q}_L^{(i)} \tilde{\Phi}^{(2)}) \tilde{u}_R^{(j)} + \text{h.c.} \right\}. \quad (1) \end{aligned}$$

Parameters and quark fields are in an arbitrary  $SU(2)_L$ -basis, as indicated by tildes above them. Superscript ( $E$ ) at the Lagrangian density means

<sup>1</sup> Low energy experiments show that those flavor-changing neutral coupling parameters (FCNCP's) which don't involve  $t$  quark are suppressed in nature at low energies  $E \sim m_q$ . For FCNCP's involving  $t$ , experimental evidence is not yet available.

<sup>2</sup> Refs. [4] and Refs. [7] concentrate largely on CP-violating and FCNC-violating phenomena, respectively.

that the theory has a finite effective energy cutoff  $E$  (energy of probes). This superscript is omitted at the fields and at the parameters for simpler notation<sup>3</sup>. The following notations are used:

$$\Phi^{(k)} \equiv \begin{pmatrix} \phi^{(k)+} \\ \phi^{(k)0} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1^{(k)} + i\phi_2^{(k)} \\ \phi_3^{(k)} + i\phi_4^{(k)} \end{pmatrix}, \quad \tilde{\Phi}^{(k)} \equiv i\tau_2 \Phi^{(k)*}, \quad (2)$$

$$\tilde{q}^{(i)} = \begin{pmatrix} \tilde{u}^{(i)} \\ \tilde{d}^{(i)} \end{pmatrix} : \quad \tilde{q}^{(1)} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}, \quad \tilde{q}^{(2)} = \begin{pmatrix} \tilde{c} \\ \tilde{s} \end{pmatrix}, \quad \tilde{q}^{(3)} = \begin{pmatrix} \tilde{t} \\ \tilde{b} \end{pmatrix}, \quad (3)$$

$$\langle \Phi^{(1)} \rangle_0 = \frac{e^{i\eta_1}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi^{(2)} \rangle_0 = \frac{e^{i\eta_2}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad v_1^2 + v_2^2 = v^2. \quad (4)$$

Here,  $v \equiv v(E)$  is the usual vacuum expectation value (VEV) needed for the electroweak symmetry breaking:  $v(E_{\text{ew}}) \approx 246$  GeV. Phase difference  $\eta \equiv \eta_2 - \eta_1$  between the two VEV's in (4) may be nonzero and represents CP violation originating from the purely scalar sector.

Popular 2HDM(I) and 2HDM(II) models proposed by Glashow and Weinberg [3] are special cases (subsets) of this framework:  $U^{(1)} = D^{(1)} = 0$  in 2HDM(I), and  $U^{(1)} = D^{(2)} = 0$  in 2HDM(II). In these models, mass matrices  $M^{(U)}$  and  $M^{(D)}$  in the mass basis are proportional to the relevant nonzero Yukawa matrices  $U$ ,  $D$ , respectively. Therefore, there are no FCNYCP's in the physical (quark mass) basis, since off-diagonal elements are zero there by definition. Moreover, this remains true even when radiative corrections are included, *i.e.*, when the model is evolved from a high "bare" energy  $E = \Lambda$  to low energies  $E \sim m_q$ . This is so because the mentioned structure of the Yukawa sector is ensured by specific discrete symmetries.

There are no such symmetries in the Yukawa sector of G2HDM, so a priori it is unclear whether suppression of FCNYCP's would persist when we increase energy of probes beyond electroweak scales. We carried out such an analysis, by using one-loop renormalization group equations (RGE's) for Yukawa coupling matrices appearing in (1). Those RGE's were derived by us earlier [8].

In order to interpret more easily the RGE results at high energies of probes, as well as the initial conditions at low (electroweak) scales, it is convenient to introduce the following derived quantities:

$$\Phi'^{(1)} = \cos \beta \Phi^{(1)} + e^{-i\eta} \sin \beta \Phi^{(2)}, \quad \Phi'^{(2)} = -\sin \beta \Phi^{(1)} + e^{-i\eta} \cos \beta \Phi^{(2)}, \quad (5)$$

where

$$\tan \beta = \frac{v_2}{v_1} \Rightarrow \cos \beta = \frac{v_1}{v}, \quad \sin \beta = \frac{v_2}{v}; \quad \eta = \eta_2 - \eta_1. \quad (6)$$

<sup>3</sup> For renormalized quantities,  $E \sim 10^2$  GeV.

VEV's of the redefined scalar isodoublets are:  $e^{-i\eta_1} \langle \Phi'^{(1)} \rangle_0^T = (0, v/\sqrt{2})$ ,  $\langle \Phi'^{(2)} \rangle_0^T = (0, 0)$ . Hence, isodoublet  $\Phi'^{(1)}$  is responsible for the quark masses. Below we will see that  $\Phi'^{(2)}$  leads to FCNYCP's. We now rewrite original Yukawa Lagrangian density (1) of G2HDM in terms of these fields

$$\begin{aligned} \mathcal{L}_{\text{G2HDM}}^{(E)} = & - \sum_{i,j=1}^3 \left\{ \tilde{G}_{ij}^{(D)} (\bar{q}_L^{(i)} \Phi'^{(1)}) \tilde{d}_R^{(j)} + \tilde{G}_{ij}^{(U)} (\bar{q}_L^{(i)} \tilde{\Phi}'^{(1)}) \tilde{u}_R^{(j)} + \text{h.c.} \right\} \\ & - \sum_{i,j=1}^3 \left\{ \tilde{D}_{ij} (\bar{q}_L^{(i)} \Phi'^{(2)}) \tilde{d}_R^{(j)} + \tilde{U}_{ij} (\bar{q}_L^{(i)} \tilde{\Phi}'^{(2)}) \tilde{u}_R^{(j)} + \text{h.c.} \right\}. \quad (7) \end{aligned}$$

$\tilde{G}^{(U)}$  and  $\tilde{G}^{(D)}$  are rescaled mass matrices, and  $\tilde{U}$  and  $\tilde{D}$  the corresponding Yukawa matrices "orthonormal" to  $\tilde{G}^{(U)}$  and  $\tilde{G}^{(D)}$

$$\tilde{G}^{(X)} = \cos \beta \tilde{X}^{(1)} + e^{\mp i\eta} \sin \beta \tilde{X}^{(2)} = \sqrt{2} \tilde{M}^{(X)} / v \quad (X = U, D), \quad (8)$$

$$\tilde{X} = -\sin \beta \tilde{X}^{(1)} + e^{\mp i\eta} \cos \beta \tilde{X}^{(2)} \quad (X = U, D). \quad (9)$$

Minus sign in exponents is for  $X = U$ , and plus for  $X = D$ . Transition to the quark mass basis (at a given energy  $E$ ) is implemented by biunitary transformations involving unitary matrices  $V_{L,R}^U$  and  $V_{L,R}^D$

$$U = V_L^U \tilde{U} V_R^{U\dagger}; \quad G^{(U)} = V_L^U \tilde{G}^{(U)} V_R^{U\dagger}, \quad G_{ij}^{(U)} = \delta_{ij} m_i^{(u)} \sqrt{2}/v; \quad (10)$$

$$D = V_L^D \tilde{D} V_R^{D\dagger}; \quad G^{(D)} = V_L^D \tilde{G}^{(D)} V_R^{D\dagger}, \quad G_{ij}^{(D)} = \delta_{ij} m_i^{(d)} \sqrt{2}/v; \quad (11)$$

$$u_L = V_L^U \tilde{u}_L, \quad u_R = V_R^U \tilde{u}_R, \quad d_L = V_L^D \tilde{d}_L, \quad d_R = V_R^D \tilde{d}_R. \quad (12)$$

Parameters and fields without tildes are in the mass basis. Superscripts  $(E)$  are omitted for simpler notation. CKM mixing matrix is  $V \equiv V_L^U V_L^{D\dagger}$ . Neutral part of Lagrangian density (7) in the quark mass basis is then

$$\begin{aligned} \mathcal{L}_{\text{G2HDM}}^{(E)\text{neutr.}} = & -\frac{1}{\sqrt{2}} \left\{ G_{ii}^{(D)} \tilde{d}_L^{(i)} d_R^{(i)} [\phi_3'^{(1)} + i\phi_4'^{(1)}] + G_{ii}^{(U)} \tilde{u}_L^{(i)} u_R^{(i)} [\phi_3'^{(1)} - i\phi_4'^{(1)}] \right. \\ & \left. + D_{ij} \tilde{d}_L^{(i)} d_R^{(j)} [\phi_3'^{(2)} + i\phi_4'^{(2)}] + U_{ij} \tilde{u}_L^{(i)} u_R^{(j)} [\phi_3'^{(2)} - i\phi_4'^{(2)}] + \text{h.c.} \right\}, \quad (13) \end{aligned}$$

where summation is performed over repeated flavor indices. We see from (13) that the  $U$  and  $D$  matrices, defined by (9)–(11) through the original matrices  $\tilde{U}^{(j)}$  and  $\tilde{D}^{(j)}$  of G2HDM (1), allow the model to possess in general FCNYCP's. This is so because in the quark mass basis only the (rescaled) quark mass matrices  $G^{(U)}$  and  $G^{(D)}$  of (10)–(11) [*c.f.* also (8)] are diagonal, while additional matrices  $U$  and  $D$  are in general not.

As mentioned earlier, CSY [5] argued that  $U$  and  $D$  matrices in the quark mass basis and at low energies  $E$  have the form

$$U_{ij}(E) = \xi_{ij}^{(u)} \frac{\sqrt{2}}{v} \sqrt{m_i^{(u)} m_j^{(u)}} , \quad D_{ij}(E) = \xi_{ij}^{(d)} \frac{\sqrt{2}}{v} \sqrt{m_i^{(d)} m_j^{(d)}} , \quad (14)$$

with

$$\xi_{ij}^{(u)}, \xi_{ij}^{(d)} \sim 1 \quad \text{for } E \sim E_{\text{ew}} (\sim M_Z) . \quad (15)$$

This form, at least for the diagonal elements, is suggested by the existing quark mass hierarchies and the requirement that there is no fine-tuning in which large Yukawa terms  $U_{jk}^{(i)}$  (and:  $D_{jk}^{(i)}$ ) would add together via (9) to result in much smaller terms  $U_{jk}$  ( $D_{jk}$ ) — for this, inspect Eqs. (8)–(9), but this time in the quark mass basis (no tildes). Moreover, this ansatz turns out to be phenomenologically viable, *i.e.*, the known suppression of flavor-changing neutral processes at low energies (not involving on-shell top quarks) can be reproduced. Similar (but not identical) ansätze have been proposed by the authors of [6] (AHR), motivated by their requirement that the Yukawa interactions have approximate  $U(1)$  flavor symmetries.

Now we are prepared to present some typical examples of the RGE evolution of Yukawa parameters in G2HDM's. For simplicity of analysis, we neglect contributions of the first quark generation. Further, we assume that all original four Yukawa matrices  $\tilde{U}^{(j)}$ ,  $\tilde{D}^{(j)}$  are real and the VEV phase difference  $\eta$  is zero (no CP violation).

For the boundary conditions to the RGE's, at the evolution energy  $E = M_Z$ , we first took the CSY ansatz (14)–(15), with  $\xi_{ij}^{(u)} = 1 = \xi_{ij}^{(d)}$  or  $\xi_{ij}^{(u)} = 2 = \xi_{ij}^{(d)}$ , for all  $i, j = 1, 2$ . We emphasize that  $i = 1$  refers now to the second quark family ( $c, s$ ), and  $i = 2$  to the third family ( $t, b$ ). For the  $(2 \times 2)$  orthonormal CKM mixing matrix  $V$  we take  $V_{12}(M_Z) = 0.045 = -V_{21}(M_Z)$ . Values of other parameters at  $E = M_Z$  were chosen to be:

$\tan \beta = 1.0$ ;  $v \equiv \sqrt{v_1^2 + v_2^2} = 246.22$  GeV;  $\alpha_3 = 0.118$ ,  $\alpha_2 = 0.332$ ,  $\alpha_1 = 0.101$ ;  $m_c = 0.77$  GeV,  $m_s = 0.11$  GeV,  $m_b = 3.2$  GeV, and  $m_t = 171.5$  GeV. These quark mass values correspond to:  $m_c(m_c) \approx 1.3$  GeV,  $m_s(1\text{GeV}) \approx 0.2$  GeV,  $m_b(m_b) \approx 4.3$  GeV, and  $m_t^{\text{phys.}} \approx 174$  GeV [ $m_t(m_t) \approx 166$  GeV]. For  $\alpha_3(E)$  we used two-loop evolution formulas, with threshold effect at  $E \approx m_t^{\text{phys.}}$  taken into account; for  $\alpha_j(E)$  ( $j = 1, 2$ ), we used one-loop evolution formulas.

This simplified framework resulted in 18 coupled RGE's [for 18 real parameters:  $v^2$ ,  $\tan \beta$ ,  $\tilde{U}_{ij}$ ,  $\tilde{D}_{ij}$ ,  $\tilde{G}_{ij}^{(U)}$ ,  $\tilde{G}_{ij}^{(D)}$ ], with the mentioned boundary conditions at  $E = M_Z$ . The RGE system was solved numerically, using Runge-Kutta subroutines with adaptive step size control (given in [9]).

The results for the ratios of FCNYCP's  $X_{ij}(E)/X_{ij}(M_Z)$  ( $X = U, D$ ;  $i \neq j$ ) are given for the case of  $\xi_{ij}^{(u)} = 1 = \xi_{ij}^{(d)}$  in Fig. 1. The Figure shows

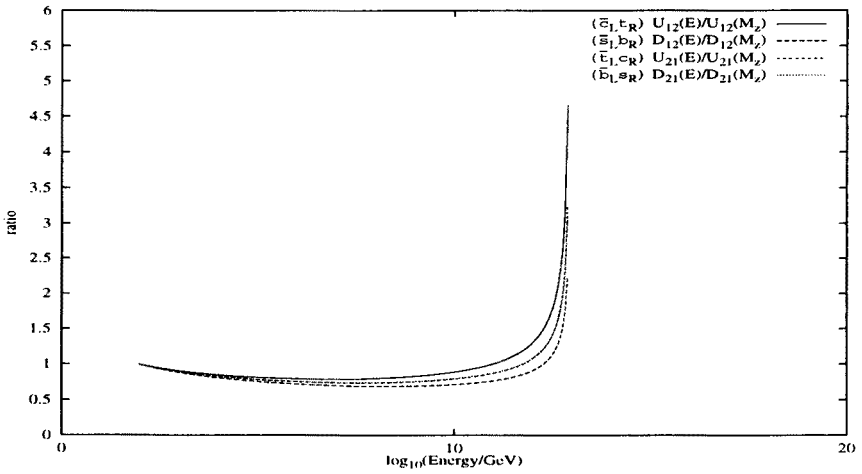


Fig. 1. Evolution of FCNYCP ratios  $U_{ij}(E)/U_{ij}(M_Z)$ ,  $D_{ij}(E)/D_{ij}(M_Z)$  ( $i \neq j$ ), when  $\xi_{ij}^{(u)} = \xi_{ij}^{(d)} = 1$  (for all  $i, j = 1, 2$ ).

that all the FCNYCP's are remarkably stable as the evolution energy increases. Even the up-type FCNYCP's, appearing in couplings involving the heavy  $t$  quark, remain rather stable. Only very close to the  $t$ -quark-dominated Landau pole ( $\sim 10^{13}$  GeV) these parameters start increasing substantially. For example, in the down-type FCN sector ( $b$ - $c$ ) the corresponding ratio  $D_{21}(E)/D_{21}(M_Z)$  acquires its double initial value (*i.e.*, value 2) at  $E \approx 0.7 E_{\text{pole}}$ , which is very near the pole. About the same is true also for  $U_{21}(E)/U_{21}(M_Z)$ . For the ratio  $D_{12}(E)/D_{12}(M_Z)$  the corresponding energy is even closer to  $E_{\text{pole}}$ , while for the  $t$ -quark-dominated  $U_{12}(E)/U_{12}(M_Z)$  it is somewhat lower. We draw the conclusion that the case  $\xi_{ij}^{(u)} = 1 = \xi_{ij}^{(d)}$  shows no "unnaturality" concerning the behavior of FCNYCP's at high energies. We may also compare this stability with that of other ratios. For example, in Ref. [8] we showed also behavior of those ratios of neutral Yukawa parameters which don't involve flavor-changing couplings:  $U_{jj}(E)/U_{jj}(M_Z)$  and  $D_{jj}(E)/D_{jj}(M_Z)$ ;  $G_{jj}^{(U)}(E)/G_{jj}^{(U)}(M_Z)$  and  $G_{jj}^{(D)}(E)/G_{jj}^{(D)}(M_Z)$ . We saw that several (but not all) of these coupling parameters are also reasonably stable. The results for FCNYCP's of Fig. 1 are interesting and perhaps surprising. We should bear in mind that the off-diagonal Yukawa parameters appearing in Fig. 1 are, at low energies, by several orders of magnitude smaller than the third generation diagonal parameters, due to CSY ansatz<sup>4</sup>.

<sup>4</sup> Note that  $D_{12} = D_{21}$  at  $E = M_Z$  is two orders of magnitude smaller than  $U_{22}$ .

Therefore, the fear that the latter, substantially larger, parameters would “pull up” the suppressed off-diagonal ones (by a large factor, or even by orders of magnitude) as the energy of probes increases, is intuitively justified. Fig. 1 says that this does not happen.

One may object that behavior shown in Fig. 1 is due to a special choice  $\xi_{ij}^{(u)} = 1 = \xi_{ij}^{(d)}$ . In fact, in Ref. [8], where the relevant RGE’s were derived, we performed numerical analysis only for the case  $\xi_{ij}^{(u)} = 1 = \xi_{ij}^{(d)}$ . We have recently performed calculations for variations of the CSY ansatz. Results are independent of the VEV ratio  $\tan \beta$  (*c.f.* also discussion in [8]). In Fig. 2,

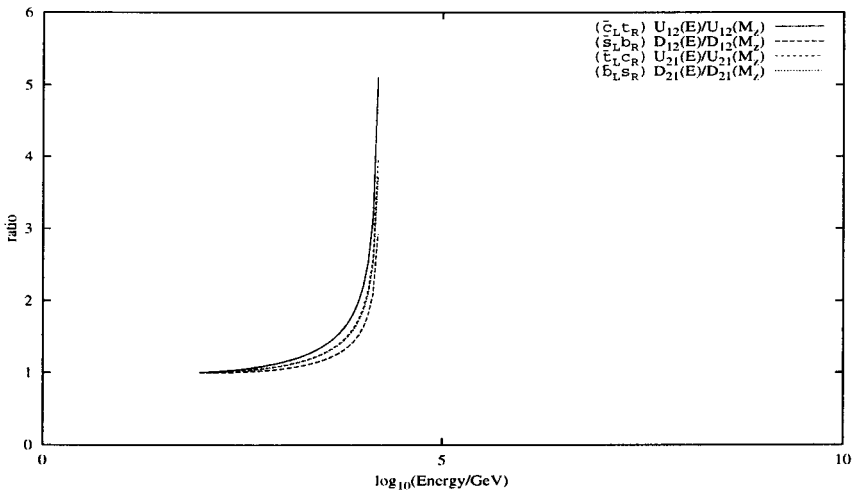


Fig. 2. Same as in Fig. 1, but for the choice  $\xi_{ij}^{(u)} = \xi_{ij}^{(d)} = 2$  (for all  $i, j = 1, 2$ ).

we show results for  $\xi_{ij}^{(u)} = 2 = \xi_{ij}^{(d)}$  at  $E = M_Z$  (for all  $i, j = 1, 2$ ). The Landau pole is now of course lower ( $\sim 10$  TeV), but the behavior with respect to the stability of FCNYCP’s remains qualitatively the same. When we vary some of the  $\xi_{ij}$  parameters, the results remain qualitatively the same, while the Landau pole is influenced largely by the (*t*-quark-dominated) up-type CSY parameter  $\xi_{22}^{(u)}$  [ $U_{22}(M_Z)$ , *c.f.* Eqs. (14)–(15)]. We investigated also cases which go beyond CSY ansatz. For example, we chose to suppress the up-type off-diagonal parameter even more drastically, by taking  $\xi_{12}^{(u)} = \xi_{21}^{(u)} = 0.05163$  and all other  $\xi_{ij}$  parameters equal to 1. For such a choice, we have  $D_{12} = D_{21} = U_{12} = U_{21}$  at  $E = M_Z$ . The results are given in Fig. 3, and are very close to those of Fig. 1.

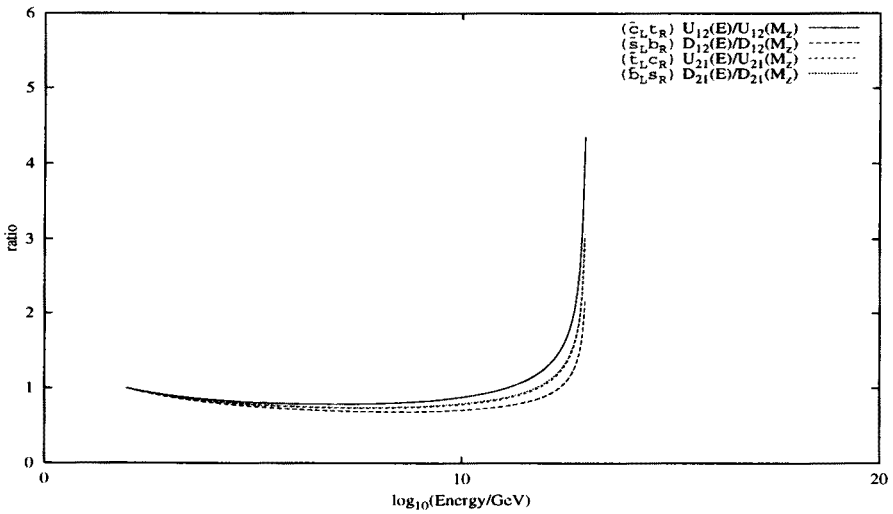


Fig. 3. Same as in Fig. 1, but for the choice  $\xi_{12}^{(u)} = \xi_{21}^{(u)} = 0.05163$  (other  $\xi_{ij}$ 's are 1).

We conclude that in G2HDM's, with effects of the first light family neglected, the flavor-changing neutral Yukawa coupling parameters (FC-NYCP's) remain remarkably stable as the energy of probes increases up to the vicinity of the (top-quark-dominated) Landau pole. The conclusion is insensitive to any specific variation of the CSY ansatz (14)–(15) for Yukawa parameters at low energies, and apparently survives even when going beyond this ansatz by suppressing low energy FCNYCP's even more. Thus, in the presented framework, the fear that the suppressed FCNYCP's at low energies are pulled up drastically by the much larger diagonal Yukawa coupling parameters as the energy of probes increases doesn't materialize. The described framework appears to behave naturally in this respect, FCNYCP's remain remarkably suppressed even at higher energies although there is no explicit exact symmetry which would ensure complete suppression of these parameters. In this sense, we have an indication that the G2HDM's are not unnatural, in contrast with the widely held beliefs.

One may still argue that our conclusions might change when contributions of the first quark generation are included. We intend to perform this extension of numerical analysis in the near future.



## Abbreviations frequently used in the article:

CSY — Cheng, Sher and Yuan; FCNC — flavor-changing neutral currents; FCNYCP — flavor-changing neutral Yukawa coupling parameter; G2HDM — general two-Higgs-doublet (Standard) Model; RGE — renormalization group equation; VEV — vacuum expectation value.

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