A NONHOMOGENEOUS INFLATION*

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A nonhomogeneous inflation in the scalar field theory is studied. The nonhomogeneity of the Universe is used to determine the scalar field potential. Connection of the model with the elementary particles theory is suggested.

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Inflation of the nonhomogeneous field configuration leads to a state which locally looks like an axially symmetric one. In the inflationary model this effect is very small and is difficult to measure since inflation levels smoothen all the differences. In April 1997 Nodland and Ralston [1] found an axial symmetry by measuring the polarization plane of the electromagnetic radiation on cosmological distances. Even if this report [2] is doubtful it is very interesting to consider the consequences of the nonhomogeneous field configuration before inflation.

Motivation of this investigation is twofold. Firstly, it is clear that there exists a much bigger number of nonhomogeneous states than homogeneous ones and it is more likely that such a configuration is a fair starting point for the inflation studies. Secondly, *a priori* it is possible to find some remnants of the world before inflation, and an assumption of the nonhomogeneous inflation can lead to indications how to find them. In other words, it gives the opportunity of testing our elementary particles theory on an extremely large energy level. To simplify the consideration we assume that the state of

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the Universe before inflation may be described by one self interacting scalar field φ . The chosen potential of the self interaction is the simplest possible. It is assumed that the field configuration has the smallest possible energy and can take all values of the field permitted by the inflation theory. The potential leading to the inflation has the shape shown in Fig 1.

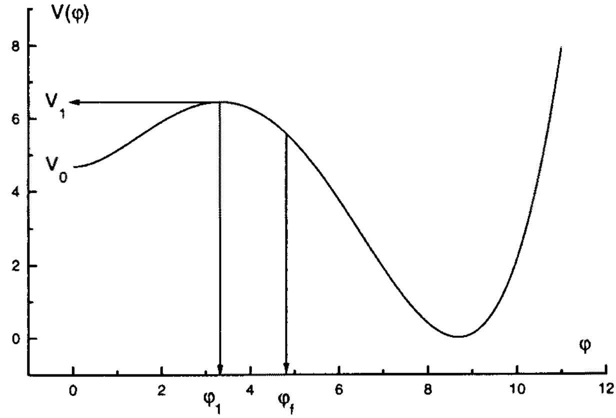

Fig. 1. The potential $V(\varphi)$ leading to the inflation.

We can approximate this potential by

$$V(\varphi) = \begin{cases} A\varphi + V_0 & \text{for} \quad \varphi \in (0;\varphi_1)\,, \\ V_1 - \frac{1}{2}M^2\varphi^2 & \text{for} \quad \varphi \in (\varphi_1;\varphi_f)\,, \end{cases} \tag{1}$$

where $A = \frac{V_1 - V_0}{\varphi_1}$, and $V_0, V_1, \varphi_1, \varphi_f$ are parameters of the model. V_0 is the value of the potential in the false vacuum, V_1 and φ_1 are the height of the potential barrier and the value of φ at the top of the potential barrier, respectively, φ_f is the value of the field at which the inflation stops. In the standard inflation theory [3] the field φ "slowly rolls down" in potential during the phase transition [4] . This rolling down takes place from the top, or in some models, from the point near the top [3,4] of the potential barrier to the value φ_f — to the place where "rolling down" is too fast to give considerable inflation [3]. We have attempted to find a smooth field configuration, starting from the false vacuum and ending in φ_f as a solution to the spatially dependent equation of motion. The time evolution of this field configuration gives the inflation of the Universe. From the "slow roll" condition it is clear that locally for every value of φ, the inflation occurs in the same way as like in the standard theory. Provided the solution has the smallest possible energy we can assume the spherical symmetry of the field configuration can be assumed. This leads to the equation

$$\varphi'' + \frac{2}{r}\varphi' = \frac{\partial U}{\partial \varphi}\,, \tag{2}$$

where prim denotes differentiation with respect to the space radius r. It is simple to solve equation (2) with the potential (1). We obtain

$$\varphi = \begin{cases} \frac{A}{6}r^2 + B + \frac{C}{r} & \text{for} \quad \varphi \in (0;\varphi_1)\,, \\ K\frac{\sin Mr}{r} + L\frac{\cos Mr}{r} & \text{for} \quad \varphi \in (\varphi_1;\varphi_f)\,. \end{cases} \tag{3}$$

Solution with the smallest energy and with $\varphi \in (0;\varphi_f)$ must fulfill the conditions

$$\frac{\partial\varphi}{\partial r}\,|_{\varphi=0} = 0\,,$$
$$\frac{\partial\varphi}{\partial r}\,|_{\varphi=\varphi_f} = 0\,. \tag{4}$$

Using (4) and the condition that φ must be continuous with its derivative in all points, we can show that B and C in (3) must vanish and that coefficients K and L are

$$K = \frac{3\varphi_1}{M}\cos Mr_1 + \sqrt{\frac{6}{V_1 - V_0}}\varphi_1^2 \sin Mr_1\,, \tag{5}$$

$$L = \sqrt{\frac{6}{V_1 - V_0}}\varphi_1^2 \cos Mr_1 - \frac{3\varphi_1}{M}\sin Mr_1\,,$$

where $r_1 = \sqrt{\frac{6}{V_1 - V_0}}\varphi_1$ is the space radius of the field at the top of the potential barrier. There is a connection between the potential and the field derivative for the solution of Eq. (2)

$$\frac{1}{3}\varphi'^2 - V(r) + \int \frac{\varphi'^2}{r}dr = \text{const}\,. \tag{6}$$

Using (6) it is easy to prove that

$$\frac{1}{3}(V_1 - V_0) = \frac{1}{2}M^2\varphi_f^2 + \int_{r_1}^{r_f} \frac{\varphi'^2}{r}dr\,. \tag{7}$$

The integral in (7) is elementary and can readily be calculated. Equation (7) gives the relationship between V_1 and V_0. At this level of accuracy we can estimate this relation to be $V_1 = aV_0$ where a is of the order of 10. This estimation stems from the properties of the well known exact solutions of (2). In fact Eq. (7) gives only the value of $V_1 - V_0$ and this value depends on M. Inflation of the Universe arises from the rolling down in the part of the potential which is described by $V_1 - \frac{1}{2}M^2\varphi^2$ .It is possible to find the estimation of M^2 from the "slow roll condition" [3]

$$\varepsilon = \frac{m^2}{16\pi}\left(\frac{V'}{V}\right)^2 \leq 1\,, \tag{8}$$

where m is the Planck mass. Using (4), (5) and (7) we can obtain the connection between φ_1 and φ_f. It may be proved that $\varphi_f = b\varphi_1$, where $b \leq 3$. Finally using above formulated conditions and solution (3) we can find relations between the parameters of the model. We have seven parameters $(V_0, V_1, \varphi_1, \varphi_f, r_1, r_f, M^2)$ and five equations

$$\begin{aligned} r_1 &= \sqrt{\frac{6}{V_1 - V_0}}\varphi_1\,, \\ M^2 &= \sqrt{\frac{16\pi}{m^2}}\frac{V_1}{\varphi_f}\,, \\ \varphi_f &\approx b\varphi_f \quad (b \approx 3)\,, \\ r_f &\leq \frac{\pi}{2M}\,, \\ V_1 &= V_0 a \quad (a \approx 10)\,. \end{aligned} \tag{9}$$

To obtain numerical values of these parameters it is necessary to find parameters of the false vacuum (in our model φ_1 and V_0). These parameters may be estimated from the observed properties of the Universe. One of them can be found from the homogeneity of our Universe. This homogeneity leads to the number of e-folds during inflation [3]

$$N(\varphi) = \frac{8\pi}{m^2}\int \frac{V}{V'}d\varphi\,, \tag{10}$$

where $N \approx 60$ [3]. Using this equation and relations between parameters we can find the value of φ_1

$$\varphi_1 = \frac{Nm}{2\sqrt{\pi}b\ln b}\,. \tag{11}$$

We ignore the fact that in this model the Universe is slightly nonhomogeneous, because potential (1) is almost flat. Thus, nonhomogeneity may cause only negligible changes in N. Value of the V_0 can be estimated from the assumption that the effect found by Nodland and Ralston is real. The nonhomogeneity reported in [1] may be caused by the remnant of the Universe before inflation. For a rough estimation we can assume that the particles which survive inflation have the same influence on the polarization plane as standard matter. In this model the column density of these particles is bigger in the direction of small inflation. In the opposite direction density of these particles is small. From [1] we have that the rotation of the polarization plane of the electromagnetic radiation is of several degrees when the radiation passes the cosmological distance. The same rotation may be caused by the few meter passage of radiation through optically active matter with density $\varrho_0 = \frac{1\mathrm{kg}}{\mathrm{m}^3}$. From this we can estimate the column density of the remnant particles $\varrho_N \approx 10^{-25}\varrho_0$. Using the number N we can find the density of these particles in the epoch before inflation $\varrho_A \approx \varrho_0 10^{-25} e^{3N}$ Assuming that $V_0 \approx P\varrho_A$, where P is the factor connecting V_0 with ϱ_A, we find all parameters of the potential (1). For example

$$r_1 \approx \frac{N}{b \ln b\sqrt{(P\varrho_0)}} 10^{26} e^{-\frac{3}{2}N} m \qquad V_0 \approx P\rho_0 10^{-8} e^{3N} \frac{\mathrm{kg}}{s^2 m}, \tag{12}$$

where P, and b are numbers of the order of 10. We see that in this very simple model it is possible to find the properties of the false vacuum. This is not surprising since we used one more condition than is typically used. The model discussed above has some interesting advantages. The predicted nonhomogeneity can be compared with observations. Especially, if someone wants to find particles from ancient Universe, it is clear in what direction one should look for them. For example magnetic monopoles may have bigger density in the direction found in [1]. If the inflation arises in the way described above then the Universe splits into two parts. One of them is our Universe. The other part, unfortunately under the horizon, has the properties of the world before inflation. This latter part of the world is created by that part of the potential which lies on the left side of the potential barrier. The sphere of radius r_1 is the "gate" to this fascinating Universe. We see that our model leads to observationally verifiable predictions.

REFERENCES

[1] B. Nodland, J.P. Ralston, *Phys. Rev. Lett.* **78**, 3034 (1997).

[2] J.F.C. Wardle, R.A. Perley, M.H. Cohen, *Phys. Rev. Lett.* **79**, 1801 (1997).

[3] W.H. Kinney, K.T. Mahanthappa, *Phys. Rev.* **D53**, 5455 (1996); E.J. Copeland, E.W. Kolb, A.R. Liddle, J.E. Lidsey, *Phys. Rev.* **D48**, 2529 (1993); J. Sładkowski, *Int. J. Theor. Phys.* **30**, 1071 (1991).

[4] A.D. Linde, *Nucl. Phys.* **B216**, 421 (1983); S. Coleman, *Phys. Rev.* **D15**, 2929 (1977); M. Gleiser, E.W. Kolb, R. Watkins, *Nucl. Phys.* **B364**, 411 (1991).