NUCLEON STRUCTURE FUNCTIONS FROM THE INSTANTON VACUUM: LEADING AND NON-LEADING TWISTS*

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(Received September 23, 1997)

We review the description of nucleon structure functions in the instanton vacuum. This includes the calculation of the twist-2 parton distributions at a low normalization point as well as higher-twist matrix elements. The instanton vacuum with its inherent small parameter, the packing fraction of the instanton medium, $\bar{\rho}/R$, provides a consistent picture of the non-perturbative gluon degrees of freedom at the scale $\bar{\rho}^{-1} \simeq 600\,\mathrm{MeV}$. The twist-2 quark and antiquark distribution are of order unity, while the twist-2 gluon distribution is of order $(\bar{\rho}/R)^4$. Twist-4 matrix elements determining power corrections to the Bjorken, Ellis–Jaffe and Gross–Llewellyn-Smith sum rules are found to be of order $(\bar{\rho}/R)^0$. We present numerical estimates for the parametrically large quantities.

PACS numbers: 12.38. Lg, 13.60. Hb, 11.15. Kc, 12.39. Ki

In these talks we give a brief summary of recent progress in understanding the deep-inelastic structure of the nucleon, both at leading and non-leading twist level, in the instanton vacuum [1–4]. Our aim is to show that the instanton vacuum, with its inherent small parameter — the packing fraction of the instanton medium, $\bar{\rho}/R$ — provides a basis for a consistent and quantitative description of nucleon structure functions.

* Presented at the XXXVII Cracow School of Theoretical Physics, Zakopane, Poland, May 30 – June 10, 1997.

1. Leading and non-leading twist

The non-perturbative information which enters in the QCD description of deep-inelastic scattering and other related experiments is contained in nucleon matrix elements of operators of twist-2 and higher. The moments of the non-power suppressed part of the structure functions are given by matrix elements of operators of leading twist; in the unpolarized case these are the twist-2 quark and gluon operators

$$i^{n-1} \sum_{\text{spin}} \langle P| \, \bar{\psi} \tau^{NS,S} \gamma_{\{\mu_1} \nabla_{\mu_2} \ldots \nabla_{\mu_n\}} \psi \, \Big|_\mu \, |P\rangle$$

$$= 2A^{(n)}_{NS,S}(\mu) \; [P_{\mu_1} \ldots P_{\mu_n} - \text{traces}], \tag{1}$$

$$i^{n-1} \sum_{\text{spin}} \langle P| \, F_{\{\mu_1 \alpha} D_{\mu_2} \ldots D_{\mu_{n-1}} F^{\alpha}{}_{\mu_n\}} \, \Big|_\mu \, |P\rangle$$

$$= 2A^{(n)}_{G}(\mu) \; [P_{\mu_1} \ldots P_{\mu_n} - \text{traces}], \tag{2}$$

and similarly for the polarized case, see [5]. Here, $\tau^S = 1, \tau^{NS} = \tau^3$ are flavor matrices. Alternatively, one may work with non-local (light-cone) operators, which serve as generating functions of the series of local twist-2 operators. In the partonic language, the matrix elements of these operators can directly be interpreted as parton distribution functions[1], the scale dependence of which is described by the DGLAP evolution equation:

$$q_f(x,\mu) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \, \mathrm{e}^{i\lambda x}$$

$$\times \sum_{\text{spin}} \langle P| \, \bar{\psi}_f(0) \not{n} \left\{ \mathrm{P} \exp \left[-i \int_0^\lambda d\lambda' \; n \cdot A(\lambda' n) \right] \right\} \psi_f(\lambda n) \, \Big|_\mu \, |P\rangle, \tag{3}$$

$$g(x,\mu) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \, \mathrm{e}^{i\lambda x}$$

$$\times \sum_{\text{spin}} n_\mu n_\nu \langle P| \, F_{\mu\alpha}(0) \left\{ \mathrm{P} \exp \left[-i \int_0^\lambda d\lambda' \; n \cdot A(\lambda' n) \right] \right\} F^{\alpha}{}_{\nu}(\lambda n) \, \Big|_\mu \, |P\rangle, \tag{4}$$

where n denotes a light-like four-vector, $n^2 = 0$. Operators of higher twist arise in the description of power corrections [5]. For example, in the unpolarized case the $1/Q^2$-power corrections to the Gross–Llewellyn-Smith sum

[1] Here, $q_f(x)$ corresponds to the quark distribution at positive x, and to minus the antiquark distribution at negative x.

rule are governed by the matrix element of the twist-4, spin-1 operator (M_N is the nucleon mass)

$$\tfrac{1}{2}\sum_{\text{spin}}\langle P|\bar{\psi}\gamma_\alpha\gamma_5\tilde{F}^{\beta\alpha}\psi|P\rangle \;=\; 2M_N^2\; C_S^{(2)}\; P^\beta. \tag{5}$$

In polarized scattering the $1/Q^2$-power corrections to the isovector and isosinglet combinations of the first moment of the polarized structure function g_1 — the Bjorken and Ellis–Jaffe sum rules — involve the matrix elements of the twist-3, spin-2 operators [6]:

$$\begin{aligned}&\langle PS|\bar{\psi}\tau^{NS,S}\left(\gamma^\alpha\tilde{F}^{\beta\gamma}+\gamma^\beta\tilde{F}^{\alpha\gamma}\right)\psi|PS\rangle - \text{traces}\\ &= 2M_N\; d_{NS,S}^{(2)}\;\left[2P^\alpha P^\beta S^\gamma - P^\gamma P^\beta S^\alpha - P^\alpha P^\gamma S^\beta + (\alpha\leftrightarrow\beta) - \text{traces}\right],\end{aligned} \tag{6}$$

(the same matrix elements contribute also at leading twist level to the third moment of the structure function g_2), and the matrix elements of the twist-4, spin-1 operators,

$$\langle PS|\bar{\psi}\tau^{NS,S}\gamma_\alpha\tilde{F}^{\beta\alpha}\psi|PS\rangle \;=\; 2M_N^3\; f_{NS,S}^{(2)}\; S^\beta. \tag{7}$$

Here, S is the nucleon polarization vector, $S^2 = -1$. The operators here are assumed to be normalized at scale μ; the scale dependence of the matrix elements is described by the renormalization group equation. The twist-2 quark-, antiquark- and gluon-distributions at a low normalization point have been determined by fits to data from a variety of experiments [7,8]. The twist-3 matrix element $d^{(2)}$ has recently been extracted from measurements of the structure function g_2 [9]. Experimental knowledge of the higher-twist matrix elements entering only in power corrections, such as the twist-4 matrix element $f^{(2)}$, is still rather poor [10].

Any attempt to calculate the matrix elements mentioned here from first principles requires an understanding of the non-perturbative effects giving rise to the structure of the nucleon. While the gross features of the leading-twist quark distributions can be understood in phenomenological models like the quark model or the bag model, to describe the gluon distribution or higher-twist matrix elements explicitly involving the gluon field one needs a theory of the non-perturbative fluctuations of the gluon field. Lattice calculations of structure functions have been making steady progress during the last years; however, they are still far from giving a satisfactory quantitative description [11].

2. Instanton vacuum

A microscopic picture of the non-perturbative fluctuations of the gluon field is provided by the instanton vacuum. For an introduction to the instanton vacuum and its applications to hadronic physics we refer to the extensive literature on this subject, *e.g.* the recent reviews [12,13]; we can here touch upon only those aspects directly relevant to structure functions. Instantons and antiinstantons (I and $\bar{I}$ for short) are particular field configurations which are solutions of the Euclidean Yang–Mills equations, characterized by a size, ρ, center, z and color orientation given by an SU(N_c) matrix, U,

$$A^a_\mu(x;\, z,U)_{I(\bar{I})} \;=\; f_\nu(x-z)\, O^{ab} (\eta^{\mp})^b_{\mu\nu},$$
$$f_\nu(x) \;=\; \frac{2\rho^2}{(x^2+\rho^2)x^2} x_\nu, \qquad O^{ab} \;=\; \tfrac{1}{2}\mathrm{tr}\,[\lambda^a U \lambda^b U^\dagger]. \quad (8)$$

Here, $(\eta^{\mp})^b_{\mu\nu} = \bar{\eta}^b_{\mu\nu}, \eta^b_{\mu\nu}$ are the 't Hooft symbols. Instantons have many special properties; not all of them are of interest to us here. Let us note that a single $I(\bar{I})$ is an $O(4)$-symmetric field configuration — this fact makes important for differences between instanton contributions to operators of different spin, see below.

In the instanton vacuum one considers non-perturbative effects due to field configurations with a finite density of I's and $\bar{I}$'s. The medium of I's and $\bar{I}$'s stabilizes itself due to instanton interactions [15], meaning that the average size of the instantons in medium is finite,

$$\bar{\rho} \simeq (600\,\mathrm{MeV})^{-1}. \quad (9)$$

The coupling constant is fixed at a scale of order $\bar{\rho}^{-1}$, so when we evaluate matrix elements of QCD operators below it is implied that the operators are normalized at $\mu \simeq \rho^{-1}$. It should be stressed that no external scale is introduced here; all parameters of the instanton medium are obtained in terms of the QCD scale parameter, Λ_{QCD}. Hence this approach preserves the essential renormalization properties of QCD.

The most important property of the instanton vacuum is the small packing fraction of the medium, *i.e.*, the small ratio of the average size of the instantons in the medium to the average separation between nearest neighbors, $\bar{\rho}/\bar{R} \simeq 1/3$ [14,15]. This small parameter is the starting point for a systematic analysis of non-perturbative phenomena in this picture.

In particular, the instanton vacuum explains the dynamical breaking of chiral symmetry. The Dirac operator in the background of one $I(\bar{I})$ has a localized zero mode,

$$\left[i\partial\!\!\!/ + A\!\!\!/_{I(\bar{I})}(x;z,U)\right] \Phi_\pm(x;z,U) = 0. \quad (10)$$

In the medium the zero modes associated with the individual instantons delocalize [16], resulting in a finite fermion spectral density at zero eigenvalue, which by the Banks–Casher theorem is equivalent to the chiral condensate [17]. Alternatively, one can derive the effective action of fermions in the instanton medium in the $1/N_c$-expansion, integrating over the instanton coordinates in the ensemble [18]. An individual $I(\bar{I})$ interacts with the fermion field through the zero mode, *i.e.*, through a "potential"

$$V_{I(\bar{I})}[\bar{\psi}_f, \psi_f] = \int d^4x \int d^4y \, \bar{\psi}_f(x) \, \partial\!\!\!/ \Phi_\pm(x; z, \mathcal{U}) \bar{\Phi}_\pm(y; z, \mathcal{U}) \, \partial\!\!\!/ \psi_f(y) \,. \quad (11)$$

In leading order in $\bar{\rho}/R$ the effective action exhibits a many-fermionic interaction, which is given by the one-instanton average of Eq. (11) and has the form of the 't Hooft determinant in flavor indices [19],

$$\int d^4z \, d\mathcal{U} \prod_f^{N_f} V_{I(\bar{I})}[\bar{\psi}_f, \psi_f] \propto \det_{fg} \, \bar{\psi}_f \frac{1 \pm \gamma_5}{2} \psi_g. \quad (12)$$

In addition, there is a form factor (not written here for brevity) related to the finite size of the zero-mode wave function, which makes the interaction vanish for quark momenta larger than $\bar{\rho}^{-1}$. Chiral symmetry is spontaneously broken due to this many-fermionic interaction: the quarks acquire a dynamical (momentum-dependent) mass, a quark condensate develops, and a massless pion appears as a collective excitation. In the $1/N_c$-expansion one can easily construct the effective action in the chirally broken phase. It can be formulated as a theory of massive quarks, interacting with the pion field in a chirally invariant way.

$$Z = \int \mathcal{D}\pi \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp \int d^4x i\bar{\psi}(x) \left[i\partial\!\!\!/ + iM \, e^{i\gamma_5 \tau^a \pi^a(x)} \right] \psi(x). \quad (13)$$

The effective theory applies for quark momenta up to the inverse instanton size, $\bar{\rho}^{-1} \simeq 600\,\mathrm{MeV}$, which acts as a cutoff. For the applicability of this effective theory it is crucial that the ratio of the dynamically generated quark mass to the cutoff is proportional to the packing fraction of the instanton medium,

$$M\bar{\rho} \propto \left(\frac{\bar{\rho}}{R} \right)^2. \quad (14)$$

Hence the diluteness of the instanton medium guarantees that the picture of massive "constituent" quarks applies in a parametrically wide range of momenta. Finally, we note that the nucleon is obtained in the $1/N_c$-expansion as a chiral soliton of the effective theory, Eq. (13) [20]. This picture of the

nucleon gives a very reasonable description of a variety of hadronic properties such as the N–Δ splitting, electromagnetic formfactors, axial coupling constants *etc.* [21].

The instanton vacuum, with the resulting effective chiral theory, allows to evaluate hadronic matrix elements of QCD operators involving the gluon field. In Ref. [1] a method was developed by which "gluonic" operators can systematically be represented as effective operators in the effective chiral theory. Let $\mathcal{F}[A]$ be a gluonic operator, *i.e.*, some function of the gauge field. To leading order in the packing fraction, $\bar{\rho}/R$, the interaction of the gluon operator with the fermion field is mediated by single instantons. The effective operator, denoted by "$\mathcal{F}$", is obtained by substituting in $\mathcal{F}[A]$ the gauge field of one $I(\bar{I})$ and integrating over the collective coordinates,

$$\text{“}\mathcal{F}\text{”}[\bar{\psi},\psi] \propto \sum_{I+\bar{I}} \int d^4z\, d\mathcal{U}\; \mathcal{F}\left[A_{I(\bar{I})}(z,U)\right] \prod_f^{N_f} V_{I(\bar{I})}[\bar{\psi}_f,\psi_f]. \qquad (15)$$

In higher orders of $\bar{\rho}/R$ one needs to take into account many-instanton contributions to the effective operator. Below we shall need the effective operator corresponding to the gauge field itself, which is given by (*cf.* Eq. (8)) [4]:

$$\text{“}A\text{”}(x)^a_\mu \propto \sum_{\pm} \int d^4z\; f_\nu(x-z)\,\bar{\psi}(z)\,\frac{\lambda^a}{2}\sigma_{\mu\nu}\frac{1\pm\gamma_5}{2}\;\mathrm{e}^{\pm i\pi^a(z)\tau^a}\;\psi(z). \qquad (16)$$

Again we have suppressed the form factors coming from the zero mode wave function. Note the presence of the pion field, as a result of which the effective operator Eq. (16) is chirally invariant — as it should be, since the gluon field is flavor neutral.

It is important to note that the representation of QCD operators as effective operators relies entirely on the approximations already inherent in the effective theory — the diluteness of the instanton medium and the $1/N_c$-expansion; no additional assumptions are required. It was shown in [1] that this approach preserves the essential renormalization properties of QCD; for example, the QCD trace and U(1) anomalies are realized at the level of hadronic matrix elements. This method is thus well suited for computing matrix elements of the QCD operators of twist-2 and higher-twist of interest here.

3. Twist-2 matrix elements in the instanton vacuum

Let us first consider the twist-2 gluon operators, Eq. (2). Since we are interested only in forward matrix elements we may average the operator position over the 4-volume. The definition of the effective operator, Eq. (15),

implies the integral over the instanton coordinate, z. Since the instanton field is O(4) symmetric the only tensor one has at hand to construct the effective operator is the Kronecker delta, but it is impossible to construct a traceless symmetric tensor from it! (When working with light-like components this follows from $\delta_{++} = 0$.) Thus, the effective operators for the twist-2 gluon operators, Eq. (2), or, more generally, the nonlocal operator, Eq. (4), vanish at one-instanton level. We conclude that the twist-2 gluon distribution is parametrically suppressed in $\bar{\rho}/R$.

To determine the twist-2 gluon distribution quantitatively one has to include (at least) the two-instanton contribution to the effective operator. Preliminary results indicate that the gluon distribution is, in fact, proportional to $(M\bar{\rho})^2$, which is parametrically of order $(\bar{\rho}/R)^4$. It is interesting to note that, numerically, a suppression of the gluon relative to the singlet quark momentum fraction by a factor $(M\bar{\rho})^2 \simeq 0.3$ is consistent with the GRV parametrization of the data at a normalization point of $\mu \simeq 600\,\mathrm{MeV}$ [8].

In the twist-2 quark operators, Eq. (1), the gauge field enters through the covariant derivative, or, equivalently, through the path-ordered exponential in the non-local operator, Eq. (3). It is interesting to ask how much the gauge field contributes to the moments of the quark distribution functions in the instanton vacuum, which is formulated in the so-called singular gauge in which the instanton field has the form of Eq. (8). For simplicity, let us consider the second moment of the singlet unpolarized quark distribution, $A_S^{(2)}$, which is given by the matrix element of the operator

$$i\langle P|\bar{\psi}\gamma_{\{\mu_1}\left(\partial_{\mu_2\}} - i\frac{\lambda^a}{2}A^a_{\mu_2\}}\right)\psi|P\rangle - \text{traces} = 2A_S^{(2)}P_{\mu_1}P_{\mu_2} - \text{traces}. \quad (17)$$

It is instructive to compute the matrix element not immediately in the nucleon, but first in a "constituent" quark, *i.e.*, the massive quark of the effective chiral theory. It is easy to see that the short derivative in Eq. (17) makes an order unity contribution to $A_S^{(2)}$. Computing the gauge field contribution to the matrix element using the effective operator, Eq. (16), one finds that it is of order $(M\bar{\rho})^2 \propto (\bar{\rho}/R)^4$, *i.e.*, parametrically suppressed relative to the short derivative. Thus we have

$$A^{(2)}_{S,\mathrm{quark}} = 1 + O\left[\left(\frac{\rho}{R}\right)^4\right]. \quad (18)$$

This is consistent with the fact that the gluon distribution is of higher order in the packing fraction: To order $(\bar{\rho}/R)^0$ the quarks carry the entire momentum, and the gluon distribution is zero. (To compute the full $(\bar{\rho}/R)^4$ contribution would, again, requires to take into account two-instanton contributions to the effective operators, as well as to the effective quark action.)

The above statements are easily generalized to higher moments. In fact, speaking of parton distribution functions "inside the constituent quark" one may say that

$$\sum_f^{N_f} q_f(x)_{\text{quark}} = \delta(x-1) + O\left[\left(\frac{\bar\rho}{R}\right)^4\right], \tag{19}$$

$$g(x)_{\text{quark}} = O\left[\left(\frac{\bar\rho}{R}\right)^4\right]. \tag{20}$$

To leading order in $\bar\rho/R$ the constituent quark has no structure. Consequently, in leading order in $\bar\rho/R$ it is justified to identify the distribution of "constituent" quarks and antiquarks in the nucleon with the actual parton distribution at the scale $\mu \simeq \bar\rho^{-1}$. In another way of saying, when computing the twist-2 quark distribution function in the effective theory in leading order in $\bar\rho/R$ one can identify the QCD quark fields normalized at $\mu \simeq \bar\rho^{-1}$ with the quark fields of the effective chiral theory and put the path-ordered exponential in Eq. (3) to unity. This is the "quarks–antiquarks only" approximation which was employed to compute the quark and antiquark distributions of the nucleon in the chiral quark soliton model [2]. We have thus seen that this approximation has a parametric justification in the instanton vacuum.

The twist-2 quark and antiquark distributions of the nucleon computed in the chiral quark soliton model satisfy all general requirements, such as positivity, proper normalization *etc.* This is a fully field-theoretic description of the nucleon, which, in particular, makes possible a consistent calculation of the antiquark distributions. All partonic sum rules (baryon number, isospin, momentum, Bjorken sum rule) are satisfied within the model. Computed so far were the leading distribution functions in the large-N_c limit, the isosinglet unpolarized and isovector polarized [2], as well as the isovector transverse polarized distribution [3]. The isosinglet unpolarized quark and antiquark distributions are shown in Fig. 1, together with the GRV parametrization [8]; for the polarized distributions see the original papers.

To order $(\bar\rho/R)^0$ the constituent quarks have no structure, and the constituent quarks and antiquarks carry the entire nucleon momentum. As a result the singlet quark distribution calculated in this approximation is generally larger than the GRV parametrization, which includes gluons at the low normalization point, see Fig.1. In higher orders of $\bar\rho/R$ one starts to systematically resolve the structure of the constituent quark in terms of the original QCD degrees of freedom, and the nucleon momentum gets distributed among quarks and gluons at the low scale. It is interesting that the 30% gluon momentum fraction at the low scale obtained by GRV [8] is con-

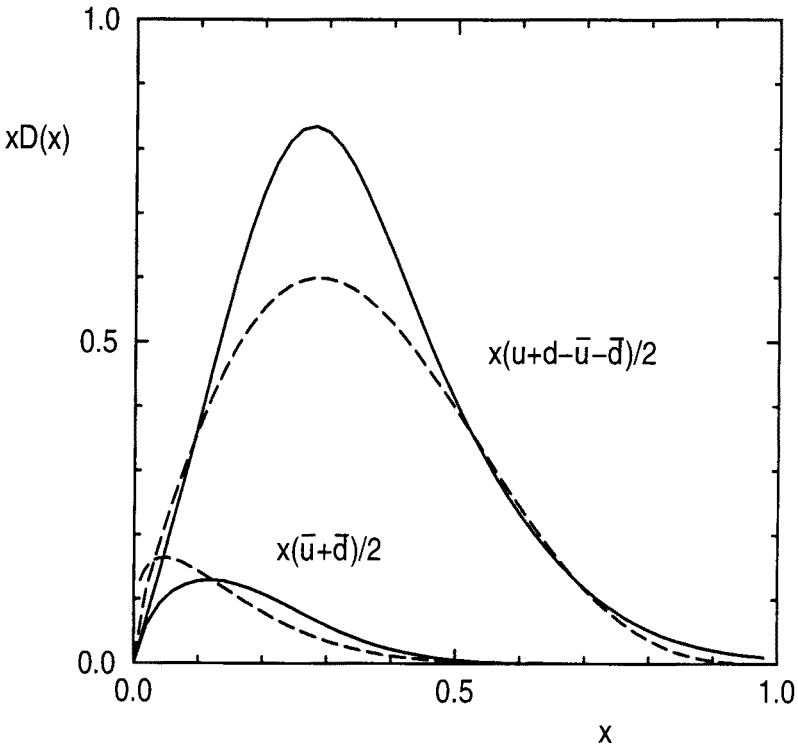

Fig. 1. *Solid lines*: The isosinglet unpolarized valence quark and antiquark distributions computed in the chiral soliton model of the nucleon [2]. *Dashed lines*: NLO-parametrization of GRV ($\mu^2 = 0.34\,\mathrm{GeV}^2$) [8].

sistent with the gluon distribution in the instanton vacuum being suppressed by a factor $(M\bar{\rho})^2 \simeq 0.3$.

To summarize the discussion of twist-2 operators, one may say that at twist-2 level the effects of the instanton medium are essentially contained in the dynamical quark mass generated in the dynamical breaking of chiral symmetry. Instanton contributions to the twist-2 operators, in the sense of effective operators, Eq. (15), are parametrically suppressed. This is what one could call a "constituent" quark picture. We note that, contrary to other approaches where the "constituent" quark is a largely philosophical object, in the instanton vacuum this term has a well-defined meaning, thanks to the parameter $\bar{\rho}/R$ [22].

4. Higher-twist matrix elements in the instanton vacuum

From the above discussion one may have the impression that in the instanton vacuum "gluonic" contributions to operators are always parametrically suppressed relative to quark operators. This is not so — instantons can make order $(\bar{\rho}/R)^0$ in operators of *higher twist*. The most immediate way to convince oneself of this is to consider a particular higher-twist operator whose matrix elements vanishes by the QCD equations of motion. In addition, this exercise provides a beautiful check for the consistency of the effective operator method. Consider the twist-4 matrix element obtained by projecting the operator in Eq. (17) not on spin two but on spin zero:

$$i\langle P|\bar{\psi}\not{\nabla}\,\psi|P\rangle \equiv \langle P|\bar{\psi}\,(i\not{\partial} + \not{A}\,)\,\psi|P\rangle = 0. \tag{21}$$

In QCD the matrix element is zero by virtue of the QCD equations of motion. In the instanton vacuum we find, computing the contribution of the gauge field in Eq. (21) using the effective operator, Eq. (16), one may show that

$$\langle P|\bar{\psi}\,(i\not{\partial} + \text{“}\not{A}\text{”})\,\psi|P\rangle = \langle P|\bar{\psi}\left(i\not{\partial} + iM\,\mathrm{e}^{i\gamma_5\tau^a\pi^a(x)}\right)\psi|P\rangle = 0. \tag{22}$$

(The calculation is actually rather involved; see [4] for details.) Here the gauge field contribution to the operator is of order unity, *i.e.*, of the same order as that of the short derivative. As a result the QCD operator of Eq. (21) reduces to an effective operator which vanishes identically due to the equations of motion of the effective chiral theory. From this we learn two things: First, instantons can make order unity contributions in twist-4 operators. Second, the method of effective operators preserves a principal feature of QCD: matrix elements of operators which are zero in QCD due to the QCD equations of motion are automatically zero in the effective theory. We note that also other operators whose forward matrix elements vanish in QCD,

$$\langle P|\bar{\psi}\gamma_\alpha F^{\beta\alpha}\psi|P\rangle, \qquad \langle PS|\bar{\psi}\gamma_\alpha\gamma_5 F^{\beta\alpha}\psi|PS\rangle, \tag{23}$$

give zero matrix elements when translated to the effective chiral theory.

We can now turn to the calculation of the matrix elements of twist-3 and 4 operators appearing in power corrections, Eqs (5), (6), (7). Again, the qualitative features can be seen by studying the matrix elements in "constituent" quark states. Computing the matrix elements of the effective operators corresponding to Eqs (5), (6), (7) one finds that twist-3 and 4 matrix elements are of different order in the packing fraction:

$$\text{twist-4:}\quad f^{(2)}, C^{(2)} \sim (M\bar{\rho})^0 \sim \left(\frac{\bar{\rho}}{R}\right)^0,$$

$$\text{twist-3:}\quad d^{(2)} \sim (M\bar{\rho})^2 \log M\bar{\rho} \sim \left(\frac{\bar{\rho}}{R}\right)^4 \log\left(\frac{\bar{\rho}}{R}\right).$$

Again, the reason for this can be seen in the $O(4)$ symmetry of the individual $I(\bar{I})$. Note that the parametric order of the matrix elements is determined by two factors: *i)* the number of instantons participating in the effective operator (here we have included only the one-instanton contribution), and *ii)* the dependence of the quark loop integrals in the matrix element (obtained by closing the quark lines on the many-fermionic effective operator) on the cutoff, $\bar{\rho}^{-1}$, keeping in mind that $(M\bar{\rho})^2 \sim (\bar{\rho}/R)^4$. An order unity contribution can come only from integrals which are "quadratically divergent", meaning they are proportional to $\bar{\rho}^{-2}$. Incidentally, this last fact implies that, in the framework of our effective theory, the dominant contributions to higher-twist matrix elements come from "divergent" loop diagrams where the effective many-fermionic operator couples to a *single* constituent quark, not from diagrams describing interactions of more than one constituent quark mediated by the many-fermionic effective operator. In the latter all momenta are cut by the bound-state wave function of the nucleon, not by the cutoff, $\bar{\rho}^{-1}$. Losely speaking, one may thus say that in our picture higher-twist matrix elements measure properties of the individual constituent quarks, not correlations between them.

We have computed the leading higher-twist nucleon matrix elements in the large-N_c limit; see [4] for details. These are the flavor nonsinglet spin-dependent ones ($d_{NS}^{(2)}, f_{NS}^{(2)}$) and flavor singlet spin-independent ones ($C_S^{(2)}$). Results are shown in Table I.

TABLE I

Numerical results for the flavor-nonsinglet spin-dependent twist-4 and 3 matrix elements $f_{NS}^{(2)}$ and $d_{NS}^{(2)}$, Eqs (6, 7), and the flavor-singlet spin-independent twist-4 matrix element $C_S^{(2)}$, Eq. (5). Shown are the results obtained from the instanton vacuum [4], from QCD sum rule calculations [23, 24, 26], from the bag model [25], and from lattice calculations [11]. Also shown are estimates of $d_{NS}^{(2)}$ based on measurements of the structure function g_2 [9], and estimates of $f_{NS}^{(2)}$ from an analysis of power corrections to g_1 [10].

	$f_{NS}^{(2)}$	$d_{NS}^{(2)}$	$C_S^{(2)}$	scale/GeV^2
Instanton vacuum [4]	-0.10	$\sim 10^{-3}$	0.36	~ 0.4
Sum rules [23]	-0.20	0.072	—	1
Sum rules [24]	-0.072	0.072	—	1
Bag model [25]	0.11	0.063	—	5
Lattice [11]	—	-0.13	—	4
Sum rules [26]	—	—	0.37	1
E142, E143, E154 [9]	—	0.043 ± 0.046	—	3
Power corr. [10]	0.10 ± 0.28	—	—	1

The result for $d_{NS}^{(2)}$ should be taken as an order of magnitude estimate; to compute it accurately at level $(\bar{\rho}/R)^4$ one needs to include the two-instanton contribution to the effective operator. Note that the results for $f^{(2)}$ and $C^{(2)}$ agree well with estimates from QCD sum rules [23,26]; our value for $d^{(2)}$ is consistent with estimates based on measurements of the structure function g_2 [9].

5. Summary

The predictions of the instanton vacuum for nucleon matrix elements relevant to structure functions can be summarized as follows:

- At twist-2 level, the quark and antiquark distributions are of order unity in the packing fraction. In this order they can be computed in the effective chiral theory without including instanton contributions to the twist-2 operators, *i.e.*, replacing covariant by short derivatives in Eq. (1), or dropping the path-ordered exponential in the light-cone operator, Eq. (3). The twist-2 gluon distribution is of order $(\bar{\rho}/R)^4$; to compute it one needs to take into account at least two-instanton contributions to the effective operators.
- The instanton vacuum implies a *hierarchy of twists*: Large — that is, $(\bar{\rho}/R)^0$ — contributions are found in operators of lowest spin (twist-4, in our case), while the contributions to operators of higher spin (twist-3 and 2) are suppressed. The reason for this pattern is the $O(4)$-symmetry of the single instanton.

The instanton vacuum provides a consistent framework for describing the non-perturbative input necessary for a complete understanding of DIS experiments. The key element, which makes possible a systematic approach to non-perturbative phenomena, is the small parameter $\bar{\rho}/R$ inherent in this picture. The "quarks–antiquarks only" approximation for twist-2 operators, in connection with the chiral quark soliton model of the nucleon, gives a very successful description of the twist-2 quark and antiquark distributions of the nucleon, both polarized and unpolarized [2]. As to higher twists, one may hope that increasing accuracy of the measurements of polarized and unpolarized structure functions (power corrections) or, possibly, semi-inclusive measurements, will allow to test the specific predictions of the instanton vacuum more accurately.

Much remains to be done on the theoretical side. In particular, one should refine the effective operator approach to be able to compute also parametrically small matrix elements, first of all the twist-2 gluon distribu-

tion [2]. In addition to taking into account the two-instanton contributions to the effective operator this requires to compute also the effective quark action to higher orders in $M\bar{\rho}$, since many properties — for example, the correct realization of the QCD equations of motion — depend on the consistency of the definitions of effective operators and the effective action. Work in this direction is in progress.

It is a pleasure to thank J. Balla, D.I. Diakonov, V.Yu. Petrov, and P.V. Pobylitsa for a stimulating collaboration and many interesting discussions, as well as K. Goeke for encouragement and multiple support.

The work reported here has been supported in part by the Deutsche Forschungsgemeinschaft (DFG), by a joint grant of the DFG and the Russian Foundation for Basic Research, and by COSY (Jülich). M.V.P. is supported by the A.v. Humboldt Foundation.

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