ON SPACE-TIME EVOLUTION OF NUCLEAR COLLISIONS AT CERN–SPS ENERGIES*

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We make an attempt to combine information from different CERN–SPS data on nuclear interactions to get a rough picture of the space-time evolution of these collisions. Topics discussed include: formation time of hadrons, energy densities reached in heavy-ion interactions, dependence of longitudinal radii R_L on nucleon numbers of colliding nuclei, e^+e^- and strangeness production and J/Ψ suppression. We argue that the CERN–SPS data on heavy-ion collisions might be consistent with the picture of the space-time evolution consisting of a stage of intermediate gluons, with duration of 0.5–2,0 fm/c depending on the size of the system, followed by the stage of interacting hadron gas. The behaviour of hadron gas is supposed to be strongly influenced by the formation time of hadrons and its Lorentz dilation.

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1. Introduction

Experiments on heavy-ion interactions at the CERN–SPS has brought quite a lot of most interesting data and new data and increase of statistics are expected in the near future. The data include among others, J/Ψ suppression [1,2], production of dileptons [3–6] ratios of heavy strange baryons Ω/Ξ, and Ξ/Λ [7,8] and Bose–Einstein correlations of identical particles [9,10].

It is quite natural to ask whether these data give hints to what is the space-time development of heavy-ion collisions in the CERN–SPS energy range. Of course, the question has be to posed in a different way, one has to formulate first a qualitative, or if possible, also quantitative picture of the process and then ask whether the picture is consistent with data.

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It is frequently assumed that Quark–Gluon Plasma (QGP) is formed in heavy-ion collisions even in the CERN–SPS energy region. But in order to understand general features of nuclear collisions in this energy region as a function of nucleon numbers A and B of colliding ions it is difficult to avoid a discussion of the dynamics by which QGP is formed. Without the dynamics we can hardly answer the questions like: "Starting with what values of A, B and of the transverse energy E_{T} is the QGP produced?" or "what is the life-time and size of QGP at given A, B and E_{T}?". In the near future experiments on heavy-ion collisions at the CERN–SPS will be probably performed at a lower beam energy of 40 or 60 GeV per nucleon. The preceding questions will again be repeated in a slightly different form, *e.g.* "at what beam energy will QGP be produced in central collisions with given A and B?" or "What is the life-time of QGP formed in the central collision of A and B at a given beam energy?" Without answering such questions it would be difficult to have a picture of the evolution of the collision permitting to analyse data on different signatures from a unique point of view.

Of course, assuming that QGP, or some intermediate gluonic stage is formed in heavy-ion collisions at a given beam energy is equivalent to assuming that gluon (parton) degrees of freedom play an important role in these collisions. If, for instance, we assume that gluonic stage is not present in the BNL energy range, but it is present in the CERN–SPS one, we would need to understand why partonic degrees of freedom are important in the latter case and hadronic d.o.f. in the former one. We shall discuss this issue from the point of view of the Landau–Pomeranchuk–Migdal (LPM) effect in Sect. 2.

This discussion will give hints supporting a picture of ion–ion collisions at the CERN–SPS energies consisting of the following three stages

(i) gluonic stage of 0.5–2.0 fm/c, depending on the nucleon numbers and the impact parameter of collision,

(ii) hadronization,

(iii) interactions of hadrons, influenced by the formation time of hadrons and its Lorentz dilation.

Each type of data is influenced by all the three stages but some stage can play a dominant role. Let us give a few examples. According to arguments in Refs [11, 12] the cross-section for disintegration of J/Ψ by hadrons is rather small near threshold and J/Ψ is thus influenced mostly by stage *(i)*,(and by Gerschel–Hüfner mechanism [13]) the dilepton production is rather small in the gluonic stage and is thus most sensitive to the third stage, ratios Ω/Ξ and Ξ/Λ are given by the process of hadronization and to lesser extent by interactions of final state hadrons, total strangeness production as given by the average number $\langle s\bar{s}\rangle$ of $s\bar{s}$ pairs in final state hadrons is essentially given by the first stage and Bose–Einstein correlation functions reflect the total expansion of the system, summarizing thus all the three stages. In Table I we indicate the influence of the three stages on some of the signatures. A strong influence of a stage on a signature is denoted by **, some influence by *.

TABLE I

Contribution of stages to processes

	J/Ψ suppr.	Ψ'	dilepton production	Ω/Ξ Ξ/Λ	$\langle s\bar{s}\rangle$	BE correl.
gluonic stage	**	**			**	**
hadronization			*	**		*
inter. of hadrons	*	**	**	*		**

We believe that in order to find a reasonable qualitative picture of the evolution of heavy-ion collisions it is more important to have a rough agreement with all available types of data than to have a very good agreement with only one piece of data.

The purpose of this contribution is to look at a rather broad set of data and to try to find features of the evolution of nuclear collisions which could be at least qualitatively consistent with most or all of these data.

The contribution is organized as follows. In the next Section we shall discuss what are the relevant degrees of freedom, Sect. 3 deals with the relationship between the formation time of hadrons and the energy density in the hadronic stage of heavy ion collisions, in Sect. 4 we try to analyze implications of data on longitudinal radii as obtained from Bose–Einstein correlations as a function of nucleon numbers, in Sect. 5 we discuss the effect of the formation time of pions on e^+e^- production, in Sect. 6 we try to understand the mechanism behind the strangeness production and in Sect. 7 we describe a picture of J/Ψ suppression by the intermediate gluonic stage. Comments and conclusions are presented in Sect. 8.

Our discussion is based mostly on the recent work done in Bratislava, not because we consider it better or more important than work by other groups, but because we are more familiar with it.

2. What are the relevant degrees of freedom in ion–ion collisions

For the sake of simplicity we shall consider here proton–nucleus (pA) interaction. For a nucleus at rest the mean free path of the proton in the nuclear matter is

$$\triangle z \approx \frac{1}{\rho_0 \sigma_{\rm in}} \approx 2,7\ {\rm fm}\,, \tag{1}$$

where $\rho_0 \approx 0.15$ fm^{-3} is the density of nuclear matter and $\sigma_{\rm in} \approx 2,5$ fm^2 is the inelastic proton–nucleon cross-section. In the c.m. frame of proton–nucleon collisions the mean free path is Lorentz contracted and the distance and time interval between two subsequent collisions becomes

$$\triangle z \approx \frac{\lambda}{\gamma}, \qquad \triangle t \approx \frac{\triangle z}{c}, \qquad \gamma = \frac{\sqrt{s}}{2M}\,. \tag{2}$$

According to the arguments based on the uncertainity principle and the LPM effect , the processes occurring in proton–nucleon interactions which do not obey the inequality

$$\triangle p_{\rm L} \triangle z \geq \frac{\hbar}{2}, \qquad \triangle E \triangle t \geq \frac{\hbar}{2} \tag{3}$$

do not have enough time and space to materialize and are suppressed by the forthcoming interaction. Typical values for AGS, SPS and RHIC energy ranges is given in Table II.

TABLE II

	γ	$\triangle z$/fm	$\triangle p_{\rm L}$/[GeV/c]	$\triangle E$/GeV
AGS	3	0.8	0.25	0.25
SPS	10	0.27	0.8	0.8
RHIC	100	0.027	8.0	8.0

According to Table II, soft processes connected with $\triangle p_{\rm L} < 0.25$ GeV/c in individual nucleon–nucleon collisions are not fully repeated in each proton–nucleus (pA) collision. Simplifying a bit we can say that soft processes with $\triangle p_{\rm L} < 0.25$ GeV/c occur rather as processes of proton on the nucleus as a whole. On the other hand processes with $\triangle p_{\rm L} > 0.25$ GeV/c are repeated in each nucleon–nucleon interaction. Rephrasing it in a different way: processes with $\triangle p_{\rm L} < 0.25$ GeV/c are proportional to the number of participants whereas those with $\triangle p_{\rm L} > 0.25$ GeV/c are proportional to the number of nucleon–nucleon collisions. In the context of the LPM effect this point has been made by Feinberg and Pomeranchuk [14] some 40 years ago.

We shall refer to processes which occur in each nucleon–nucleon collision as to the "cascade" and to those which happen only after the nuclei pass through each other as to the fragmentation of wounded nucleons [15]. As follows from Table II the cascade processes at AGS correspond to $\triangle p_{\rm L} \geq 0.25$ GeV/c, these processes are rather soft and the relevant degrees of freedom are hadronic ones. On the other hand at RHIC only the processes corresponding to $\triangle p_{\rm L} > 8$ GeV/c are proportional to the number of nucleon–nucleon interactions and the appropriate d.o.f. in this case are gluons and quarks. In the CERN–SPS energy region the cascade part of the process is given by interactions with $\triangle p_{\rm L} \geq 0.8$ GeV/c. Taking into account that the value of 0.8 GeV/c is not strict in the sense that processes with $\triangle p_{\rm L}$ lower than 0.8 GeV/c are completely forbidden in the cascade part. Processes with $\triangle p_{\rm L}$ somewhat lower will be somewhat but not completely suppressed. So the cascade part may contain $\triangle p_{\rm L} \geq 0.5$ GeV/c which are interactions at the border between partonic and hadronic d.o.f.

Assuming, optimistically, that such processes can be approximately described by the Perturbative QCD, we can use for the cascade part at the CERN–SPS the partonic language.

To illustrate the mechanism of suppression of soft processes we shall consider a few simple cases.

Example 1. Rutherford scattering and Debye screening

The amplitude for scattering of a charged particle on the static Coulomb potential

$$V(r) \sim \frac{1}{r} \tag{4}$$

is given as

$$A(q) \sim \int \mathrm{e}^{i\vec{q}\cdot\vec{r}} V(r) d^3r \sim \frac{1}{\vec{q}^2}\,, \tag{5}$$

where $\vec{q} = \vec{k}_{\rm f} - \vec{k}_{\rm i}$ and $\vec{k}_{\rm f}$, $\vec{k}_{\rm i}$ are particle momenta of the final and initial state particle. For scattering on the screened Coulomb potential

$$V(r) \sim \frac{1}{r}\,\mathrm{e}^{-\mu r}, \tag{6}$$

we obtain

$$A(q) \sim \frac{1}{q^2 + \mu^2} \equiv \frac{1}{q^2 + (\frac{\hbar}{r_0})^2}, \tag{7}$$

where $\mu = \hbar/r_0$ and r_0 is the radius of the screening. When comparing the shapes of Eq. (5) and Eq. (7) it is easy to see that for q much larger than $(\hbar/r_0)$ both amplitudes are almost identical, whereas for q much smaller than $(\hbar/r_0)$ the amplitude in Eq. (7) is much smaller than the one in Eq. (5). Qualitatively the same effect happens for any cut-off of the Coulomb potential at larger distances, although rapid cutt-offs may lead to additional oscillations.

Example 2. Bremsstrahlung radiation in Tritium decay

In the Tritium decay an electron with velocity $\vec{v}$ is produced in time $t = 0$. The amplitude for producing a soft bremsstrahlung photon with energy ω much smaller than that of the electron is given by the classical formula [16]

$$\vec{A} \sim \vec{n} \times (\vec{n} \times \vec{v}) \int_0^\infty e^{i[\omega t - \vec{k}\cdot\vec{r}]}\, dt\,, \tag{8}$$

where $\vec{r} = \vec{v}t$ and $\vec{k}$ is the wave vector $\vec{k} = \vec{n}\omega$. Making use of $\vec{r} = \vec{v}t$ we obtain

$$\vec{A} \sim \vec{n} \times (\vec{n} \times \vec{v}) \int_0^\infty e^{i\omega t(1-\vec{n}\cdot\vec{v})}\, dt \sim \vec{n} \times (\vec{n} \times \vec{v}) \frac{i}{\omega(1 - \vec{n}\cdot\vec{v})} \tag{9}$$

and the amplitude contains the factor $1/\omega$ typical for bremsstrahlung processes.

Suppose now that the electron from the tritium decay is absorbed in time $t = T$ after the decay. Using Eq. (8) and integrating up to T the bremsstrahlung amplitude is given as

$$A(\omega) \sim \vec{n} \times (\vec{n} \times \vec{v}) \frac{i}{\omega(1 - \vec{n}\cdot\vec{v})} \left[1 - e^{i\omega T(1-\vec{n}\cdot\vec{v})}\right], \tag{10}$$

for $\omega T(1 - \vec{n}\cdot\vec{v}) \ll 1$ expansion of the exponent shows that the $1/\omega$ singularity for $\omega \to 0$ is cancelled. The exponent in Eq. (10) can be written as $\exp[iT/t_{\rm f} - 1]$ and

$$t_{\rm f} = \frac{1}{\omega(1 - \vec{n}\cdot\vec{v})} \tag{11}$$

is referred to as the formation time of the bremsstrahlung photon [17]. Further discussion and references can be found in Ref. [18].

When comparing amplitudes in Eqs (9) and (10) it is easy to see that for

$$T < t_{\rm f}, \qquad \text{or} \qquad \omega < \frac{T}{1 - \vec{n}\cdot\vec{v}}$$

the amplitude in Eq. (10) is suppressed with respect to that in Eq. (9). For large values of ω the two amplitudes differ by the rapidly oscillating factor which is due to the immediate absorption of the electron. More decent dependence of the absorption on time would tame the oscillations in the way similar to that in the first Example.

3. Formation time of hadrons and energy densities reached in heavy ion collisions at the CERN SPS

Formation time of hadrons in pA and AB interactions at high energies has been introduced [19] in order to describe in phenomenological models the absence of cascades induced by secondary hadrons produced in nucleon–nucleon collisions. An excellent review by Kisielewska can be found in Ref. [20].

In this sense the formation time is the time during which the secondary hadron is unable to interact. This time may consist of two components, the former corresponding to the duration of the nucleon–nucleon collision and the formation of the "naked" hadron and the latter one to the "dressing up" of the hadron. The formation time can be clearly defined in electrodynamics where it corresponds to the time during which a charged particle after large angle scattering builds up its electromagnetic field [21]. In hadronic collisions one can imagine a meson being produced as a narrow wave packet consisting of a quark and an antiquark which expands, builds up the gluon component and the cross-section for its interaction is gradually increasing.

Since the formation time in this sense depends on processes related to the produced hadron, this formation time is Lorentz dilated. Denoting the value of the formation time in the rest frame of the hadron as $\tau_{\rm f}$, its value in the frame in which the hadron has energy E is

$$t_{\rm f} = \frac{E}{m}\tau_{\rm f}\,, \tag{12}$$

where m is the mass of the hadron.

In most models it is assumed that the formation time $\tau_{\rm f}$ is roughly 1 fm/c. In heavy ion collisions the value of the formation time influences very strongly the energy density of interacting hadronic matter — that means that one does not count those hadrons which are not yet formed. The dependence of the energy density on the formation time has been studied in [22] and it has turned out that the energy densities in Pb+Pb collisions at the CERN–SPS become larger than critical provided that the formation time is smaller than about 1 fm/c. In this situation models which do not include a gluonic intermediate stage become inconsistent. The dependence of energy density on the formation time for S+Pb and Pb+Pb collisions is shown in Figs 1 and 2.

Although being model dependent, the formation time in a particular model, can be, in principle, determined from Bose–Einstein correlation data in πp, pp or e^+e^- interactions. We have studied this question in Ref. [23] where we have assumed that in πp interactions one produces resonances like $\rho, \omega, K^*, \Delta$, and "direct" pions. The formation time of resonances permits them to propagate before the decay starts. The distance of pions produced

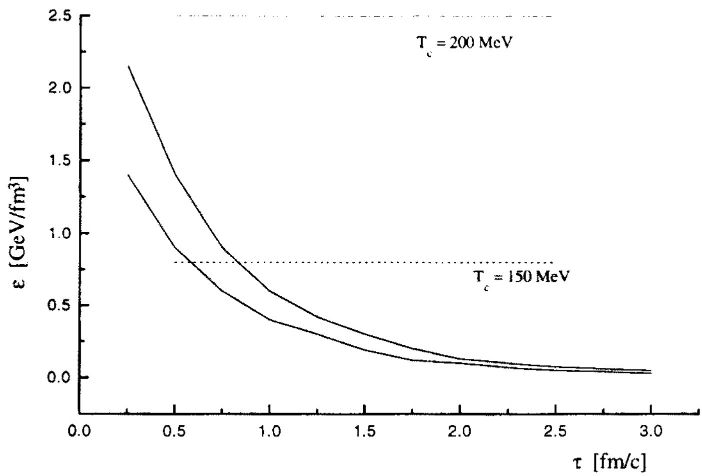

Fig. 1. Energy density in central S–Pb interactions as estimated in Ref. [22]. The upper curve gives the maximal, the lower one the average energy density. Dash-dot constants show the energy densities of phase transitions at $T_c = 150$ and $T_c = 200$ MeV.

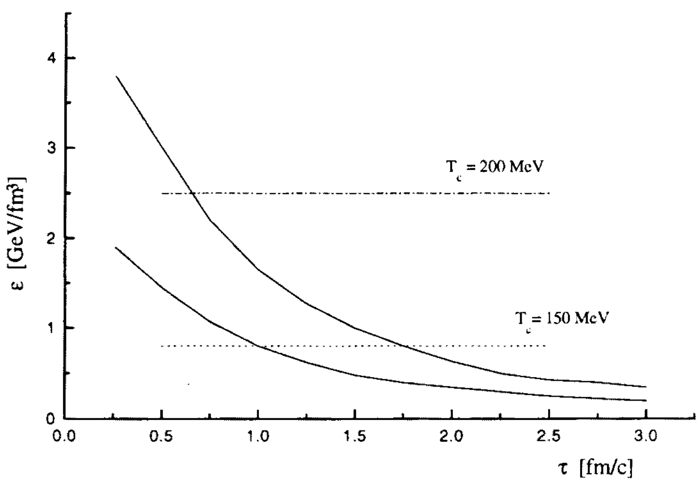

Fig. 2. Energy densities in central Pb–Pb interactions, notation is the same as in Fig. 1.

by decay of a resonance from the point, where the resonance has been produced is then given by both the resonance formation time τ_f and the mean life-time of the resonance τ_d taken from data on the width of the resonance. When extracting the resonance production cross-sections from the data and comparing the calculated Bose–Einstein correlations with the data of EHS/NA22 collaboration [24] we have found surprisingly short formation times of 0.2–0.4 fm/c [23]. It should be stressed that one should be careful in combining results of Refs. [23] and [22] to the conclusion of the presence of super-critical energy densities in S–Pb and Pb–Pb collisions at the CERN–SPS at least for two reasons. Firstly, the formation time as revealed by HBT data is related to the ability of the resonance to decay, and the formation time as defined by the ability of the resonance to interact with the full cross-section, need not be two absolutely identical concepts, although it would be strange if they were very different. Secondly, the model used in Ref. [23] has been drastically simplified and used to analyse a limited set of data.

Detailed analysis of data obtained at the CERN–ISR has been performed by Lednický and Progulova [25]. In their model they do not consider the formation time of resonances as used in Ref. [23] but introduce instead as a parameter the effective radius r_0 of the region in which resonances are produced. Their analysis has lead to the value

$$r_0 = 0.55 \pm 0.08(\text{stat.}) \pm 0.10(\text{syst.})\ \text{fm}\,.$$

This can be compared with results of Ref. [23], where all resonances are produced in the same point but due to the formation time they decay in the distance roughly given by

$$z_\text{f} \approx \sinh(y_R)\tau_\text{f}\,,$$

where y_R is the rapidity of a resonance which gives as a decay product a pion with low momentum in the cms of collision. For the ρ meson we have [23] $y_R \approx 1.67$ and taking $z_\text{f} \approx 0.55$ we obtain $\tau_\text{f} \approx 0.2$ fm/c which corresponds to results obtained in Ref. [23]. More studies along these lines are certainly desirable.

As can be seen from Fig. 1 the formation time of hadrons of 0.2–0.4 fm/c leads to maximal energy densities in Pb+Pb interactions which exceed the critical value of the phase transition corresponding to both $T_\text{c} = 150$ MeV and $T_\text{c} = 200$ MeV.

Both models used in Refs [22,23] contain simplifications, nevertheless the combination of results indicates that energy densities in Pb+Pb interactions at the CERN SPS might be larger than the critical value. Since these densities are reached for only a short time and in a small volume, this does not mean that QGP is necessarily formed but it certainly indicates the presence of a new state of matter which cannot be described as a gas of interacting hadrons.

4. Longitudinal radii in AB interactions

In Ref. [26] we have studied the contribution of nuclear geometry to the longitudinal radii as observed via Bose–Einstein correlations of identical pions. It has turned out that the contribution of geometry is rather small and the effects due to geometry are unable to explain the data [9,10] obtained by NA44 and NA49 collaborations at the CERN–SPS.

The argument is very simple if one considers pA or AB collisions as a superposition of individual nucleon–nucleon interactions. Assume that in the experiment one studies the correlation of identical pions with $y \sim 0$ in the c.m.s. of nucleon–nucleon collision. Such pions are produced either directly or by decays of resonances with rapidity within one or two units larger or smaller than $y \sim 0$. Suppose now that the production of direct pions and resonances within this rapidity interval is the same for any nucleon–nucleon interaction. The correlation function for two identical pions with momenta $\vec{k_1}$ and $\vec{k_2}$ is written in the standard way

$$C(\vec{k_1}, \vec{k_2}) = 1 + \lambda \left| \int e^{i(\vec{k_1}-\vec{k_2})\cdot\vec{r}} \rho(\vec{r}, \vec{K}) d^3r \right|^2 , \tag{13}$$

where $\vec{K} = (\vec{k_1}+\vec{k_2})/2$ and $\rho(\vec{r}, \vec{K})$ is the density of sources of pions normalized to 1. To simplify the discussion we have assumed that both pions have the same energy, and that only the z-component of the difference $\vec{q} = \vec{k_1} - \vec{k_2}$ is non-vanishing. In this case Eq. (13) is simplified to

$$C(q) = 1 + \lambda \left| \int e^{iqz} \rho(z) dz \right|^2 , \tag{14}$$

where $\rho(z, K)$ is the density of pion sources integrated over x, y and t and we have suppressed K in $\rho(z, K)$.

Let $\rho_{nn}(z)$ denote the distribution of sources for an individual nucleon–nucleon collision and $\rho_{AB}(z)$ be the distribution of z-coordinates of nucleon–nucleon collisions in the interaction of A and B. The total density of the source of pions is then given by the convolution

$$\rho(z) = \int dz' \rho_{nn}(z - z') \rho_{AB}(z') . \tag{15}$$

Inserting this into Eq. (14) we find

$$C(q) = 1 + \lambda |\tilde{\rho}_{nn}(q)|^2 |\tilde{\rho}_{AB}(q)|^2 , \tag{16}$$

where $\tilde{\rho}_{nn}(q)$ is the Fourier transform of $\rho_{nn}(z)$ and $\tilde{\rho}_{AB}(q)$ is related in the same way to $\rho_{AB}(z)$. The whole dependence of $C(q)$ on A and B is given by the factor $|\tilde{\rho}_{AB}(q)|^2$. In Ref. [26] we have used the experimental values of longitudinal radii obtained by the Gaussian parametrization of the data [9, 10] and parametrized $|\tilde{\rho}_{AB}|^2$ also by a Gaussian with $\langle z^2 \rangle$ given either by the result of Monte Carlo calculations or from a simplified picture of the effects of nuclear geometry. Although Gaussian parametrizations are too crude for both the correlation function $C(q)$ and the distribution $\rho_{AB}(z)$, the net result is simple. The dependence of the correlation function $C(q)$ on A, B cannot be explained by the factor $\tilde{\rho}_{AB}(q)$ calculated from nuclear geometry. The results are shown in Fig. 3.

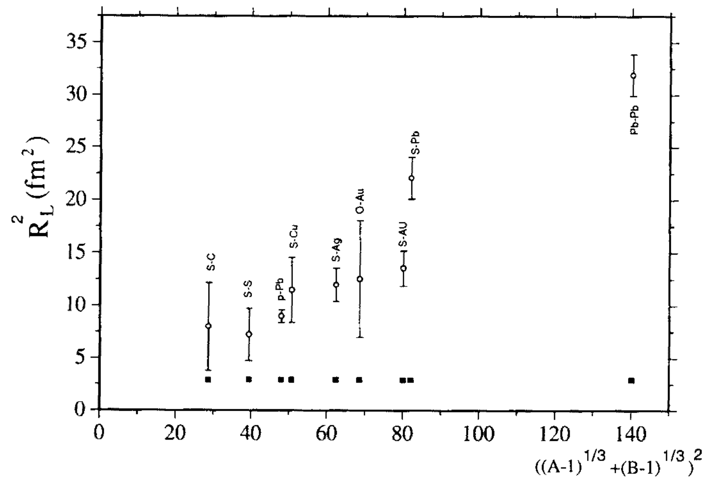

Fig. 3. Data on $R_L^2(A, B)$ as a function of $[(A-1)^{1/3} + (B-1)^{1/3}]^2$ and the results of calculations [26] (squares) obtained via Eq. (16) and nuclear geometry only.

If the nuclear geometry cannot explain the increase of R_L with A, B what may be the reason? In Ref. [26] we have offered one possibility, namely that the ratio of direct pions to pions coming from resonance decays is not the same for all of the nucleon–nucleon interactions. The last collision of a nucleon is followed by the fragmentation which is not disturbed by further collisions and as discussed in Sect. 2 all soft processes can materialize. On the other hand for those nucleon–nucleon collisions which are followed by further collisions, soft processes are to some extent suppressed and the final states may contain higher resonance fractions. So far there are no data which could support or exclude this possibility. Results are presented in Fig. 4, where R_L is plotted vs. variable

$$p_c = \frac{\nu_{AB}}{\nu_{AB} + w_A + w_B},$$

where ν_{AB} is the number of nucleon–nucleon collisions and w_A, w_B are numbers of wounded nucleons in nuclei A, B. But even if increased resonance production were present, the value of $R_{\rm L}$ (Pb,Pb) and perhaps also that for $R_{\rm L}$(S,Pb) is larger than extrapolations from light ion induced collisions, indicating the onset of some new dynamics in Pb+Pb — what we have already learned from the NA50 data on J/ψ suppression.

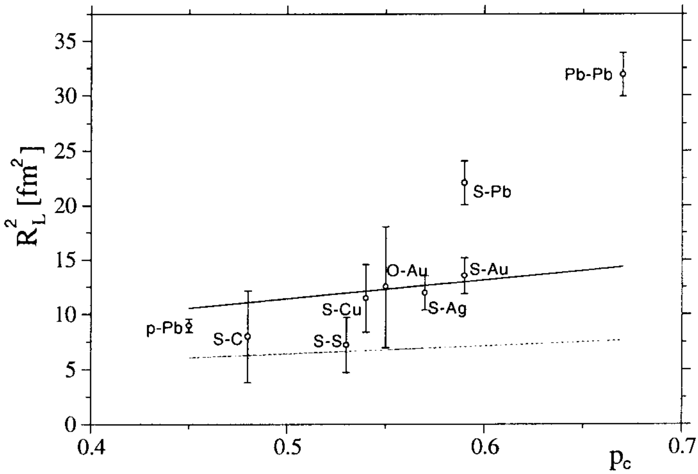

Fig. 4. $R^2_{\rm L}(A,B)$ compared with two parametrizations of the model [26] in which resonances are more copiously produced in nucleon–nucleon collisions not followed by fragmentation. Variable $p_{\rm c}$ is described in the text.

Apart of changing resonance fractions, there is also another possibility of understanding the increase of $R_{\rm L}(A,B)$ with A, B. Suppose that in pA and AB collisions at the CERN–SPS the gluonic cascade is present, like in models of Geiger and Müller [27–30] and Gyulassy, Pang and Zhang [31]. If any nucleon–nucleon collisions contributes in the average the same amount of energy the energy density reached will be proportional to $A^{1/3}B^{1/3}$, numbers of nucleons in colliding tubes being proportional to $A^{1/3}$ and $B^{1/3}$. This energy density is decreasing by longitudinal (and partly also by transverse) expansion. When reaching the threshold corresponding to hadronization the system hadronizes and expands further as hadron gas. The time of expansion of the intermediate gluonic stage thus increases with A and B in accordance with the trend of data. Such attempts at explaining the increase of $R_{\rm L}(A,B)$ with A and B are in our opinion very attractive and numerical analyses remain to be done.

5. e^+e^- production and pion formation time

The search of an excess of dilepton production in πp, pp, pA and AB interactions over the expected conventional sources like resonance decays, Dalitz decays and bremsstrahlung of virtual photons has a rather long history. The experimental indications of a possible dilepton excess appeared first in about 1976, followed by the Bjorken–Weisberg suggestion [32] of the possible contribution due to annihilations of quarks and antiquarks present in the intermediate stage, before recombination to hadrons. We have studied this question in Bratislava [33] but it has later turned out that the excess is not present in data on hadronic collisions. The simplest qualitative explanation of this fact is that there are practically no quarks and antiquarks in the intermediate stage of a hadronic collision. In the same way the anomalous dilepton should be originated by a dense and interacting hadronic matter, *e.g.* via $\pi^+\pi^- \to e^+e^-$ as suggested by Kapusta and Lichard [34] in connection with the DLS data [35]. Of course an intermediate stage dominated by gluons is not excluded by the absence of anomalous dileptons. In this sense the negative result of the search of anomalous dileptons indicates that in hadronic collisions there is no or only very short quark–antiquark intermediate stage. In combination with the increase of longitudinal radii as obtained from Bose–Einstein correlations data already for relatively light ion interactions, like S+S, p+Pb, S+Cu this gives an indirect support for an intermediate gluonic stage, or for the modified resonance fractions in individual nucleon–nucleon collisions.

Experimental evidence of an excess of dilepton production over conventional sources has been found by the CERES [3,4] and HELIOS [5,6] groups in S–Au, S–W and Pb–Au interactions. There are two interesting features of the excess [3,4] : the magnitude and the shape of the excess. The average multiplicity of final state hadrons in the CERES data is about $dn_{ch}/d\eta \sim 270$. Assuming that all of these secondary hadrons are pions the magnitude of the excess corresponds to about 3–5 pion–pion interactions for each final state pion [36]. The shape is interesting by the absence of the ρ-peak with usual position and width and the maximum of a rather flat excess seems to be shifted to about 600 MeV$/c^2$.

The main purpose of the study in Ref. [36] has been to learn what are the modifications of e^+e^- mass distributions caused by the formation time of pions. The idea is rather simple. Suppose that in the final state of S–Au or Pb–Au interactions only pions produced in nucleon–nucleon collisions are present. The momenta of pions have been generated by the Monte Carlo model written by Závada [37]. We have also assumed that the intermediate gluonic stage is absent or rather short, and the pions start to interact (and annihilate to dileptons) only after their formation time. Because of the Lorentz dilation, Eq. (12), pions with smaller energies are able to interact earlier then those with larger ones, and a fraction of more energetic pions leaves the pionic gas with one or no interaction. Softer pions will populate the dilepton mass region close to the threshold, and the contribution to the ρ-mass region will be relatively suppressed.

As expected for smaller values of the formation time, like $\tau_f = 0.2$ fm/c the ρ-peak is much higher than the dilepton spectrum at lower masses, whereas for larger formation times the ρ-peak is suppressed relative to the continuum below the ρ.

A typical feature of dilepton production via $\pi^+\pi^- \to e^+e^-$ is a faster than linear dependence of dilepton yield on the multiplicity of pions in the final state. Increasing density leads to an increasing number of collisions of each of pions and that means also an increase of the number of dileptons per final state pion. The trend of results obtained in Ref. [36] is rather similar to that of data [3, 4].

It should be noted that the model we have used is very simplified, it does not contain other dilepton sources like $\pi N \to N^* \to \rho N$ followed by the ρ decay to e^+e^-, contributions of other vector mesons, *etc.*

The question whether the formation time can cause deformations of the dilepton spectrum consistent with data, or whether the shift of the ρ-mass and width is necessary remains open. The shift of the ρ-mass and width connected with a partial restauration of the chiral symmetry would be a most interesting effect. The effect has been discussed [38] and shown to be able to fit the data [39], but before accepting this possibility we should be sure that the observed dilepton mass distribution cannot be naturally explained by other, more mundane effects.

6. Strangeness production in heavy-ion interactions

The enhanced production of strangeness has been suggested as one of possible signatures of QGP formation in heavy ion collisions [40]. We have recently studied [41] the dependence of $\langle s\bar{s}\rangle$ on nucleon numbers A and B of colliding nuclei. The basic idea has been rather simple. We have aimed at compiling the data for proton and light ion induced reactions and extrapolating them to the case of Pb–Pb interactions. The difference between such an extrapolation and the data on $\langle s\bar{s}\rangle$ for Pb–Pb could give an indication of the onset of a new dynamical mechanism of Pb–Pb interactions, possibly completing the information already available on the basis of data on J/Ψ suppression by the NA50 collaboration [1, 2]. The compilation of data on proton and lighter ion-induced reactions has been done in a similar way as in Bialkowska *et al.* [42], some differences with respect to their approach are described in Ref. [41].

The parametrization and extrapolation of data obtained with lighter ion beams has been performed by two simple models, both based on the model of multiple nucleon–nucleon collisions. In the former (Model 1) we have first parametrized the $\langle s\bar{s}\rangle$ in nucleon–nucleon interactions as a function of the energy squared (s) as

$$\langle s\bar{s}\rangle = 0.38(\ln\sqrt{s} - 1.6) + 0.11 \tag{17}$$

and taken into account the degradation of nucleon energy in each collision by Δy.

In the latter version of our model (referred to as Model 2) we have simply assumed that $s\bar{s}$ production in i-th collision of one nucleon and j-th of the other one is proportional to

$$\langle s\bar{s}\rangle_{ij} \sim (1-\beta)^{i-1}(1-\beta)^{j-1}\,. \tag{18}$$

Such a dependence of $\langle s\bar{s}\rangle_{ij}$ on i, j would be obtained *e.g.* in models like that of Hwa *et al.* [43] where structure functions of partons, in particular gluons are attenuated during the passage of the nucleon through nucleus.

By using a simple Monte Carlo model describing ion–ion collisions as superposition of nucleon–nucleon interactions, see *e.g.* Ref. [44] we have found that for light ion induced reactions

$$\Delta y \approx 0.35; \qquad \beta \approx 0.12\,. \tag{19}$$

Using Model 1 (rapidity shifts) for the extrapolation to the Pb–Pb case we get $\langle s\bar{s}\rangle \approx 270$ and using Model 2 (attenuation by subsequent collisions) we find $\langle s\bar{s}\rangle \approx 330$ As a net result of the extrapolation by using Models 1 and 2 and estimating the error of the extrapolation via the difference between the two models we have

$$\langle s\bar{s}\rangle_{\mathrm{Pb-Pb}} \approx 300 \pm 30\,. \tag{20}$$

Observation of a larger value of $\langle s\bar{s}\rangle$ in Pb–Pb collisions would give an indication of the onset of a new dynamics.

7. J/Ψ suppression

The story has started with the paper by Matsui and Satz [46] where the J/Ψ suppression has been suggested as a signature of QGP formation. The data obtained soon afterwards by the NA38 Collaboration [47] has shown the suppression already for O+U interactons. Alternative explanations have followed, including the suppression by nucleons present originally in nuclei [13] and by the hadronic gas [48]. By about 1992 a paradigma has been accepted, according to which most of suppression in O and S induced interactions with heavy targets is due to the Gerschel–Hüfner mechanism [13] and the p_{T} dependence of the produced J/Ψ is caused by the multiple scattering of gluons prior to J/Ψ formation [49]. The NA50 data obtained in Pb–Pb interactions [1,2] have changed the picture dramatically.

On the theoretical side a novel development since the period around 1990 consists in the argument [11, 12] that collisions of J/Ψ with hadrons in the hadron gas formed in the nuclear interaction presumably does not give a sufficiently strong suppression. In that case the threshold observed in Pb–Pb interactions should be due to gluons. In order to understand the effect one would need to have a space-time evolution of gluon densities in the phase-space. Such a picture can be provided only by the cascade models like those in Refs [27–31]. In such models the suppression of J/Ψ is influenced by the evolution of gluon structure functions both before [50] and after the J/Ψ production.

In the rest of this section we shall describe a simple model of Ref. [51] which illustrates in a qualitative way the main issues of J/Ψ suppression by intermediate stage gluons after the J/Ψ has been produced.

A qualitative picture of the evolution of gluon densities in heavy-ion collisions in the 200 AGeV energy range

For the sake of simplicity we shall picture the colliding nuclei A and B as cylinders of lengths $2L_A$ and $2L_B$ and radii R_A and R_B. These parameters are fixed for each nucleus by requiring that the volume of the cylinder and the volume of the sphere are the same and that the value of $\langle z^2 \rangle$ where z is the distance from the centre of the sphere (from the centre of the cylinder) along the z-axis, identical with the axis of rotational symmetry for the cylinder. In this way we find

$$L_A = \sqrt{\frac{3}{5}} R_A, \qquad r_A^2 = \frac{2}{3}\sqrt{\frac{5}{3}} R_A^2 \,. \tag{21}$$

Space-time evolution of two colliding rows of nucleons, each row consisting of a tube along the z-axis, can be visualized simply as shown in Fig. 5, where one can see the position of both rows at any value of time. Taking a particular value of z and making in that point straight line parallel to the t-axis we find the time interval during which nucleons of both rows collide at this value of z. The time when nucleons start to collide in point z is denoted as $t_1(z)$ and the time when collisions cease as $t_2(z)$.The rate $(dn/dt)_0$ at which gluons are produced in the point z vanishes for $t \le t_1(z)$ as well as for $t \ge t_2(z)$. Within the time interval $t_1(z) < t < t_2(z)$ the rate is given as

$$\alpha \equiv \left(\frac{dn}{dt}\right)_0 = v_{\rm rel}\gamma\rho_0\gamma\rho_0\sigma 2C \,, \tag{22}$$

where $v_{\rm rel} \approx 2c$ is the relative velocity of colliding nucleons, σ is the cross-section for semi-hard nucleon–nucleon collision leading to production of two gluons with momenta larger than about 0.5 GeV, ρ_0 is the nucleon density in a nucleus at rest and factor 2 stands for the production of two gluons in one sub-collision. The factor C takes into account that partons which are responsible for most of semi-hard interactions are not Lorentz contracted. Positions of collisions within the nucleus A will not be distributed within the distance $2L_A/\gamma$ but over a larger distance $(2L_A/\gamma + \Delta)$ where Δ is about 1 fm. The correction factor C thus becomes

$$C = \frac{R_A}{R_A + \delta} \frac{R_B}{R_B + \delta}, \tag{23}$$

where

$$\delta = \frac{1}{2}\sqrt{\frac{5}{3}}\gamma\Delta \approx 6,5\Delta\,. \tag{24}$$

Note that this correction leads to rather lower densities for collisions of lighter ions. To get a feeling for the numbers, consider Pb–Pb collision with the following parameters: $v_{\rm rel} \approx 2c$, $\gamma \approx 10$ (this corresponds to $E_{\rm lab} \approx$200 AGeV), $\sigma \approx 20$ mb $= 2$ fm^2, $\rho_0 \approx 0.15$ fm^{-3}, $\Delta \approx 1$ fm, $R_{\rm Pb} \approx 7$ fm. The resulting $(dn/dt)_0$ becomes about (4,8 gluons) fm^{-3} (fm/c)$^{-1}$. For an S–S collision due to the C-factor the corresponding $(dn/dt)_0$ is only about a half of the value for Pb–Pb.

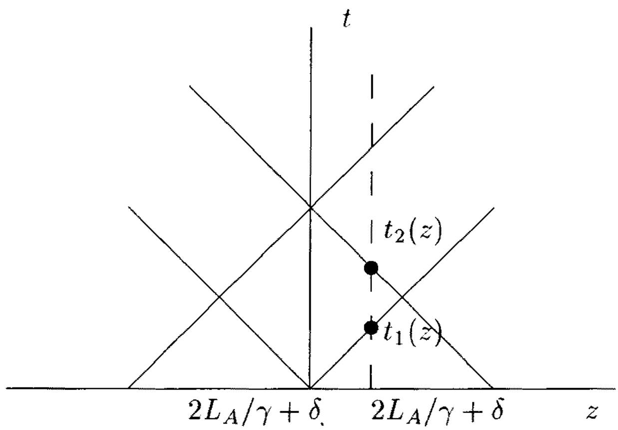

Fig. 5. The z–t diagram of the space-time evolution of nucleon–nucleon collision. Nucleons of two equal mass nuclei,modelled as cylinders, collide within the square indicated. At a particular value of z nucleon–nucleon collisions start at $t = t_1(z)$ and end at $t = t_2(z)$. Length of each cylinder is $2L_A/\gamma + \delta$ and the symbols used are explained in the text.

By estimating $\sigma \approx 20$ mb we make the crucial assumption that a large part of nucleon–nucleon interaction in ion–ion collision is due to semi-hard interactions even at $E_{\rm lab} \approx 200$ AGeV. For this assumption the role of principle of uncertainity, as discussed above is essential.

The second decisive parameter is the time τ which a gluon spends in the system as a gluon. This parameter can be estimated as follows. In proton–proton collision one can imagine that the two gluons separate extending a colour string between themselves, the tension of the string being 1 GeV/fm, and after a time of about 0.5 fm/c, when the energy of gluons is converted to that of the string tension, the hadronization starts. In a heavy-ion collision the situation might be rather different. The region in which nucleons collide contains coloured partons what screens interactions between gluons. Gluons produced in one of semi-hard interactions may interact with softer gluons of incoming nucleons what would increase their number. They can also interact with gluons originated by other semi-hard collisions what makes their lifetime longer. We shall lump all these effects into a single parameter τ and consider its value as a free parameter with a value of about 0.5–1.5 fm/c, depending on the gluon density and on the size of the intermediate gluonic system.

We assume that the density of gluons increases due to semi-hard collisions in the region of nucleon–nucleon collisions shown in Fig. 5 and that this density is decreasing due to the hadronization of gluons and their escape from this region. Characterizing the loss of gluons by a characteristic time τ we can write the equation for the time evolution of gluon density in point z as:

$$\frac{dn}{dt} = \left(\frac{dn}{dt}\right)_0 - \lambda n, \qquad \lambda = \frac{1}{\tau} \tag{25}$$

with $\alpha \equiv (dn/dt)_0$ given by Eq. (22). The solution of Eq. (25) is

$$n(t) = \tau\alpha[1 - \mathrm{e}^{-\lambda(t-t_1)}] \tag{26}$$

for

$$t_1 \equiv t_1(z) \leq t \leq t_2 \equiv t_2(z)$$

and

$$n(t) = \tau\alpha[1 - \mathrm{e}^{-\lambda(t_2-t_1)}]\,\mathrm{e}^{-\bar{\lambda}(t-t_2)}, \qquad t \geq t_2\,, \tag{27}$$

where $\bar{\lambda} = 1/\bar{\tau}$ describes the rate at which gluons disappear after nucleons ceased to interact in the point z. In making the estimates we shall first assume that $\bar{\tau} = 0$, what corresponds to $n(t) = 0$ for $t > t_2$. Note,however,that $\bar{\tau}$ may become large when the density of gluons approaches or exceeds the critical density corresponding to the phase transition to QGP. We shall return to this point below.

Suppression by intermediate stage gluons

When the time dependence of gluon density is known we can calculate the survival probability of J/Ψ by standard procedures, see e.g. Ref. [48]. For J/Ψ at rest in the c.m.s. of nucleon–nucleon collision the survival probability S due to interactions with gluons is given as

$$S = \left\langle \exp(-\int \langle\sigma_d\rangle n(t') v_{\rm rel} dt' \right\rangle , \qquad (28)$$

where $\langle\sigma_d\rangle$ is the mean value of the cross-section for J/Ψ disintegration in $g + J/\Psi$ collisions obtained by averaging over momenta of gluons. Most of gluons produced in semi-hard nucleon–nucleon collisions have momenta within the range $0.5\ \mathrm{GeV}/c \leq p \leq 1.5\ \mathrm{GeV}/c$. According to estimates of the disintegration cross-section for $g + J/\Psi$ collisions given by Kharzeev and Satz [11] we shall take $\langle\sigma_d\rangle = 2$ mb $= 0.2$ fm^2. For massless gluons $v_{\rm rel} = c$.

We shall now calculate the J/Ψ survival probability, by using Eq. (28) and Eqs (26), (27).

Averaging in Eq. (28) goes over all positions (z,t) in which J/Ψ can be created and the integral in Eq. (28) has as the lower limit the time $t_1(z)$. Some results are summarized in Table III. As can be seen from this Table there is no combination of relaxation times τ and $\bar{\tau}$ which would give the rather small suppression for S–S and about 0.5–0.6 for Pb–Pb interactions.

TABLE III

The dependence of J/Ψ survival probabilities on relaxation times τ and $\bar{\tau}$

τ [fm/c]	$\bar{\tau}$ [fm/c]	S(S+S)	S(S+Pb)	S(Pb+Pb)
0.3	0.3	0.93	0.89	0.82
0.5	0.5	0.89	0.83	0.73
0.5	1.0	0.83	0.76	0.64
1.0	0.5	0.86	0.78	0.67
1.0	1.0	0.79	0.70	0.56
0.5	1.5	0.79	0.69	0.56
1.0	1.5	0.74	0.62	0.48
0.5	2.0	0.74	0.63	0.50
1.0	2.0	0.68	0.56	0.42
2.0	2.0	0.65	0.52	0.37

These patterns can be obtained provided that we assume that for a system like Pb–Pb where the dimensions and the gluon density are larger the relaxation time is also larger.

As seen in Table III combinations with $\tau \approx 0.5$ fm/c and $\bar{\tau} \approx 1.5$ fm/c and $\tau \approx 1$ fm/c and $\bar{\tau} \approx 1$ fm/c give additional suppression of J/Ψ in Pb–Pb collisions of about 0.56. Combinations with $\tau \approx 0.5$ fm/c and $\bar{\tau} \approx 0.5$

fm/c keep J/Ψ suppression at acceptable levels of S(S–S)$\approx$ 0.89 and S(S–Pb)$\approx$ 0.83 in S–S and S–Pb collisions. However an increase of relaxation times τ and $\bar{\tau}$ with increasing nucleon numbers is required.

Dependence of the life-time of the system of intermediate gluons on nucleon numbers of colliding nuclei

Hadronization of intermediate stage gluons can not be described by PQCD. Still, it is the process which occurs in hadronic, nuclear, ep and e^+e^- interactions and there exist various phenomenological ways of describing it, *e.g.* [52, 53].

We shall now present arguments indicating that the life-time of the system of intermediate stage gluons is increasing with nuclear numbers of colliding nuclei. In our model the life-time of this system is parametrized by relaxation times τ and $\bar{\tau}$ and the increase of the life-time of the gluonic system is expressed via the increase of relaxation times.

The increase of nucleon numbers leads to the increase of the density of gluons. As discussed below Eq. (24) the rate with which gluons are produced is $(dn/dt)_0 \approx 4.8$ gluons fm^{-3}(fm/c)$^{-1}$ for Pb+Pb and about 2.4 fm^{-3}(fm/c)$^{-1}$ for S+S interactions. Eqs (26) and (27) show that even if the relaxation times were the same in both cases, densities would differ at least by a factor of two. Taking $\tau \approx 0.5$ fm/c we get for the density of gluons in Pb+Pb about 2.4 fm^{-3} whereas for S+S we get about 1.2 fm^{-3}. Various models of hadronizaion differ in details but they have also much in common. We shall base our argumentation on the model by Ellis and Geiger [52]. In this model two partons with Lorentz invariant distance

$$L = \sqrt{(\vec{x}_1 - \vec{x}_2)^2 - (t_1 - t_2)^2}\,,$$

form a colour neutral cluster with probability

$$\Pi(L) \approx 1 - \exp\left(\frac{L_0 - L}{L_c - L}\right)$$

if $L_0 \le L \le L_c$. For $L \le L_0$ the probability $\Pi(L) = 0$ and $\Pi(L) = 1$ if $L \ge L_c$. In the c.m.s. of nucleon–nucleon collision we take $t_1 = t_2$ and take $L = |\vec{x}_1 - \vec{x}_2|$. This picture when applied to the system of intermediate gluons indicate that there are two characteristic densities: $n_0 \approx L_0^{-3}$ and $n_c \approx L_c^{-3}$. For densities above n_0 there is no hadronization, for densities below n_c the hadronization is very rapid and for $n_c \le n \le n_0$ the hadronization depends on the density.

Ellis and Geiger use values $L_c \approx 0.8$ fm corresponding to $n_c \approx 1.95$ fm^{-3} and $L_0 \approx 0.6$ fm corresponding to $n_0 \approx 4.62$ fm^{-3}. Our estimates for gluon densities in Pb+Pb and S+S collisions are $n_{\mathrm{Pb+Pb}} \approx 2.4$ fm^{-3} and $n_{\mathrm{S+S}} \approx$

1.2 fm^{-3}. These numerical values depend very strongly on the assumed value of cross-sections for production of gluons in nucleon–nucleon subcollisions but we find it quite possible that in the case of Pb+Pb interactions the density of gluons is larger than n_c, whereas the density of gluons in S+S interactions is lower than this value. The system of gluons formed in Pb+Pb interactions expands before hadronization and this makes relaxation times in Pb+Pb larger than in S+S. This in turn leads to larger suppression of J/Ψ in Pb+Pb interactions than in S+S ones.

In spite of the crudeness of approximations made above we shall now discuss what might be the relation of the system of gluons produced in semi-hard nucleon–nucleon collisions to QGP. For Pb+Pb interactions the energy density ε_g, of these intermediate stage gluons can be simply estimated as

$$\varepsilon_g \approx \langle\varepsilon\rangle\tau\left(\frac{dn}{dt}\right)_0 \approx \langle\varepsilon\rangle\tau 4.8\ \mathrm{fm}^{-3}\,,$$

where $\langle\varepsilon\rangle$ is the average energy of a semi-hard gluon.

Taking 0.5 GeV $\leq \langle\varepsilon\rangle \leq$ 1 GeV and $1 \leq \tau \leq 2$ fm/c we obtain

$$1.2\ \mathrm{GeVfm}^{-3} \leq \varepsilon_g \leq 4.8\ \mathrm{GeVfm}^{-3}\,.$$

Note that the energy density estimated in this way is proportional to the cross-section σ for production of semi-hard gluons, and that even for smaller σ the energy density of intermediate stage gluons would be rather large.

According to lattice calculations the critical temperature for the phase transition to QGP is between 150 MeV and 200 MeV,the lower values being preferred. The corresponding energy densities are

$$0.8\ \mathrm{GeVfm}^{-3} \leq \varepsilon_{\mathrm{crit}} \leq 2.5\ \mathrm{GeVfm}^{-3}\,.$$

The comparison of these two regions of energy density indicates that the effect observed by the NA50 Collaboration [1,2] might be connected with the closeness of the energy density of semi-hard gluons to the critical one. In our model it is natural to assume that the relaxation time of the gluonic system increases substantially when the density of gluons approaches the critical one. The abrupt increase of J/Ψ suppression could mean that the NA50 Collaboration have observed the effects due to system of gluons approaching or forming for a short time the QGP.

8. Comments and conclusions

A qualitative picture of the space-time evolution of nuclear interactions in the CERN–SPS energy region will be most likely obtained by combining information from various pieces of data. A very preliminary attempt at such an analysis has been done here. No firm conclusions have been obtained, but there are a few hints:

– Arguments based on the LPM effect indicate that processes with $\Delta p_{\rm L}$ larger than about 0.8 GeV/c may be repeated in all individual nucleon–nucleon collisions in the sense of the Glauber model. Softer processes, like fragmentation, take place only after nuclei have passed through each other. Processes with $\Delta p_{\rm L}$ larger than about 0.8 GeV/c can be, at least in a rough approximation, described by PQCD.

– Rather short formation times of resonances (0.2–0.4 fm/c) following from data on HBT correlations in hadronic collisions combined with the estimated dependence of energy density on the formation time of hadrons indicate that in Pb–Pb and perhaps already in S–Pb is larger than the critical value corresponding to the phase transition between QGP and hadronic matter. This is indicated both by the analysis of energy densities in Sect. 3 and by the discussion of a possible mechanism of J/Ψ suppression in Sect. 7.

– Additional evidence for the formation of a new state of matter could be brought by data on the total strangeness $\langle s\bar{s}\rangle$ production in Pb–Pb interaction if these data turn out to be higher than extrapolations from collisions of nuclei with lower values of A and B, as discussed in Sect. 6.

– A very interesting piece of information is given by the CERES and HELIOS data on e^+e^- enhancement. If effects due to the formation time of pions as discussed in Sect. 5 or mechanisms like a_1 decay or $N^* \to \rho N$ with $\rho \to e^+e^-$ decays are unable to explain the shape of the dilepton mass spectrum, shifts of the ρ-meson mass and width due to a partial restauration of chiral symmetry remain as a possible and most interesting option.

– We have not discussed the increase of ratios Ω/Ξ and Ξ/Λ observed by the WA97 Collaboration for the simple reason that we do not understand it. The understanding of what happens in collisions of heaviest nuclei should be combined with that of why these ratios are increasing when going from pp and $p\bar{p}$ to pA interactions.

– A qualitative picture which might be consistent with the data discussed above consists of an intermediate gluonic stage followed by the interacting hadron gas. Such a picture is consistent with that proposed and studied by Geiger and Müller and by Gyulassy, Pang and Zhang.

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