# ELECTRON SCATTERING AND NUCLEAR STRUCTURE*

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*(Received June 1, 1997)*

These lectures are divided into two parts. First, an elementary introduction to elelctron scattering is presented, starting with the scattering of a non-relativistic lepton from a charge distribution, the extending to a relativistic Dirac electron, and finally including the quantum dynamics of the target. The relation to gamma decay is discussed. The analysis is repeated in a covariant manner and target structure functions defined. The parity violating asymmetry in the scattering of longitudinally polarized electrons arising from the exchange of a $Z$ is calculated. The structure functions are evaluated for deep-inelastic scattering in the quark-parton model where they exhibit Bjorken scaling. The asymmetry arising from scattering polarized nucleons on polarized nucleons is calculated in this model, as is the parity-violating asymmetry. The second part of the lecture series presents an overview of the current status of electron scattering, including a description of CEBAF. This latter material already appears in the published literature.

PACS numbers: 25.30. -c

## 1. Introduction

This School is on the "Dynamics of Strong Interactions." The nucleus is the principal laboratory for investigating the consequences of these dynamics. Electron scattering provides a microscope for examining the behavior of the nucleus. This set of three lectures will be concerned with "Electron Scattering and Nuclear Structure" [1,2].

* Presented at the XXXVII Cracow School of Theoretical Physics, May 30–June 10, 1997, Zakopane, Poland.

These lectures are divided into two parts. First, I will give an elementary introduction to electron scattering. This material is based on lectures I gave at the 6th Annual Summer School in Nuclear Physics Research held at North Carolina State University in Raleigh, North Carolina and at a summer school in La Rábida, Spain [6]. I have written up the first part for the proceedings of this school. Then I will give an overview of the present status of electron scattering, including a description of CEBAF.[1] The second part is based on two talks I have given at conferences and on an article I just wrote for *Physics News in 1996*. Since the material in the second part appears in the published literature in Refs. [3–5], I will simply refer students to that published material.

## 2. Basic nuclear physics

**Non-Relativistic Scattering of a Charged Lepton — Born Approximation.** Suppose one scatters a non-relativistic lepton of charge $ze_p$ with $z = \pm 1$ from the nucleus. The interaction takes place through the Coulomb potential

$$V(x) = \frac{ze^2}{4\pi}\int \frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|}\rho_N(\boldsymbol{x}')d^3x'\,. \tag{2.1}$$

The scattering amplitude is given in first Born Approximation by [14]

$$f_{\rm B.A.}(\boldsymbol{k}',\boldsymbol{k}) = -\frac{2\mu}{4\pi\hbar^2}\int \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{x}}\,V(x)d^3x\,. \tag{2.2}$$

Here $\mu$ is the reduced mass, $\hbar\boldsymbol{k}$ is the initial momentum, $\hbar\boldsymbol{k}'$ is the final momentum, and $\hbar\boldsymbol{q}$ with $\boldsymbol{q} = \boldsymbol{k}'-\boldsymbol{k}$ is the three-momentum transfer whose magnitude is given for elastic scattering by $\boldsymbol{q}^2 = 2\boldsymbol{k}^2(1-\cos\theta) = 4\boldsymbol{k}^2\sin^2\theta/2$. For a spherically symmetric nuclear charge density $\rho_N(x)$, the Fourier transform of the potential in Eq. (2.1) yields[2]

$$\begin{aligned}\int \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{x}}\,\frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|}\rho_N(\boldsymbol{x}')d^3x d^3x' &= \frac{4\pi}{\boldsymbol{q}^2}\int \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{y}}\,\rho_N(y)d^3y \\ &= \frac{4\pi}{\boldsymbol{q}^2}F(\boldsymbol{q}^2)\,. \end{aligned}\tag{2.3}$$

Here $F(\boldsymbol{q}^2)$ is the nuclear "form factor". Now use $e^2/\hbar c = 4\pi\alpha$ where $\alpha \approx 1/137.0$ is the fine-structure constant. The differential cross section then

[1] Now known as the Thomas Jefferson National Accelerator Facility (TJNAF).

[2] Use $\int \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{x}}(\mathrm{e}^{-\lambda x}/x)d^3x = 4\pi/(\boldsymbol{q}^2+\lambda^2)$; now let $\lambda \to 0$.

follows from the square of the modulus of the scattering amplitude as

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{4\mu^2}{\hbar^4}(\hbar c\alpha)^2\frac{1}{\boldsymbol{q}^4}|F(\boldsymbol{q}^2)|^2 \\ &= \frac{(\hbar c\alpha)^2}{16E_0^2\sin^4\theta/2}|F(\boldsymbol{q}^2)|^2 \\ &= \sigma_{\text{Rutherford}}|F(\boldsymbol{q}^2)|^2 \,. \end{aligned} \tag{2.4}$$

Here $E_0 = \hbar^2\boldsymbol{k}^2/2\mu$ is the incident energy and $\sigma_{\text{Rutherford}}$ is the familiar cross section for scattering from a point charge. Experimental measurement of this cross section evidently determines the Fourier transform of the nuclear charge density[3]

$$\begin{aligned}F(\boldsymbol{q}^2) &= \int \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{y}}\,\rho_N(y)d^3y \\ &= \int \frac{\sin qy}{qy}\rho_N(y)d^3y \,. \end{aligned} \tag{2.5}$$

Note that $F(0) = Z$, the total nuclear charge.

**Nuclear Physics.** Suppose now that one extends the analysis to deal with the internal quantum dynamics of the nuclear target. The nuclear charge density then becomes an operator in the nuclear Hilbert space

$$\rho_N(\boldsymbol{x}) \to \hat{\rho}_N(\boldsymbol{x}) \,; \qquad \text{Nuclear Density Operator}\,. \tag{2.6}$$

In first quantization, for example, with a collection of structureless nucleons, the nuclear density operator takes the form

$$\hat{\rho}_N(\boldsymbol{x}) = \sum_{j=1}^{Z} \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_j)\,. \tag{2.7}$$

The analysis of the scattering amplitude in Eq. (2.3) indicates that one now requires the nuclear transition matrix elements of the operator

$$\hat{F}(\boldsymbol{q}) = \int \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{y}}\,\hat{\rho}_N(\boldsymbol{y})d^3y\,. \tag{2.8}$$

Take the momentum transfer $\boldsymbol{q}$ to define the $z$-axis and expand the plane wave appearing in this expression according to

$$\mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{x}} = \sum_{J=0}^{\infty} \sqrt{4\pi(2J+1)}(-i)^J j_J(qx)Y_{J0}(\Omega_x)\,. \tag{2.9}$$

[3] One actually measures the square of the modulus of the form factor, but since it is real here and $F(0) = Z$, one can track through the zeros and determine both the sign and magnitude.

This gives

$$\hat{F}(\boldsymbol{q}) = \sum_{J=0}^{\infty} \sqrt{4\pi(2J+1)}(-i)^J \hat{M}_{J0}(q)\,,$$
$$\hat{M}_{JM}(q) \equiv \int j_J(qx) Y_{JM}(\Omega_x)\hat{\rho}_N(\boldsymbol{x}) d^3x\,. \tag{2.10}$$

The quantities $\hat{M}_{JM}(q)$ are now *irreducible tensor operators* (ITO) in the nuclear Hilbert space. The general proof depends on the fact that the nuclear density is a scalar under rotations; in the case where Eq. (2.7) holds, these multipoles consist of a sum of single particle radial functions multiplied by spherical harmonics and the result is evident. The great advantage of identifying an ITO is that one can now use the general theory of angular momentum (Ref. [13]), in particular the Wigner–Eckart theorem states that the matrix element of an ITO taken between nuclear eigenstates of angular momentum results in[4]

$$\langle J_f M_f|\hat{M}_{JM}|J_i M_i\rangle = (-1)^{J_f - M_f}\begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix}\langle J_f||\hat{M}_J||J_i\rangle\,. \tag{2.11}$$

This result has two invaluable features: it gives the explicit dependence on the nuclear orientation (all $M$'s), and it contains the angular momentum selection rules (the $J$'s must satisfy the triangle inequality). The average over initial states and sum over final states $\overline{\sum}_i\sum_f$ for a nuclear transition to a discrete state (this can include elastic scattering) is then immediately performed using the orthogonality property of the 3-$j$ symbols

$$\frac{1}{2J_i+1}\sum_{M_i}\sum_{M_f}\begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix}\begin{pmatrix} J_f & J' & J_i \\ -M_f & M' & M_i \end{pmatrix}$$
$$= \frac{1}{2J_i+1}\frac{1}{2J+1}\delta_{JJ'}\delta_{MM'}\,. \tag{2.12}$$

Hence the nuclear physics is now contained in the following expression

$$\overline{\sum}_i\sum_f |\langle J_f M_f| \int e^{-i\boldsymbol{q}\cdot\boldsymbol{y}}\,\hat{\rho}_N(\boldsymbol{y}) d^3y|J_i M_i\rangle|^2 = \frac{4\pi}{2J_i+1}\sum_{J=0}^{\infty}|\langle J_f||\hat{M}_J(q)||J_i\rangle|^2\,. \tag{2.13}$$

This sum is actually finite since the nuclear matrix elements vanish unless the selection rules are satisfied.

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[4] We assume here that the nuclear target is heavy and localized and that the nuclear eigenstates can be characterized by their angular momentum.

**Relativistic (Massless) Electrons.** The cross section for the scattering of relativistic (massless) electrons through the Coulomb interaction can now be obtained from the previous results through the following modifications:

1) Replace the transition matrix element for the projectile $e^{-i\boldsymbol{q}\cdot\boldsymbol{x}}$ by $e^{-i\boldsymbol{q}\cdot\boldsymbol{x}}$ $u^\dagger(\boldsymbol{k}')u(\boldsymbol{k})$ which includes the overlap of the Dirac spinors for the electron.
A simple calculation with the Dirac wave functions then gives (Ref. [14])[5]

$$\tfrac{1}{2}\sum_{s_1}\sum_{s_2}|u^\dagger(\boldsymbol{k}')u(\boldsymbol{k})|^2 = \tfrac{1}{2}(1+\cos\theta) = \cos^2\frac{\theta}{2}\,. \tag{2.14}$$

2) Replace $\mu c^2$ in the numerator of the scattering amplitude by the full final electron energy $\hbar k'c$; this factor arises from the appropriate incident flux and density of final states in Fermi's Golden Rule.
3) Make use of the four-momentum transfer $q_\mu^2 = \boldsymbol{q}^2 - q_0^2$ where $q_0 = k' - k$ to write the point cross section. This quantity satisfies

$$q_\mu^2 = 4kk'\sin^2\frac{\theta}{2} \tag{2.15}$$

for both elastic and inelastic transitions.

The resulting differential cross section then takes the form

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \sigma_{\text{Mott}}\frac{q_\mu^4}{\boldsymbol{q}^4}\frac{4\pi}{2J_i+1}\sum_{J=0}^{\infty}|\langle J_f||\hat{M}_J(q)||J_i\rangle|^2\,,\\ \sigma_{\text{Mott}} &\equiv \frac{\alpha^2\cos^2\theta/2}{4k^2\sin^4\theta/2}\,.\end{aligned} \tag{2.16}$$

Here $\sigma_{\text{Mott}}$ is the cross section for scattering a Dirac electron from a fixed, point charge. Note that this quantity can also be written

$$\sigma_{\text{Mott}} = \frac{4\alpha^2 k'^2\cos^2\theta/2}{q_\mu^4}\,. \tag{2.17}$$

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[5] Use

$$\begin{aligned}\frac{1}{2}\sum_{s_1}\sum_{s_2}|\bar{u}(\boldsymbol{k}')\gamma_4 u(\boldsymbol{k})|^2 &= \frac{1}{2}\frac{1}{4kk'}\operatorname{Tr}\gamma_4(-i\gamma_\mu k_\mu)\gamma_4(-i\gamma_\nu k'_\nu)\\ &= -\frac{1}{2kk'}(2k_4k'_4 - k_\mu k'_\mu)\\ &= \frac{1}{2kk'}(\boldsymbol{k}\cdot\boldsymbol{k}' + kk') = \frac{1}{2}(1+\cos\theta).\end{aligned}$$

**Long-Wavelength Limit.** In the limit that the momentum transfer goes to zero, an expansion of the spherical Bessel functions reduces the multipole operators in Eq. (2.10) to the form

$$\hat{M}_{JM}(q) \to \frac{q^J}{(2J+1)!!}\int x^J Y_{JM}(\Omega_x)\hat{\rho}_N(\boldsymbol{x})d^3x\,. \tag{2.18}$$

If $R$ characterizes the size of the nuclear target, then the multipole operators go as $(qR)^J$ and the lowest allowed multipole dominates in the limit $qR \to 0$. In electron scattering, the product $qR$ can be made arbitrarily large by going first to larger scattering angles at fixed incident energy, and then by going to higher and higher energy electrons.

Recall it is a property of Fourier transforms that the equivalent wavelength at which the system is examined bears an inverse relation to the momentum transfer

$$|\boldsymbol{q}| \equiv \frac{2\pi}{\lambda}\,. \tag{2.19}$$

At CEBAF, we will be interested in wavelengths which probe the nucleus at distance scales of tens of Fermis down to tenths of Fermis.[6]

**Gamma Decay.** Consider a nuclear transition $|J_iM_i\rangle \to |J_fM_f\rangle$ with the emission of a photon. The hamiltonian governing this electromagnetic process is

$$\begin{aligned} H' &= -\frac{e_p}{c}\int \hat{\boldsymbol{J}}_N(\boldsymbol{x})\cdot \boldsymbol{A}(\boldsymbol{x})d^3x\,, \\ \boldsymbol{A}(\boldsymbol{x}) &= \sum_{\boldsymbol{k}}\sum_{\lambda=1,2}\left(\frac{\hbar c^2}{2\omega_k\Omega}\right)^{1/2}(a_{\boldsymbol{k}\lambda}\boldsymbol{e}_{\boldsymbol{k}\lambda}\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \mathrm{h.c.})\,. \end{aligned} \tag{2.20}$$

Here $\boldsymbol{A}$ is the vector potential for the quantized radiation field and the hamiltonian is written in the Schrödinger picture. In this expression $\boldsymbol{e}_{\boldsymbol{k}1,2}$ are a set of unit vectors orthogonal to $\boldsymbol{k}$, $\omega_k = kc$, $a^\dagger$ $(a)$ are the creation (destruction) operators for the photons, and we use periodic boundary conditions in a big box of volume $\Omega$.

It is convenient to first make a canonical transformation to photon states with circular polarization. This leads to an expression for the vector potential where one now replaces $\sum_{\lambda=1,2} \to \sum_{\lambda=\pm 1}$ with $\boldsymbol{e}_{\boldsymbol{k},\pm 1} \equiv \mp(\boldsymbol{e}_{\boldsymbol{k}1} \pm i\boldsymbol{e}_{\boldsymbol{k}2})/\sqrt{2}$.

The nuclear matrix element for photoemission then takes the form

$$H'_{\mathrm{fi}} = -\frac{e_p}{c}\left(\frac{\hbar c^2}{2\omega_k\Omega}\right)^{1/2}\langle f|\int \boldsymbol{e}^\dagger_{\boldsymbol{k}\lambda}\,\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\cdot\hat{\boldsymbol{J}}_N(\boldsymbol{x})d^3x|i\rangle\,. \tag{2.21}$$

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[6] 1 Fermi = $10^{-13}$ cm.

Now introduce the following expansion for the plane wave times the unit vector (Ref. [8, 13])

$$\begin{aligned} &\boldsymbol{e}_{\boldsymbol{k}\lambda}^{\dagger}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} = \\ &-\sum_{J\geq 1}\sqrt{2\pi(2J+1)}(-i)^J\left\{\frac{1}{k}\boldsymbol{\nabla}\times[j_J(kx)\mathcal{Y}_{JJ}^{-\lambda}] + \lambda j_J(kx)\mathcal{Y}_{JJ}^{-\lambda}\right\}. \end{aligned} \tag{2.22}$$

Here the vector spherical harmonics are defined by $(\boldsymbol{e}_{\boldsymbol{k}0} \equiv \boldsymbol{k}/|\boldsymbol{k}|)$.

$$\mathcal{Y}_{lJ}^{M} \equiv \sum_{m_l m_s} \langle l m_l 1 m_s | l 1 J M\rangle Y_{l m_l}(\Omega_x)\boldsymbol{e}_{m_s} \tag{2.23}$$

Equation (2.22) is simply an algebraic identity. Its great utility lies in the fact that it allows one to again make an expansion of the required nuclear transition operator in ITO

$$\begin{aligned} &\int \boldsymbol{e}_{\boldsymbol{k}\lambda}^{\dagger}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \cdot \hat{\boldsymbol{J}}_N(\boldsymbol{x})d^3x = \\ &-\sum_{J\geq 1}\sqrt{2\pi(2J+1)}(-i)^J[\hat{T}^{\mathrm{el}}_{J,-\lambda}(k) + \lambda\hat{T}^{\mathrm{mag}}_{J,-\lambda}(k)]. \end{aligned} \tag{2.24}$$

The electric and magnetic multipole operators are defined by

$$\begin{aligned} \hat{T}^{\mathrm{el}}_{JM}(k) &= \frac{1}{k}\int\{\boldsymbol{\nabla}\times[j_J(kx)\mathcal{Y}_{JJ}^{M}(\Omega_x)]\}\cdot\hat{\boldsymbol{J}}_N(\boldsymbol{x})d^3x\,, \\ \hat{T}^{\mathrm{mag}}_{JM}(k) &= \int[j_J(kx)\mathcal{Y}_{JJ}^{M}(\Omega_x)]\cdot\hat{\boldsymbol{J}}_N(\boldsymbol{x})d^3x\,. \end{aligned} \tag{2.25}$$

The decay rate now follows from Fermi's Golden Rule

$$d\omega_{\mathrm{fi}} = \frac{2\pi}{\hbar}|H'_{\mathrm{fi}}|^2\delta(E_f + \hbar\omega_k - E_i)\frac{\Omega d^3k}{(2\pi)^3} \tag{2.26}$$

Since the electromagnetic multipoles have opposite parity, it follows that the good parity of the nuclear states implies

$$|\langle J_f||\hat{T}^{\mathrm{el}}_J + \lambda\hat{T}^{\mathrm{mag}}_J||J_i\rangle|^2 = |\langle J_f||\hat{T}^{\mathrm{el}}_J||J_i\rangle|^2 + |\langle J_f||\hat{T}^{\mathrm{mag}}_J||J_i\rangle|^2\,. \tag{2.27}$$

We leave it as an exercise to show that a combination of the above results leads to the following expression for the decay rate for photon emission [8]

$$\omega_{\mathrm{fi}} = 8\pi\alpha k c\frac{1}{2J_i+1}\sum_{J=1}^{\infty}(|\langle J_f||\hat{T}^{\mathrm{el}}_J(k)||J_i\rangle|^2 + |\langle J_f||\hat{T}^{\mathrm{mag}}_J(k)||J_i\rangle|^2)\,. \tag{2.28}$$

In fact, this is a general expression for the decay rate for photon emission for any heavy, localized quantum mechanical system; it is exact to order $\alpha$. The multipole operators appearing in this expression now contain a factor $c^{-1}$ and are dimensionless.

The amplitude for the scattering of a relativistic electron from a nuclear target [9–12] can be calculated to order $\alpha$ in time-independent perturbation theory by combining the first-order Coulomb amplitude arising from Eq. (2.1) with the second-order amplitude for the exchange of a transverse photon of momentum $\hbar \boldsymbol{q}$ coming from Eq. (2.20) (and its analog for the electron). Since the Coulomb and transverse multipoles carry different amounts of angular momentum along the $\boldsymbol{q}$ axis, they do not interfere after the sum and average over nuclear orientations. It should therefore not be too surprising that the differential cross section can be written in the following form (see *e.g.* Ref. [2])

$$\frac{d\sigma}{d\Omega} = 4\pi\sigma_{\text{Mott}}\frac{1}{2J_i+1}\left\{\frac{q_\mu^4}{\boldsymbol{q}^4}\sum_{J=0}^{\infty}|\langle J_f||\hat{M}_J(q)||J_i\rangle|^2 \right.$$
$$\left. + \left(\frac{q_\mu^2}{2\boldsymbol{q}^2} + \tan^2\frac{\theta}{2}\right)\sum_{J=1}^{\infty}(|\langle J_f||\hat{T}_J^{\text{el}}(q)||J_i\rangle|^2 + |\langle J_f||\hat{T}_J^{\text{mag}}(q)||J_i\rangle|^2)\right\} . \quad (2.29)$$

Several features of this result are of interest:

- The nuclear matrix elements obey all the selection rules discussed above; in particular they vanish unless $J_f + J_i \geq J \geq |J_f - J_i|$.
- Because of the unit helicity of the photon, the sum over the transverse multipoles starts with $J = 1$; in contrast, there is a $J = 0$ Coulomb monopole.
- The momentum transfer $\hbar|\boldsymbol{q}|$ can take any value in electron scattering.
- There are 3 lepton variables in electron scattering $(k, k', \theta)$ or equivalently $(\boldsymbol{q}^2, \omega, \theta)$ where the energy transfer $\hbar\omega$ is given by $\omega/c \equiv k - k'$. The Coulomb contribution and that arising from transverse photon exchange can be separated by keeping the first two variables $(\boldsymbol{q}^2, \omega)$ fixed and varying the electron scattering angle $\theta$, or by working at $\theta = 180°$ where only the transverse term contributes.
- It has been assumed here that the nucleus is heavy and this is the laboratory cross section. If nuclear recoil is included in the density of final states, the result is to multiply this expression for the cross section by a factor $r$ where $r^{-1} = 1 + (2\hbar k/M_T c)\sin^2\theta/2$. We leave the demonstration of this result to the reader.

Construction of the nuclear current at various levels of the description of the nucleus and calculation of nuclear matrix elements is described in Refs [1,2].

## 3. Covariant analysis

**Covariant Analysis.** Let us revisit our analysis of electron scattering and start from the beginning in an explicitly covariant manner. The S-matrix with one-photon exchange can be written in the form [1, 2, 25, 26] [7]

$$S_{\mathrm{fi}} = -\frac{ee_p}{\Omega}\bar{u}(k')\gamma_\mu u(k)\frac{1}{q^2}\int \mathrm{e}^{-iq\cdot x}\langle p'|J_\mu(x)|p\rangle d^4x\,. \tag{3.1}$$

The momenta appearing in this expression are now all four-vectors and the four-momentum transfer satisfies the relation $q = k' - k$. [8] One can use translational invariance on the nuclear matrix element to write in the continuum limit

$$\int \mathrm{e}^{-iq\cdot x}\langle p'|J_\mu(x)|p\rangle d^4x = (2\pi)^4\delta^{(4)}(p' + q - p)\langle p'|J_\mu(0)|p\rangle\,. \tag{3.2}$$

The T-matrix is then identified from the expression

$$S_{\mathrm{fi}} = -\frac{(2\pi)^4}{\Omega}i\delta^{(4)}(p' + q - p)\overline{T}_{\mathrm{fi}}\,. \tag{3.3}$$

The cross section follows in the standard manner

$$d\sigma = \overline{\sum_i}\sum_f 2\pi|\overline{T}_{\mathrm{fi}}|^2\delta(W_f - W_i)\frac{\Omega d^3k'}{(2\pi)^3}\frac{1}{I_{\mathrm{inc}}}\left[\frac{(2\pi)^3}{\Omega}\delta^{(3)}(\Delta\boldsymbol{p})\right]\,. \tag{3.4}$$

The last factor takes into account the fact that up to the final step, one is really working in a big box with periodic boundary conditions so that

$$(2\pi)^3\delta^{(3)}(\Delta\boldsymbol{p}) = \int_{\mathrm{box}} \mathrm{e}^{i\Delta\boldsymbol{p}\cdot\boldsymbol{x}} \equiv \Omega\delta_{\boldsymbol{p}_f,\boldsymbol{p}_i}\,. \tag{3.5}$$

where the last term is a Kronecker delta satisfying

$$[\delta_{\boldsymbol{p}_f,\boldsymbol{p}_i}]^2 = \delta_{\boldsymbol{p}_f,\boldsymbol{p}_i}\,. \tag{3.6}$$

We leave it as an exercise for the reader to show that the incident flux in any frame where $\boldsymbol{k}||\boldsymbol{p}$ can be written for a massless electron as

$$I_{\mathrm{inc}} = \frac{1}{\Omega}\frac{\sqrt{(k\cdot p)^2}}{\varepsilon E_p}\,. \tag{3.7}$$

[7] We now revert to units where $\hbar = c = 1$. We use a metric with $x_\mu = (\boldsymbol{x}, it)$. Our gamma matrices are Hermitian and satisfy $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$. Also $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$.

[8] The quantity $q^2$ now denotes the four-momentum transfer; the three-momentum transfer will henceforth be explicitly denoted by $\boldsymbol{q}^2$.

This relation is immediately verified in the laboratory frame where $E_p = M_T$ and $k \cdot p = -\varepsilon M_T$.

The square of the T-matrix then leads to the cross section in the form

$$d\sigma = \frac{1}{\sqrt{(k \cdot p)^2}} \frac{4\alpha^2}{q^4} \eta_{\mu\nu} W_{\mu\nu} \frac{d^3k'}{2\varepsilon'} \,. \tag{3.8}$$

As an element of transverse area, this cross section must take the same value in any frame where $\boldsymbol{k} || \boldsymbol{p}$, and indeed, it has now been written in an explicitly Lorentz invariant form.

The lepton tensor appearing in this expression is defined by

$$\begin{aligned} \eta_{\mu\nu} &= -\tfrac{1}{2} 2\varepsilon\varepsilon' \sum_{s_1} \sum_{s_2} \bar{u}(k)\gamma_\nu u(k')\bar{u}(k')\gamma_\mu u(k) \\ &= k_\mu k'_\nu + k_\nu k'_\mu - k \cdot k' \delta_{\mu\nu} \,. \end{aligned} \tag{3.9}$$

The hadronic target contribution is similarly summarized in a tensor of the form

$$W_{\mu\nu} = (2\pi)^3 \overline{\sum_i} \sum_f \delta^{(4)}(q + p' - p) \langle p | J_\nu(0) | p' \rangle \langle p' | J_\mu(0) | p \rangle (\Omega E_p) \,. \tag{3.10}$$

This Lorentz tensor can be analyzed through the following observations:

- Conservation of the electromagnetic current implies $q_\mu W_{\mu\nu} = W_{\mu\nu} q_\nu = 0$.
- The only remaining four-vectors with which to construct this tensor are $p_\mu$ and $q_\mu$.
- The only remaining Lorentz invariant variables are $q^2$ and $p \cdot q$.

As a result, the target response tensor must take the form

$$\begin{aligned} W_{\mu\nu} &= W_1(q^2, q \cdot p) \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \\ &+ W_2(q^2, q \cdot p) \frac{1}{M_T^2} \left( p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left( p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) \,. \end{aligned} \tag{3.11}$$

Note that in the laboratory frame the Lorentz invariants take the form

$$\begin{aligned} q^2 &= 4\varepsilon\varepsilon' \sin^2 \frac{\theta}{2} \,, \\ \frac{p \cdot q}{M_T} &= \varepsilon - \varepsilon' \,. \end{aligned} \tag{3.12}$$

We again leave it to the reader to show that a combination of these results in a laboratory cross section of the form

$$\frac{d^2\sigma}{d\Omega' d\varepsilon'} = \sigma_{\text{Mott}} \frac{1}{M_T} \left[ W_2(q^2, q\cdot p) + 2W_1(q^2, q\cdot p)\tan^2\frac{\theta}{2} \right] . \quad (3.13)$$

**An Example.** As an example, consider elastic scattering from a spin zero nucleus. In this case, Lorentz invariance and current conservation imply that the nuclear matrix element must have the form [2]

$$\langle p-q, 0^+ | J_\mu(0) | p, 0^+ \rangle = \frac{1}{M_T}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right) F_0(q^2) \left(\frac{M_T^2}{E_p E_{p'} \Omega^2}\right)^{1/2} . \quad (3.14)$$

Hermiticity of the electromagnetic current implies the form factor $F_0(q^2)$ is real. The response functions are immediately evaluated in this case to give

$$\begin{aligned} W_1 &= 0 , \\ W_2 &= |F_0(q^2)|^2 \frac{M_T^2}{E_{p'}} \delta(W_f - W_i) . \end{aligned} \quad (3.15)$$

The cross section then takes the form

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} |F_0(q^2)|^2 r . \quad (3.16)$$

Here $r$ is the previously discussed recoil factor.

**Parity Violation.** Consider now a longitudinally polarized electron beam scattered from a target which is unpolarized and unobserved. If one does nothing more than reverse the electron helicity, then the parity violating asymmetry

$$\mathcal{A} = \frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow} , \quad (3.17)$$

must vanish since the electromagnetic interaction conserves parity to all orders. Parity violation is present in electron scattering to a small extent due to interference with the weak amplitude arising from the exchange of the $Z^0$ — the heavy, neutral, weak vector boson.[9] If $Z^0$ exchange is added to $\gamma$ exchange, the above S-matrix is extended to [1, 18] [10]

$$\begin{aligned} S_{\text{fi}} = &-\frac{ee_p}{\Omega} \bar{u}(k')\gamma_\mu u(k) \frac{1}{q^2} \int e^{-iq\cdot x} \langle p' | J_\mu^{(\gamma)}(x) | p \rangle d^4x \\ &- \frac{G}{\Omega\sqrt{2}} \bar{u}(k')(a\gamma_\mu + b\gamma_\mu\gamma_5)u(k) \int e^{-iq\cdot x} \langle p' | \mathcal{J}_\mu^{(0)}(x) | p \rangle d^4x . \end{aligned} \quad (3.18)$$

[9] We are here discussing electron scattering from nuclei up to several GeV.

[10] Additional contributions to the parity-violating asymmetry can arise from parity admixtures in the nuclear states coming from weak parity-violating nucleon-nucleon interactions. These contributions are generally negligible, except perhaps at very small $q^2$ (Refs. [27, 28]).

Here $G$ is the Fermi constant and the weak neutral current is assumed to have the familiar V-A form

$$\mathcal{J}_\mu^{(0)} = J_\mu^{(0)} + J_{\mu 5}^{(0)} \,. \tag{3.19}$$

In the standard model the electron weak neutral current is given by

$$a = -(1 - 4\sin^2\theta_W)\,, \qquad b = -1\,. \tag{3.20}$$

The use of helicity projection operators for massless electrons

$$P_\uparrow = \frac{1-\gamma_5}{2}\,, \qquad P_\downarrow = \frac{1+\gamma_5}{2}\,, \tag{3.21}$$

allows one to calculate the asymmetry in a manner directly analogous to that described above for the cross section. The result is (Refs [1,2])

$$\begin{aligned} \mathcal{A}\left[\cos^2\frac{\theta}{2}W_2^\gamma + 2\sin^2\frac{\theta}{2}W_1^\gamma\right] &= \frac{Gq^2}{4\pi\alpha\sqrt{2}}\left\{b\left[\cos^2\frac{\theta}{2}W_2^{\text{int}} + 2\sin^2\frac{\theta}{2}W_1^{\text{int}}\right]\right. \\ &\left. -a\left(\frac{2W_8}{M_T}\right)\sin\frac{\theta}{2}\left(q^2\cos^2\frac{\theta}{2} + \boldsymbol{q}^2\sin^2\frac{\theta}{2}\right)^{1/2}\right\}. \end{aligned} \tag{3.22}$$

Here the nuclear target response tensors are defined in a fashion similar to that in Eq. (3.10). The response tensor arising from the interference of the electromagnetic and vector part of the weak neutral current is written as

$$\begin{aligned} W_{\mu\nu}^{\text{int}} &= (2\pi)^3\overline{\sum}_i\sum_f \delta^{(4)}(q+p'-p)\left[\langle p|J_\nu^{(0)}(0)|p'\rangle\langle p'|J_\mu^{(\gamma)}(0)|p\rangle\right. \\ &\left. + \langle p|J_\nu^{(\gamma)}(0)|p'\rangle\langle p'|J_\mu^{(0)}(0)|p\rangle\right](\Omega E_p)\,. \end{aligned} \tag{3.23}$$

We assume the weak vector current is conserved, and thus this tensor must again have the covariant form

$$\begin{aligned} W_{\mu\nu}^{\text{int}} &= W_1^{\text{int}}(q^2, q\cdot p)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \\ &+ W_2^{\text{int}}(q^2, q\cdot p)\frac{1}{M_T^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right). \end{aligned} \tag{3.24}$$

The tensor arising from the interference of the electromagnetic current and axial vector part of the weak neutral current is defined by

$$\begin{aligned} W_{\mu\nu}^{\text{V-A}} &= (2\pi)^3\overline{\sum}_i\sum_f \delta^{(4)}(q+p'-p)\left[\langle p|J_{\nu 5}^{(0)}(0)|p'\rangle\langle p'|J_\mu^{(\gamma)}(0)|p\rangle\right. \\ &\left. + \langle p|J_\nu^{(\gamma)}(0)|p'\rangle\langle p'|J_{\mu 5}^{(0)}(0)|p\rangle\right](\Omega E_p)\,. \end{aligned} \tag{3.25}$$

It must be a pseudotensor, and the only one we can make from $p$ and $q$ is

$$W_{\mu\nu}^{\mathrm{V-A}} = W_8(q^2, q \cdot p)\frac{1}{M_T^2}\varepsilon_{\mu\nu\rho\sigma}p_\rho q_\sigma \,. \tag{3.26}$$

**An Example.** Consider again the example of elastic scattering from a spin zero nucleus. In this case the matrix element of the axial vector current must vanish since one cannot make an axial vector from $p$ and $q$

$$\langle p-q, 0^+|J_{\mu 5}^{(0)}(0)|p, 0^+\rangle = 0 \,. \tag{3.27}$$

The response tensors are then evaluated as above to give

$$\begin{aligned} W_1^{\mathrm{int}} &= W_8 = 0\,, \\ W_2^{\mathrm{int}} &= 2F_0^{(\gamma)}(q^2)F_0^{(0)}(q^2)\frac{M_T^2}{E_{p'}}\delta(W_f - W_i)\,. \end{aligned} \tag{3.28}$$

As above, the form factors must be real. The asymmetry in this case thus takes the form [1, 2]

$$\mathcal{A} = \frac{Gq^2 b}{2\pi\alpha\sqrt{2}}\frac{F_0^{(0)}(q^2)}{F_0^{(\gamma)}(q^2)}\,. \tag{3.29}$$

Measurement of this asymmetry at all $q^2$ thus completely determines the distribution of weak neutral current in this nuclear system.

## 4. Deep-inelastic scattering

We next turn to the subject of deep-inelastic scattering of leptons from the nucleon [1, 2, 7, 15–17]. The process is illustrated in Fig. 1. It is characterized by two Lorentz scalars, which in the laboratory frame take the following forms

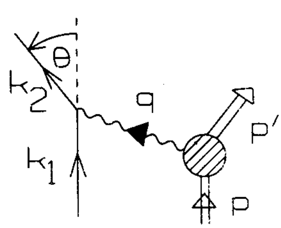


Fig. 1. Deep-inelastic lepton scattering from the nucleon.

$$q^2 = 4\varepsilon_1\varepsilon_2 \sin^2\frac{\theta}{2}\,, \qquad \nu = \frac{p\cdot q}{m} = \varepsilon_1 - \varepsilon_2\,. \tag{4.1}$$

The unpolarized differential cross section can be written in Lorentz invariant form as

$$d\sigma = \frac{4\alpha^2}{q^4}\frac{d^3k_2}{2\varepsilon_2}\frac{1}{\sqrt{(k_1\cdot p)^2}}\eta_{\mu\nu}W_{\mu\nu}\,. \tag{4.2}$$

The lepton response tensor is give by[11]

$$\begin{aligned}\eta_{\mu\nu} &= \frac{1}{4}\mathrm{Tr}\gamma_\nu(k_{2\rho}\gamma_\rho)\gamma_\mu(k_{1\sigma}\gamma_\sigma)\\ &= k_{2\mu}k_{1\nu}+k_{1\mu}k_{2\nu}-k_1\cdot k_2\,\delta_{\mu\nu}\,.\end{aligned} \tag{4.3}$$

The hadronic response tensor for unpolarized and unobserved targets is given by the Lorentz tensor

$$\begin{aligned}W_{\mu\nu} &\equiv \overline{\sum_i}\sum_f(2\pi)^3\delta^{(4)}(q+p'-p)\langle p|J_\nu(0)|p'\rangle\langle p'|J_\mu(0)|p\rangle(\Omega E_p)\\ &= W_1(q^2,q\cdot p)\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\\ &\quad +W_2(q^2,q\cdot p)\frac{1}{m^2}\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)\,.\end{aligned} \tag{4.4}$$

A combination of these results expresses the laboratory cross section as

$$\begin{aligned}\frac{d^2\sigma}{d\Omega_2 d\varepsilon_2} &= \sigma_{\mathrm{M}}\frac{1}{m}\left[W_2(\nu,q^2)+2W_1(\nu,q^2)\tan^2\frac{\theta}{2}\right]\,,\\ \sigma_{\mathrm{M}} &= \frac{\alpha^2\cos^2\theta/2}{4\varepsilon_1^2\sin^4\theta/2}\,.\end{aligned} \tag{4.5}$$

In the quark-parton model in the $|\boldsymbol{p}|\to\infty$ frame (Fig. 2), one calculates the

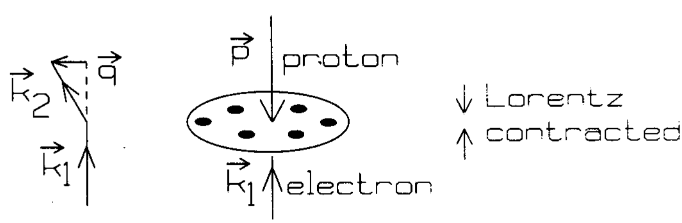


Fig. 2. Quark-parton model for deep-inelastic scattering in the $|\boldsymbol{p}|\to\infty$ frame. Here $\boldsymbol{p}=-\boldsymbol{k}_1$.

[11] We assume the ERL where the lepton mass is negligible.

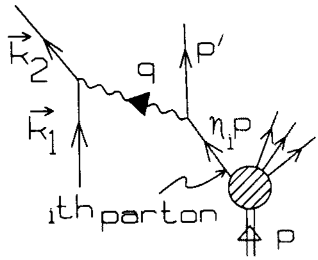


Fig. 3. Quark-parton model for deep-inelastic scattering in the impulse approximation in the $|\boldsymbol{p}| \to \infty$ frame.

Lorentz invariant structure functions by considering free, incoherent scattering from the collection of charged, pointlike constituents of the hadronic target; these are the various quarks with charges $Q_i$ [19, 20]. Each type $i$ of such constituent is assumed to carry a fraction $\eta_i p$ of the incident hadronic target four-momentum $p$ in this frame. The cross section is then calculated in the impulse approximation (Fig. 3). If the number of such constituents carrying momentum fraction between $\eta_i p$ and $(\eta+d\eta)_i p$ is given by $f_i(\eta_i)d\eta_i$, then the Lorentz invariant target response functions for scattering from the nucleon take the form [1, 19, 20, 22, 23] [12]

$$\begin{aligned} 2W_1 &= F_1(x) = \sum_i Q_i^2 f_i(x)\,, \\ \frac{\nu}{m}W_2 &= F_2(x) = \sum_i Q_i^2 x f_i(x)\,, \\ x &\equiv \frac{q^2}{2m\nu}\,. \end{aligned} \tag{4.6}$$

The response functions, which in general depend on the two variables $(\nu, q^2)$, now depend only on the single *Bjorken scaling variable* $x$, as is observed experimentally in the scaling region of $\nu \to \infty$ and $q^2 \to \infty$ at fixed $x = q^2/2m\nu$.

It is convenient for the following discussion to explicitly distinguish the number of quarks with helicity aligned and opposed to $\boldsymbol{p}$; we shall do this with a superscript; thus in the quark-parton model

$$\begin{aligned} F_1(x) &= \sum_i Q_i^2[f_i^{\uparrow}(x) + f_i^{\downarrow}(x)]\,, \\ F_2(x) &= xF_1(x)\,. \end{aligned} \tag{4.7}$$

[12] Reference [1] uses the convention $2W_1 = F_1(x)$ (which we employ here), while Ref. [21] uses $2W_1 = 2F_1(x)$

Suppose now the initial lepton beam is longitudinally polarized. Then in the ERL one can simply insert the appropriate helicity projection operators for massless fermions in the lepton traces

$$P_{\uparrow} = \tfrac{1}{2}(1-\gamma_5)\,, \qquad P_{\downarrow} = \tfrac{1}{2}(1+\gamma_5)\,. \tag{4.8}$$

The result is that the lepton traces now take the form

$$\begin{aligned} \eta^{\uparrow}_{\mu\nu} &= \tfrac{1}{2}(\eta_{\mu\nu} - \varepsilon_{\mu\nu\rho\sigma}k_{1\rho}k_{2\sigma})\,, \\ \eta^{\downarrow}_{\mu\nu} &= \tfrac{1}{2}(\eta_{\mu\nu} + \varepsilon_{\mu\nu\rho\sigma}k_{1\rho}k_{2\sigma})\,. \end{aligned} \tag{4.9}$$

Suppose, in addition, that the target is longitudinally polarized and has helicity aligned or antialigned along $\boldsymbol{p}$. Then a calculation exactly analogous to the above in the quark-parton model, with the neglect of all masses, yields an additional Lorentz covariant contribution to the response tensor for the nucleon[13]

$$\delta W^{\uparrow}_{\mu\nu} = W_{\rm h}(\nu, q^2)\frac{1}{m^2}\varepsilon_{\mu\nu\rho\sigma}p_\rho q_\sigma\,. \tag{4.10}$$

One finds, in addition, that the following combination satisfies Bjorken scaling

$$\begin{aligned} \frac{2\nu}{m}W_{\rm h} &\equiv 2g_1(x) \\ &= \sum_i Q_i^2[f_i^{\uparrow}(x) - f_i^{\downarrow}(x)]\,. \end{aligned} \tag{4.11}$$

These results can be used to compute the asymmetry for scattering of the lepton by the target in the case the helicities are aligned or antialigned

$$\mathcal{A} \equiv \frac{d\sigma_{\uparrow\uparrow} - d\sigma_{\downarrow\uparrow}}{d\sigma_{\uparrow\uparrow} + d\sigma_{\downarrow\uparrow}}\,. \tag{4.12}$$

Here the subscripts refer to the particle *helicities.* A combination of the above results then leads to

$$\begin{aligned} \mathcal{A} &= \frac{2g_1(x)}{F_1(x)}\mathcal{D}\,, \\ \mathcal{D} &= \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1^2 + \varepsilon_2^2}\,; \qquad \text{ERL}\,. \end{aligned} \tag{4.13}$$

These results hold in the deep inelastic region where $q^2 \to \infty$ and $\nu \to \infty$ at fixed $x = q^2/2m\nu$; we have assumed that this scaling limit is achieved in

[13] Overall factors of $m$ set the energy scale.

the ERL where $\varepsilon_1 \to \infty$ so that all masses are indeed negligible, and correspondingly $\theta \to 0$. In this regime, such an experiment evidently measures the spin structure function of the target nucleon defined by

$$\frac{2g_1(x)}{F_1(x)} = \frac{\sum_i Q_i^2[f_i^\uparrow(x) - f_i^\downarrow(x)]}{\sum_i Q_i^2[f_i^\uparrow(x) + f_i^\downarrow(x)]}. \tag{4.14}$$

If one retains correction terms of $O(m/\varepsilon_1)$, and correspondingly considers other directions of the polarization of the target, then the expression for the polarization asymmetry becomes more complicated, and one can, in fact, measure an additional spin structure function $g_2(x)$, whose interpretation in the quark-parton model is more ambiguous. The full response for arbitrary target polarization is given in Ref. [21], where experimental results from the scattering of very high energy polarized muons from polarized nucleon targets are also discussed.

**Parity Violation.** Suppose the incident lepton beam is longitudinally polarized, but the target nucleon is now *unpolarized* (and again unobserved). Any difference in cross section for the different helicities must now arise from parity violation. It is straightforward to combine the above results and the S-matrix in Eq. (3.18) to compute the parity-violating asymmetry arising from the interference of one-photon and $Z^0$ exchange. To simplify the presentation, we again go to the ERL for the lepton as defined above (with $\varepsilon_1 \to \infty$ and $\theta \to 0$); we also take $\sin^2\theta_W \approx 1/4$ which implies $a = 0$ in Eq. (3.20). [14] There is now an additional hadronic response tensor for the target

$$\begin{aligned} W_{\mu\nu}^{\text{int}} &\equiv \overline{\sum_i}\sum_f (2\pi)^3\delta^{(4)}(q+p'-p) \\ &\quad \times[\langle p|J_\nu^{(0)}(0)|p'\rangle\langle p'|J_\mu^\gamma(0)|p\rangle + \langle p|J_\nu^\gamma(0)|p'\rangle\langle p'|J_\mu^{(0)}(0)|p\rangle](\Omega E_p) \\ &= W_1^{\text{int}}(q^2, q\cdot p)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \\ &\quad + W_2^{\text{int}}(q^2, q\cdot p)\frac{1}{m^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right). \end{aligned} \tag{4.15}$$

In this approximation, the contributing parity-violating target response tensor arises entirely from the vector part of the hadronic weak neutral current; the full expression is given in Ref. [1].

A combination of these results now yields the parity-violating asymmetry

$$\mathcal{A}_{\text{pv}} = \frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow} = -\frac{Gq^2}{2\pi\alpha\sqrt{2}}\frac{\sum_i Q_i Q_i^{(0)}[f_i^\uparrow(x) + f_i^\downarrow(x)]}{\sum_i Q_i^2[f_i^\uparrow(x) + f_i^\downarrow(x)]}. \tag{4.16}$$

[14] It is assumed that in the nuclear domain $q^2 \ll m_W^2$.

Here $Q_i$ and $Q_i^{(0)}$ are the electromagnetic charges of the quarks, and the quark charges in the vector part of the weak neutral current, respectively.

**Quark Currents.** To the extent that the $(u, d)$ masses are equal (they are both very small in the QCD lagrangian), strong isospin is a symmetry of QCD, and the $(u, d)$ quarks form a strong isodoublet

$$\begin{pmatrix} u \\ d \end{pmatrix} \equiv \psi . \tag{4.17}$$

The charge-changing weak current can then be rewritten as

$$\mathcal{J}_\lambda^{(-)} = i\bar{\psi}\gamma_\lambda(1+\gamma_5)\tau_-\psi \cos\theta_C . \tag{4.18}$$

The electromagnetic current in the extended domain of $(u, d, s, c)$ quarks is similarly written as

$$J_\lambda^\gamma = i\bar{\psi}\gamma_\lambda\left(\tfrac{1}{2}\tau_3 + \tfrac{1}{6}\right)\psi + \tfrac{2}{3}i\bar{c}\gamma_\lambda c - \tfrac{1}{3}i\bar{s}\gamma_\lambda s . \tag{4.19}$$

It is evident that in terms of isospin (denoted with superscripts), this current has the structure

$$J_\lambda^\gamma = J_\lambda^{V_3} + J_\lambda^S . \tag{4.20}$$

It is convenient to remove the Cabibbo angle from the hadronic charge-changing current in nuclear physics and include it in the coupling constant, for then the various components of the weak and electromagnetic currents bear a simpler relation to each other. We shall henceforth do so, and we use as the coupling constant for the charge-changing hadronic processes

$$\begin{aligned} G_\pm &\equiv G\cos\theta_C \\ &= 0.974\,G . \end{aligned} \tag{4.21}$$

Since $\tau_- = (\tau_1 - i\tau_2)/2$, the charge-changing weak hadronic current now has the isospin structure

$$\mathcal{J}_\lambda^{(-)} = \mathcal{J}_\lambda^{V_1} - i\mathcal{J}_\lambda^{V_2} . \tag{4.22}$$

In this standard model, the weak current is evidently a sum of a Lorentz vector and Lorentz axial vector current

$$\mathcal{J}_\lambda = J_\lambda + J_{\lambda 5} . \tag{4.23}$$

The Lorentz vector part of the weak charge-changing current is simply a different spherical component of the isovector part of the electromagnetic

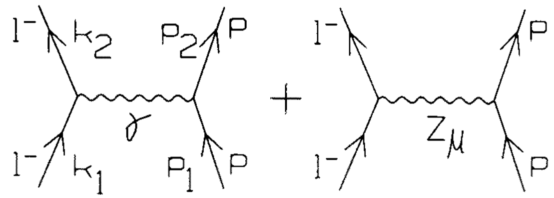


Fig. 4. One photon exchange in $(e, e')$ on the nucleon, together with the exchange of the heavy, weak neutral vector meson $Z^0$. Here $p_1 = p$ and $p_2 = p'$.

current — this is one of the foundations of the conserved vector current theory of the weak interactions (CVC).

Consider again, in addition to one photon exchange in $(e, e')$ on the nucleon, the effects of the exchange of the heavy, weak neutral vector meson $Z^0$ in the standard model as illustrated in Fig. 4. The amplitude for $Z^0$ exchange in the nuclear domain is proportional to the weak coupling G, and this amplitude is negligible unless one looks for effects which are only there due to the presence of the weak interaction; parity violation is one such effect. The S-matrix for the process in Fig. 4 is given by [1]

$$\begin{aligned} S_{\mathrm{fi}} &= -\frac{(2\pi)^4 i}{\Omega}\delta^{(4)}(k_1 + p - k_2 - p')T_{\mathrm{fi}}\,, \\ T_{\mathrm{fi}} &= \frac{4\pi\alpha}{q^2}\left\{ i\bar{u}(k_2)\gamma_\mu u(k_1)\langle p'|J_\mu^\gamma(0)|p\rangle \right. \\ &\quad \left. -\frac{Gq^2}{4\pi\alpha\sqrt{2}} i\bar{u}(k_2)\gamma_\mu(a + b\gamma_5)u(k_1)\langle p'|\mathcal{J}_\mu^{(0)}(0)|p\rangle \right\}. \end{aligned} \tag{4.24}$$

In the standard model the lepton couplings are given by

$$a = -(1 - 4\sin^2\theta_{\mathrm{W}})\,, \qquad b = -1\,. \tag{4.25}$$

The hadronic weak neutral current in the standard model in the extended domain of $(u, d, s, c)$ quarks is given in terms of the quark field in Eq. (4.17) by

$$\begin{aligned} \mathcal{J}_\mu^{(0)} &= i\bar{\psi}\gamma_\mu(1 + \gamma_5)\frac{1}{2}\tau_3\psi - 2\sin^2\theta_{\mathrm{W}} J_\mu^\gamma + \delta\mathcal{J}_\mu^{(0)}\,, \\ \delta\mathcal{J}_\mu^{(0)} &= \frac{i}{2}[\bar{c}\gamma_\mu(1 + \gamma_5)c - \bar{s}\gamma_\mu(1 + \gamma_5)s]\,. \end{aligned} \tag{4.26}$$

The last term is an isoscalar.

**An Application.** Consider the nuclear domain of $(u, d)$ quarks, for which $\delta\mathcal{J}_\mu^{(0)}$ is absent, and an isoscalar $T = 0 \to T = 0$ nuclear transition. In this case, the entire isoscalar part of the weak neutral current arises from the electromagnetic current, and nuclear strong isospin symmetry yields the proportionality, for these transitions

$$\mathcal{J}_\mu^{(0)} \doteq -2\sin^2\theta_{\rm W} J_\mu^\gamma \,. \tag{4.27}$$

Note that this relation holds to *all orders in QCD.*

In this case, Eq. (3.29) takes the simple form (recall $b = -1$)

$$\mathcal{A} = \frac{Gq^2}{\pi\alpha\sqrt{2}}\sin^2\theta_W \,. \tag{4.28}$$

This result is originally due to Feinberg [24]. In the *extended domain* of $(u, d, s, c)$ quarks, the additional isoscalar contribution $\delta\mathcal{J}_\mu^{(0)}$ in Eq. (4.26), involving the strangeness and charm currents in the nucleus, modifies this relation [1].

Also, in this case, the entire parity violation arises from the axial vector part of the lepton current (proportional to $b$), and for parity-violation purposes, the T-matrix takes the form

$$\begin{aligned} T_{\rm fi} &\doteq \frac{4\pi\alpha}{q^2} i\bar{u}(k_2)\Gamma_\mu u(k_1)\langle p'|J_\mu^\gamma(0)|p\rangle \,, \\ \Gamma_\mu &\equiv \gamma_\mu\left(1 + \frac{Gq^2\sin^2\theta_{\rm W}}{2\pi\alpha\sqrt{2}} b\gamma_5\right) . \end{aligned} \tag{4.29}$$

This expression readily yields the equivalent, effective parity-violating lepton potential in an *atom.*

## 5. CEBAF (TJNAF)

The second and third talks in this lecture series present an overview of the current status of electron scattering, including a description of CEBAF. This material appears in the published literature in Ref. [3–5].

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