SKYRMIONS IN NUCLEI* **

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(Received July 10, 1997)

Dedicated to Wiesław Czyż on his seventieth birthday

The applications of skyrmions to the derivation of the nucleon–nucleon force are now over a dozen years old, and this occasion is used to assess the degree of success of the endeavor. A very brief review is given of the use of skyrmions for determining single-baryon properties. Then their use for two-nucleon systems is described, with attention to the use of the product ansatz, the full structure of the lagrangian, baryon resonance admixtures, dilatons, and exact solutions for the $B = 2$ system in order to find the sources of attraction in the central potential. We briefly address possible insights into the behavior of the nucleon in nuclei achieved from the skyrmion approach.

PACS numbers: 12.39. Dc

* Based on lectures delivered by J.M. Eisenberg at the XXXVII Cracow School of Theoretical Physics, Zakopane, Poland, May 30–June 10, 1997.

** This work was supported in part by the Israel Science Foundation, the Deutsche Forschungsgemeinschaft, and the Yuval Ne'eman Chair at Tel Aviv University.

1. Introduction: the application of skyrmions in nuclear physics

Efforts have been made to apply the concept of the skyrmion [1] to the derivation of the nucleon–nucleon force for something like a dozen years now, since the early work of Jackson and his collaborators [2] and of Vinh Mau and his coworkers [3]. At least in part, this has been done with an eye to eventual broader applications of the skyrmion in nuclear physics. The topics involved have been reviewed in sufficient length and detail [4–15] so that further general summary seems out of order. Instead, we shall try here to give a brief sketch of the general background of this work, and then to provide an assessment of its success. This seems quite appropriate in lectures in a summer school celebrating the seventieth birthday of Wiesław Czyż. His contributions in physics have covered a broad spectrum from his early interests in nuclear structure as described by nucleon degrees of freedom to recent work in which quantum chromodynamics is applied to nuclei. With his general guidance, nuclear theory activities in Kraków have encompassed a similar range, including specific interest in skyrmions.

The philosophy of the approach is well known, and stems from the realization that in the limit of a large number of colors N_c, quantum chromodynamics becomes a theory whose degrees of freedom can be taken to be those of meson fields [16], and baryons can be viewed as arising from topological solutions for those fields [17]. In this last context, the lagrangian put forward many years ago by Skyrme [1] represents the simplest way to achieve stable topological solutions. The application to nuclear physics is particularly tempting because the nucleus requires for its minimal description degrees of freedom pertaining to nucleons, pions, and Δ isobars, all of them well handled by the skyrmion. In this regard the skyrmion represents a distinct advantage here over approaches based on bag models, since the essential pion aspects are included as an integral part of the theory.

2. Skyrmions for the single-nucleon system

The lagrangian density originally proposed by Skyrme [1] for the depiction of the nucleon through a solution in terms of meson fields with nontrivial topological properties is

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 = -\frac{F_\pi^2}{16}\,\mathrm{tr}\,(L_\mu L^\mu) + \frac{1}{32e^2}\,\mathrm{tr}\,[L_\mu, L_\nu]^2, \tag{1}$$

where F_π is the pion decay constant, with experimental value 186 MeV, and e is the Skyrme parameter which accompanies his stabilizing term with four derivatives. Here $L_\mu \equiv U^\dagger \partial_\mu U$, where $U(\boldsymbol{r}, t)$ is the chiral field, at this stage taken in SU(2). For the $B = 1$ static problem we take a time-independent hedgehog solution, $U(\boldsymbol{r}, t) = \exp[i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)]$, where $F(r)$ is referred to as the profile function.

As is well known, and is discussed in detail in the reviews referred to above, Skyrme's lagrangian guarantees stability of the single-nucleon solution. The requirement that the energy obtained from the Skyrme lagrangian remain finite leads to the boundary conditions $F(r \to \infty) \to n\pi$, $n =$ integer, and $F(0) = m\pi$, $m =$ integer for the profile function. Without regard to the particular form of the lagrangian, it is possible to construct a conserved current — not of the Noether variety — which Skyrme interpreted as the baryon current. It then emerges that $m - n$ is the baryon number of the corresponding solution.

Adkins, Nappi, and Witten [18] supplied a method of generating baryons of good internal quantum numbers for spin and isospin by carrying out a unitary rotation on the static hedgehog solution, $U(\boldsymbol{r}, t) = A(t)\, U_0(\boldsymbol{r})\, A^\dagger(t)$, where $A(t)$ is an SU(2) matrix, and then converting A into a quantized variable and using it to project out the quantum numbers needed. This then provides all the machinery necessary to construct single-nucleon observables: masses, electromagnetic radii and moments, the axial coupling constant, pion coupling constants, and $\Delta \to N$ transition moments. The usual procedure is to fit the two constants of Eq. (1) to the nucleon and Δ masses and then to calculate the remaining observables in terms of them. These prove to be in reasonable agreement with experiment, given that the theory strives for only 33 percent accuracy since it takes $N_c \to \infty$ from the start.

Subsequently, extensions of these methods to include the strangeness sector (see, *e.g.*, [19,20] and the many references in the reviews quoted above) were made quite successfully. This work assumed special meaning when experimental studies of the spin content of the nucleon were undertaken [21] and found to yield a very small value for that quantity. The skyrmion has the property of suggesting zero strangeness content for the nucleon [22], at least at the level of the lagrangian of Eq. (1). The subsequent finding [23] that the nucleon spin content — while not very large — was distinctly different from zero, required modifications to that lagrangian [24,25]. These take the form of additional terms involving six derivatives of the field variables,

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{6,1} + \mathcal{L}_{6,2} + \mathcal{L}_{SB} \\
&= -\frac{F_\pi^2}{16}\,\mathrm{tr}\,(L_\mu L^\mu) + \frac{1}{32e^2}\,\mathrm{tr}\,[L_\mu, L_\nu]^2 \\
&\quad -\epsilon_1 \frac{g_\omega^2}{m_\omega^2}\,\mathrm{tr}\,(\mathcal{B}^\mu \mathcal{B}_\mu) - \epsilon_2 \frac{g_\omega^2}{2m_\omega^2}\,\mathrm{tr}\,(\mathcal{B}^\mu)\,\mathrm{tr}\,(\mathcal{B}_\mu) \\
&\quad + \left[\frac{F_\pi^2}{32}(m_\pi^2 + m_\eta^2)\,\mathrm{tr}\,(U + U^\dagger - 2)\right. \\
&\quad \left. + \frac{\sqrt{3}F_\pi^2}{24}(m_\pi^2 - m_K^2)\,\mathrm{tr}\,(\lambda_8(U + U^\dagger))\right], \qquad (2)
\end{aligned}$$

where we have suppressed the Wess–Zumino term which also enters here. In this expression,

$$\mathcal{B}^{\mu} \equiv \frac{\epsilon^{\mu\alpha\beta\gamma}}{24\pi^2}\left[\left(U^{\dagger}\partial_{\alpha}U\right)\left(U^{\dagger}\partial_{\beta}U\right)\left(U^{\dagger}\partial_{\gamma}U\right)\right]. \tag{3}$$

The inclusion of terms with six field derivatives may seem excessive, but it has been pointed out [26] that a chiral expansion is inadequate if truncated at the level of four derivatives since one must expect the fourth-order contribution to be much less than the second-order one for that expansion to be valid, and this is violated by the energy contibutions themselves from the two terms of Eq. (1), as can easily be shown using a length-scaling arugment as in Derrick's theorem.

In order to give some impression of the success of the skyrmion in SU(3), we show several results based on the lagrangian of Eq. (2). Table I shows a sampling of possible parameters for the constants in the lagrangian, along with the masses for the baryon octet and decuplet and the root mean-square deviation of these from experiment.

TABLE I

Octet and decuplet masses in MeV. Members of octet in first line of entry; decuplet in second. Cases 1–5: $g_A \sim 1.24 \pm 0.05$; Case 6: $g_A = 1.72$.

#	Params			M				dev
	e	F_π	ϵ_1	N/Δ	Λ/Σ	Σ/Ξ	Ξ/Ω	
1	6.0	82	-1.00	938	1111	1240	1366	98
				1226	1336	1485	1649	
2	6.0	74	-1.50	936	1114	1246	1375	120
				1234	1338	1469	1624	
3	6.0	89	-0.35	928	1124	1271	1402	131
				1233	1342	1488	1659	
4	6.6	82	-0.71	933	1108	1238	1364	99
				1249	1351	1480	1633	
5	5.4	82	-1.26	919	1119	1272	1410	159
				1203	1317	1469	1643	
6	8.0	186	-1.45	939	1120	1253	1387	120
				1236	1344	1476	1634	
exp				939	1116	1193	1318	
				1232	1385	1530	1672	

The agreement over this large set of masses is excellent, although, of course, much of this may be accounted for by the correct incorporation of SU(3) symmetry. The same is true for the octet magnetic moments, shown in Table II,

TABLE II

Octet magnetic moments in units of μ_N and proton spin content.

#				μ				dev	$\Delta\Sigma$
	p	n	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-		
1	2.65	-1.73	-0.83	2.80	-1.10	-1.75	-0.89	0.73	0.17
2	2.80	-1.77	-0.90	2.98	-1.12	-1.86	-0.99	0.93	0.34
3	2.54	-1.72	-0.75	2.64	-1.13	-1.65	-0.79	0.58	0.04
4	2.49	-1.57	-0.77	2.65	-1.02	-1.62	-0.88	0.69	0.14
5	2.87	-1.98	-0.88	3.01	-1.24	-1.90	-0.89	0.93	0.17
6	2.33	-1.76	-0.78	2.42	-0.97	-1.65	-0.65	0.68	0.58
exp	2.79	-1.91	-0.61	2.46	-1.16	-1.25	-0.65		0.27 ± 0.13

and, indeed, the spin content is also within the range of the acceptable (last column in Table II). In the following, we shall make use of the lagrangian of Eq. (2) — note that in the SU(2) case the two terms $\mathcal{L}_{6,1}$ and $\mathcal{L}_{6,2}$ become equivalent — along with the further attractive term

$$\mathcal{L}_{4s} \equiv \frac{\gamma}{8e^2}\left[\mathrm{tr}\,\left(\partial^\mu U\,\partial_\mu U^\dagger\right)\right]^2 , \tag{4}$$

where γ is a new phenomenological parameter. This expression introduces terms with four time derivatives into the lagrangian, thus confusing the usual quantization procedures. It also tends to destabilize the skyrmion if γ is taken large enough so as to overwhelm the other, repulsive terms $\mathcal{L}_4$, $\mathcal{L}_{6,1}$, or $\mathcal{L}_{6,2}$. We thus entertain the expression of Eq. (4) here reluctantly as perhaps having some limited phenomenological purpose. For the discussion of NN forces they are of some interest as possible sources of additional central attraction. In the context of single-nucleon uses of the skyrmion, they serve to stress the peculiar property of the skyrmion that it has a leg in each of two camps as it were: on the one hand, the Skyrme lagrangian is originally defined in terms of meson fields, and on the other it applies to baryons through the topological aspect of the skyrmion solution. Thus one can extract [27] from $\pi\pi$ scattering data definite values for the parameters in the lagrangian $\mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{4s}$. Taking $F_\pi = 186$ MeV, one arrives at $\gamma = 0.16 \pm 0.04$ and $e = 5 \pm 2$, from which one can "predict" a nucleon mass $M_N = 880 \pm 300$ MeV, where the error noted is that arising only from uncertainties in the $\pi\pi$ data.

Before proceeding to a discussion of two-nucleon systems, is worthwhile to make explicit certain ambiguities that are bound to accompany almost any discussion of applications of skyrmions to baryons: At the level of SU(3), it proves necessary to make a large subtraction ($\sim$ 500 to 1000 MeV) of a state with the quantum numbers of the vacuum [19], so that in effect one is looking at mass splittings rather than at absolute positions. In a somewhat different approach [28], the need for an even larger subtraction is justified in terms of ambiguities in operator ordering within the lagrangian that can express themselves as a large constant term in the energy. And of course there is the question of the skyrmion Casimir energy, which is estimated [26, 29] to be quite large ($\sim$ 1000 MeV, again with large uncertainties). Thus there seems to be no possible hard and fast rule for fixing skyrmion parameters. In practice one is more or less limited to using a mixture of criteria based on masses and observables. The extreme cases that are weighted strongly in one direction or the other can then be seen as a measure of the inaccuracy of the overall approach.

3. Skyrmions for the NN system with the product ansatz

In order to set sensible goals for the skyrmion as applied to two-nucleon systems, it is important to keep in mind the advantages and the intrinsic limitations of the skyrmion approach as well as the overall expectations for our understanding of the nature of the nucleon–nucleon force. As we have noted, the skyrmion is not expected to be reliable at a level much better than 30 percent, which certainly rules it out as a method for precision calculation of NN potentials for confrontation with experimental data. This restriction is made more acute by two further concerns: The skyrmion is basically directed at the energy scale of baryons, say 1000 MeV, whereas the NN force involves features that are typically one to two orders of magnitude smaller in energy and thus may well arise from differences between much larger energy quantities, resulting in a loss of precision in the calculation. In addition, the skyrmion is a low-energy theory; it involves an expansion in derivatives, or momenta, along the lines of that used in chiral perturbation theory; short-range features of the nucleon–nucleon interaction will surely not be well represented by it.

Thus in turning to applications of the skyrmion to the NN system we expect from it insights into the qualitative features of the NN force, and we therefore wish first to sketch the kinds of NN issues that should be addressed with it. For many years studies of the NN force have been based on a picture of meson exchanges [30,31], which in more modern versions [32] also incorporates notions of chiral dynamics. The most solidly based aspect of meson exchange in the nucleon–nucleon force is of course that of one-pion exchange (OPE). As the lightest of the mesons, the pion dominates the long-range aspect of the NN potential and its role is easily verified in high-energy peripheral NN collisions. This is the feature that the skyrmion handles easily. Since the description of the pion as a Goldstone boson in QCD is complicated, it is advantageous to be able to isolate it before going on to other meson exchanges.

One-pion exchange is the major effect in deuteron binding but plays only a small role in heavier nuclei because its effects involve both spin and isospin variables which are quenched in nuclear systems with $B \geq 4$. There the crucial ingredients are the interplay between central attraction at intermediate NN separation distances and short-range repulsion. In early times, the former was thought to be provided by the exchange of a scalar σ meson, but when such a particle failed to turn up in experiment in the mass region (about 500 to 600 MeV) required to deal with intermediate ranges ($\sim$ 2 fm), a largely successful interpretation was developed in terms of two-pion exchange with intermediate states involving Δs. Short-range repulsion in the NN system was attributed to ω-meson exchange. This approach to the NN force based on two-pion exchange for attraction and omega-exchange for repulsion may well survive as supplying major ingredients for our understanding of that force, but it also seems likely that both pieces of the picture will be supplemented in an important degree by quark dynamics. Indeed it would be hard to understand how quark degrees of freedom could fail to enter in some measure at nuclear separation distances of 2 fm or less, where the individual nucleons are beginning to have sizable spatial overlap. In fact in the nonrelativistic quark model a significant source of NN central attraction at intermediate ranges has been found [33] in the form of mutual excitation of the two three-quark clusters that represent the baryons in the model. Such mechanisms must thus be entertained as possible supplements to two-pion exchange in producing central attraction. Similarly nonrelativistic quark models suggest [34] auxiliary sources of short-range repulsion stemming from the action of the color-magnetic force in QCD. This favors the occupation by quarks of the 2s states whose higher energy produces a repulsive effect at short ranges.

Against this background it is important to see what the skyrmion has to offer by way of descriptions of central attraction and short-range repulsion. The energies involved in the two-nucleon system at very short distances exceed by 1000 MeV or more the combined rest-mass energies of the baryons in these systems, and so are to be interpreted as large repulsion for small internucleon separations. Thus at the most fundamental level the main challenge for the skyrmion as applied to two-baryon systems is to offer a source of central attraction; this is the question that we shall address in the next sections. Eventually one would hope to study the skyrmion also for whatever insights it may offer into nucleon behavior in larger nuclear systems and in nuclear matter, topics to which we turn in Section 6.

3.1. The product ansatz

From the very first stage of the introduction of his topological soliton, Skyrme was interested [1] in possible applications to the two-nucleon system. Towards this end he suggested the use of the product ansatz, in which the two-baryon system is represented by a topological field which is the product of two single-baryon fields,

$$U_{B=2}(\boldsymbol{r}, \boldsymbol{R}) = U_{B=1}(\boldsymbol{r} + \tfrac{1}{2}\boldsymbol{R})\, U_{B=1}(\boldsymbol{r} - \tfrac{1}{2}\boldsymbol{R}), \tag{5}$$

where $\boldsymbol{r}$ is the general skyrmion variable and $\boldsymbol{R}$ is the separation between the two baryon centers. This ansatz has the virtues that it automatically fulfills the correct baryon number condition $B = 2$ and has well-defined baryon centers by its very construction, in contrast to other approaches we shall encounter below. It also allows for simple physical interpretation when applied in the sense of a London–Heitler calculation in $B = 2$ systems.

It is crucial for the study of the NN system that we project the product ansatz onto nucleon states, if for no other reason than that the dynamic admixture of Δs is, as noted above, often adduced as an important ingredient in providing central NN attraction, and thus we must be able to separate the nucleon from the Δ in addressing the NN force. This projection is accomplished by rotating each member of the product ansatz separately according to

$$U_{B=2}(\boldsymbol{r}, \boldsymbol{R}) = \left[A\, U_{B=1}(\boldsymbol{r} + \tfrac{1}{2}\boldsymbol{R})\, A^{\dagger}\right] \left[B\, U_{B=1}(\boldsymbol{r} - \tfrac{1}{2}\boldsymbol{R})\, B^{\dagger}\right], \tag{6}$$

where A and B are the rotation matrices of Section 2.

The product ansatz may now be used [2,3] in a London–Heitler technique by choosing a separation distance $\boldsymbol{R}$ and calculating the resulting two-nucleon energy from the lagrangian, taking care to subtract the masses of the two nucleons (*i.e.*, the energy at separation $\boldsymbol{R} \to \infty$). This requires the evaluation of integrals over the variable $\boldsymbol{r}$ and averaging over the rotation variables in expectation values formed with the nucleon wave functions of the rotationally averaged skyrmions. Details are provided in the review of Ref. [7]. At this level, and using the basic lagrangian of Eq. (1), the skyrmion yields three components of the NN potential, namely,

$$V(\boldsymbol{R}) = V_C(R) + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \, V_{SS}(R) + S_{12} \, V_T(R)\right], \tag{7}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the Pauli spin and isospin operators and S_{12} is the usual tensor operator $S_{12} \equiv 3\,(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{r}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$. The three components of the force of Eq. (7) are the central part $V_C(R)$, the spin–spin part $V_{SS}(R)$, and the tensor piece $V_T(R)$. These last two are dominated by OPE, and so should be well described by the skyrmion, as indeed they prove to be [7] when compared with phenomenological spin–spin and tensor components of the NN force. The central potential is highly repulsive in its behavior for $R < 1$ fm, and no trace of attraction is found in that component for $1 \text{ fm} < R < 2$ fm, where one knows it must occur from phenomenological potentials. In some sense it appears that the repulsion introduced through $\mathcal{L}_4$ in Eq. (1) to stabilize the skyrmion spreads beyond the 1 fm range and overcomes any possible attraction. This occurs because the lagrangian is governed by two parameters which must account for baryon masses on the scale of 1000 MeV and baryon sizes on the scale of 1 fm. The smooth tail of the baryon profile then carries sizable repulsion out to the 2 fm range.

3.2. Baryon resonance admixtures

It has long been accepted wisdom [30] that the attraction in the central NN potential represented by so-called σ-exchange in fact arises because of two-pion exchanges which can simulate the σ effects. In the application of box graphs of two-pion exchange to the construction of the NN potential, at least one of the intermediate baryon lines refer to excited states and not to the nucleon itself, because, in the use of the Schrödinger equation to solve the two-nucleon problem, one includes all iterations of graphs with two-nucleon lines, so that the construction of the potential itself must omit these in order to avoid double counting. The physical picture is then that at intermediate ranges the two-nucleon system is partly an $N\Delta$ or $\Delta\Delta$ system or indeed some other $B = 2$ system containing higher excitations of the nucleon. This suggests that in using the skyrmion for the derivation of the NN potential one should consider the admixture of $N\Delta$ and $\Delta\Delta$ states [35] into the wave function,

$$|\Psi(R)\rangle = \alpha(R)|NN(R)\rangle + \beta(R)|N\Delta(R)\rangle + \gamma(R)|\Delta\Delta(R)\rangle. \tag{8}$$

Here we once again have in mind the use of the London–Heitler technique. That is to say, the calculation [35] proceeds by fixing the internucleon separation distance R and then minimizing the system energy with respect to changes in the admixture coefficients $\alpha(R)$, $\beta(R)$, and $\gamma(R)$ at the value of R in question. For this mechanism to yield appreciable intermediate-range attraction, it is necessary [36] to treat carefully the action of the tensor force in admixing spin states. Even then only very weak attraction (~ 2 to 3 MeV) — and that only for extreme skyrmion parameters which tend to overaccentuate the role of the Δ — is achieved [36,37].

The picture in this regard is improved considerably if one entertains the admixture of the Roper $N(1440)$ resonance by extending Eq. (8) to include these. The initial motivation to consider this mechanism for central attraction in the NN force arose [38] from the realization that a similar device appears importantly in deriving central attraction in the nonrelativistic quark model, where it was found [33] that the internal excitation of the individual three-quark clusters making up each nucleon had to be allowed in order to obtain a reasonable result for $V_C(R)$. The description of the Roper excitation as a vibrational state used here is based on the specific approach of Ref. [39]. The hedgehog ansatz for the individual baryon is written as $U_\lambda(\boldsymbol{r}) = U_0(\boldsymbol{r}\,\mathrm{e}^{\lambda(t)})$, where the time-dependent scaling parameter $\mathrm{e}^{\lambda(t)}$ allows for a breathing-mode vibration of the hedgehog. This new ansatz is then substituted into the skyrmion lagrangian and the time-dependent terms are analyzed in a harmonic approximation for λ together with the rotational prescription of Section 2.

With reasonable parameters, the extension of Eq. (8) to include the admixture of these two states then yields about 5 MeV of attraction in $V_C(R)$, which is a not unreasonable well depth given the fits of modern NN potentials such as that of Paris [40] or of Argonne [41] with well depths of 5 to 20 MeV. Furthermore, it has been noted by several authors that one should make at least a minimal, algebraic correction for working with $N_c \to \infty$ in the skyrmion approach [2,42–45]. The correction in question acts to increase somewhat the central NN attraction found with the skyrmion, increasing it by a factor of $((N_c+2)/N_c)^2 = 25/9$.

There is also a certain satisfaction in the fact that the skyrmion and the nonrelativistic quark model find that the same basic feature yields attraction, namely, the internal excitation of each baryon participating in the interaction between the two. This qualitative feature drives home the intrinsic subtlety of the origins of central attraction in two-baryon systems. It also hints at a finding of the exact skyrmion calculations (Section 5): there too the departure of the skyrmion at short internucleon separations from its structure at $R \to \infty$ is what yields attraction, so that at least at a qualitative level many of the different approaches may in fact be convergent.

4. The dilaton

It is well known (see, *e.g.*, [46, 47]) that QCD with massless quarks is scale invariant when treated as a classical theory and loses this property upon quantization. Accompanying this is the trace anomaly, whereby the trace of the energy-momentum tensor $T^{\mu\nu}$, which vanishes classically, is given after quantization by

$$T^\mu_\mu = \partial_\mu D^\mu = -\frac{9\alpha_s}{8\pi} G^a_{\mu\nu} G^{a\mu\nu} \equiv \psi^4, \tag{9}$$

where $D^\mu (= T^{\mu\nu} x_\nu)$ is the dilatation current, α_s is the QCD coupling constant, $G^a_{\mu\nu}$ is the gluon field, and ψ is an order-parameter field — the dilaton — which represents the scalar glueball formed from the contraction of the two gluon fields. It was early pointed out that an effective lagrangian based on this scalar field can be used to incorporate the trace anomaly [48], and the consequences of such an approach for scalar glueballs were studied some time thereafter [49], along with modifications to the Skyrme lagrangian in order to incorporate there first the appropriate scale invariance and then the trace anomaly [50, 51]. The physical picture for the skyrmion plus dilaton is that a kind of bag is formed by the dilaton field such that the skyrmion is located within a region where the gluon condensate is suppressed. It answers for one of the obvious limitations of the skyrmion as it stands alone, namely, that there is only one length parameter in the basic model, so that there is no well-defined concept of a "surface thickness" or baryon edge. The sharpening of the baryon surface will help to reduce the problem of a repulsive force that straggles out too far.

Schechter [48] and later his coworkers [49–51] introduced a lagrangian for the dilaton field through the consideration of the trace anomaly. If terms with no more than two time derivatives appear, the most general form allowed is

$$\mathcal{L}_{\text{dilaton}} = \mathcal{L}_{2\ \text{dilaton}} - V_{\text{dilaton}}(\psi) = \tfrac{1}{2} b^2 \partial_\mu \psi \partial^\mu \psi - \frac{1}{4}\psi^4 \log\left[\frac{\psi^4}{\Lambda^4}\right], \tag{10}$$

where b is a dimensionless constant and Λ is a constant with dimensions of energy.

Given the dilaton lagrangian, it is easy to modify [50, 51] the basic Skyrme lagrangian (1) in order to incorporate the trace anomaly, and, indeed, Ref. [52] finds some 35 MeV of attraction in the central NN potential for $r \sim 1.8$ fm. This was confirmed in later calculations that included also the spin–orbit force [53]. An example of these results is shown in Fig. 1; these results are to be multiplied by the factor of 25/9, discussed at the end of the last section, in order to include $1/N_c$ corrections.

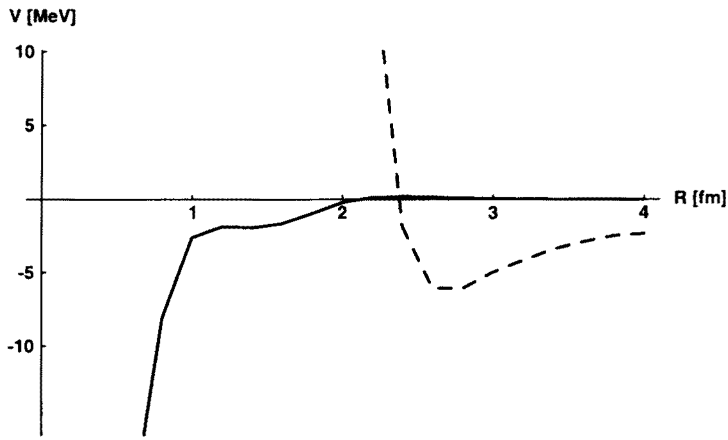

Fig. 1. Spin-orbit potential and central potential (dashed line for $r \geq 2$ fm) for skyrmion parameters $F_\pi = 143$ MeV and $e = 20.0$, and dilaton parameters $\Gamma_0 = 61.2$ MeV and $C_G = (157.3 \text{ MeV})^4$, yielding a glueball mass of 809 MeV. This result requires the presence of an $\mathcal{L}_{6,2}$ term and a dilaton.

5. The exact skyrmion solution for $B = 2$

Recently solutions of the full $B = 2$ skyrmion problem in three dimensions (or, exploiting the cylindrical symmetry of the problem, in two dimensions) have become increasingly available. The review of Walhout and Wambach [14] devotes itself in considerable degree to this topic, in which these authors have themselves been very much involved. Early exact numerical solutions [54–57] for the $B = 2$ system were obtained by relaxing the full static equations of motion on a grid within a particular geometry. These solutions have a toroidal shape, the hole in the middle being, presumably, the reflection of short-range repulsion. They do not show any residual signs of the presence of two separate nucleons, which raises the criticism noted by Manton [58] that this solution does not allow sufficient degrees of freedom to describe correctly the complete dissolution into two baryons.

Subsequent exact numerical studies were usually based on an assumption of adiabaticity, whereby one introduces constraints to fix the collective coordinates and solves the static equations of motion. The fixing of coordinates pertains both to the relative orientation of the skyrmions and to the distance between their centers. The latter may be determined, say, through Lagrange multipliers, and various relative orientations between the two skyrmions are chosen. The solutions [14, 59] then show two separated nucleons which coalesce at about 1.2 fm for the maximally attractive relative orientation of the skyrmions, and then meld into a toroid below that "separation" distance.

Central attraction may be found [60–62] in this approach when $N\Delta$, ΔN, and $\Delta\Delta$ states are admixed, along the lines discussed in Subsection 3.2 in the context of the product ansatz. Attraction is also augmented by incorporating [63] the dilaton, in which case the "bags" provided by the dilaton merge at about 1.4 fm, and an attractive central component on the scale of 20 MeV is found. This is further enhanced by again considering the admixture of states containing Δs.

The ability to carry out these difficult calculations is encouraging, and the results provide our current best understanding regarding the mechanism of central attraction in two-nucleon interactions in the skyrmion approach. They also lead to a reliable assessment as to which ingredients are essential to obtain central attraction: the admixture of excited states of the nucleon and, possibly, the dilaton device (see, however, Ref. [64]). The connections with others approaches should be pursued further in the hope of eventually constructing a model simple enough for application in more complex nuclear situations.

Quite recently Leese, Manton, and Schroers [65] have produced a skyrmion solution for the deuteron carried out on a ten-dimensional manifold which yields properties close to those of the physical deuteron. It should be emphasized that this is the first case of a realistic representation of the deuteron within the skyrmion approach. The NN system should ideally be calculated on a twelve-dimensional manifold in order to include full freedom for the locations of the two skyrmions and for their SU(2) orientations. This calculation still proves prohibitively difficult, however, and so Ref. [65] uses the lowest-dimensional manifold that still includes both a toroidal configuration of minimal energy and configurations that approximate the infinite separation of two skyrmions oriented in the maximally attractive relative direction in SU(2). To perform the calculation, an instanton assumption is again made and the maximally attractive orientation — rotation by π about the axis perpendicular to the line joining the skyrmions — is taken. The geometry that underlies the instanton is somewhat complicated in order to accommodate the limiting toroidal and separated configurations. The technical aspect of the calculation then involves evaluating the lagrangian after restricting to that geometry, and generating the hamiltonian with first-quantized collective degrees of freedom. Numerical values for deuteron properties in this approach and in that of Ref. [57] are shown in Table III, from which it is seen that results from the ten-dimensional manifold M_{10} are far closer to those of experiment than are the ones arising [57] from an eight-dimensional one M_8, because the larger dimensionality is necessary in order to allow the skyrmions to sample a space that allows also for genuine separation of the two nucleon centers.

TABLE III

Deuteron properties. Here M_{10} refers to a ten-dimensional manifold and M_8 to an eight-dimensional one; ϵ_D is the deuteron binding energy, r_c is the charge radius, Q is the quadrupole moment, and μ the magnetic moment. (From Ref. [65].)

Property	M_{10} (Ref. [65])	M_8 (Ref. [57])	Experiment
ϵ_D [MeV]	−6.18	−158	−2.225
r_c [fm]	2.18	0.92	2.095
Q [fm^2]	0.83	0.082	0.2859
μ [n.m.]	0.55	0.74	0.8574

6. The nucleon in the nucleus

In this section we wish briefly to hint at possible applications of skyrmions to problems of a baryon in a medium which are beginning to become accessible now that the two-baryon problem is more or less under control. The cases noted all relate to problems of active interest in nuclear physics at large. (We do not here address problems involving skyrmion at nonzero temperatures which are of relevance to the study of hot hadronic matter and of the quark-gluon plasma.)

6.1. Color transparency

There has been great interest in recent years (reviewed, *e.g.*, in Ref. [66]) in studying the phenomenon of color transparency, in which a nucleon is ejected from the nucleus with very high momentum transfer and experiences little final-state interaction as it exits: The high momentum transfer catches the nucleon in a small configuration, and, because the color charges of a small QCD object partially neutralize each other, the nucleon emerges while interacting weakly with the other nucleons. Thus color dynamics makes the nucleus transparent to the "small" nucleon (referred to here as a minusculon m). The crucial QCD ingredient in this process is precisely the mutual color neutralization, and thus it is of interest to examine this in QCD-based models. The skyrmion allows one to study this [67] by artificially shrinking the skyrmion and examining the consequent mN potential and total mN cross section. The shrinking is accomplished by using the scaling of Eq. (1) and taking values for the minusculon such that its inverse size parameter $(eF_\pi)_m$ becomes large. The ratio of minusculon size to the size of a normal nucleon is then $x \equiv r_m/r_N = eF_\pi/(eF_\pi)_m$. The result of the skyrmion calculation using the product ansatz is that the central potential tends to zero more or less linearly with x for $x \leq 0.3$. On the other hand, the range of the potential tends to fall very gradually until $x \sim 0.05$ and then plunges rapidly to zero. The total NN cross section, in a rough high-energy approximation, falls linearly to zero as x drops from unity to zero, showing quadratic behavior only when x is below 0.3.

6.2. Nucleon shrinking and swelling

There has been intense interest over the years [68–74] in a basic question of nucleon behavior in nuclei: Does the nucleon shrink or swell when placed in the nuclear medium? A related question is, Does the nucleon shrink or swell when in interaction with a second nucleon? Of course, if the skyrmion is used with the simple product ansatz the nucleon will surely

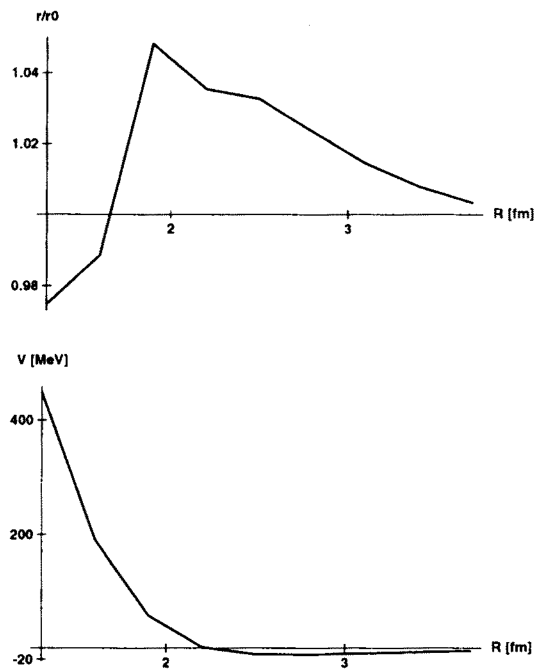

Fig. 2. The NN potential and ratio of the radius for the interacting nucleon to that of the free nucleon r/r_0 for the usual skyrmion with product ansatz and baryon-resonance admixtures. The parameters are taken to have the values $F_\pi = 130$ MeV, $e = 20$, $\gamma = 0.50$, $\epsilon_{6,2} = 2.58$, and $m_\pi = 139$ MeV, yielding the masses $M_N = 998$ MeV, $M_\Delta = 1211$ MeV, and $M_{N^*} = 1270$ MeV for the nucleon, Δ, and Roper.

shrink in a two-nucleon system, since we have already seen that this simple prescription leads to repulsion. However, when mechanisms for attraction are present, such as the admixture of higher baryon resonances (Subsection 3.2) or the inclusion of a dilaton interaction (Subsection 4), this situation changes, and one would expect a slight swelling for nucleons separated by 2 to 3 fm, followed by shrinking as they are brought closer together. This is indeed what is found in detailed calculation [75], as can be seen in Fig. 2. The swelling effect is on the scale of 3 or 4 percent, which may be sufficient to explain present EMC results [76]; below about 1.5 fm the nucleons act to reduce each other's size. Such effects should show up as changes in the form factor of the nucleon within the nucleus for different ranges of momentum within the Fermi sea, and one may hope to see such effects in high-energy electron scattering, for example at CEBAF.

7. Summary

The first decade of studies of the two-baryon system with skyrmions has yielded an understanding of several different mechanisms that may contribute to central attraction and to the attractive isoscalar spin–orbit force. Those which appear to be of lasting relevance are the admixture of nucleon resonance states, especially the $\Delta(1232)$ and the $N(1440)$, and the presence of the dilaton. In all likelihood, several candidates for this mechanism may enter together. This strengthens the one overarching conclusion to be reached from QCD-related studies of the NN interaction so far, namely, that central attraction — however crucial it may be for nuclei to exist as we know them — is the result of an extremely subtle play of QCD effects.

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