

# THE GLUON CONDENSATE IN QCD AT FINITE TEMPERATURE\*

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We begin with the discussion of the relationship between the trace of the energy momentum tensor and the gluon condensate at finite temperatures. Using the recent numerical data from the simulations of lattice gauge theory for quantum chromodynamics (QCD) we present the computational evaluations for the gluon condensate. A short discussion of the properties of deconfinement and the implications on the high temperature limit are included. We also mention the case of the massive quarks where some of the properties of the condensate appear to change. We put together these results with some ideas related to the dilatation current. We draw the conclusion that the nature of the strong interactions implies that the thermodynamics of quarks and gluons never approach even at very high temperatures that of an ideal ultrarelativistic gas.

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## 1. Introduction

*Out of the present findings based on previous knowledge new science is revealed.*

This statement particularly pertains to the recent experimental results on the Bose–Einstein condensation of atomic alkali vapors [1]. Although this talk will not be directly related to these results, it draws much inspiration from them.

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In this presentation we discuss the consequences for the gluon condensate at finite temperature of the recent high precision lattice results [2,3] for the equation of state in lattice gauge theory. We compare these results with various other calculations of the expected high temperature behavior. The essential relationship is the trace anomaly that is due to the scale variance of quantum chromodynamics (QCD). It relates the trace of the energy momentum tensor to the square of the gluon field strengths through the renormalization group beta function. Here we shall expand upon the approach investigated in [4], for which the consequences of the new finite temperature lattice data for  $SU(N_c)$  gauge theory for the gluon condensate [5] have been presented.

## 2. The trace anomaly at finite temperature

The idea of the relationship between the trace of the energy momentum tensor and the gluon condensate has been studied for finite temperature by Leutwyler [6] in relation to the problems of deconfinement and chiral symmetry. He starts with a detailed discussion of the trace anomaly based on the interaction between Goldstone bosons in chiral perturbation theory. Central to his discussion is the role of the energy momentum tensor, whose trace is directly related to the gluon field strength. It is important to note that the energy momentum tensor  $T^{\mu\nu}(T)$  can be separated into the zero temperature or vacuum part,  $T_0^{\mu\nu}$ , and the finite temperature contribution  $\theta^{\mu\nu}(T)$  as follows:

$$T^{\mu\nu}(T) = T_0^{\mu\nu} + \theta^{\mu\nu}(T). \quad (2.1)$$

The zero temperature part,  $T_0^{\mu\nu}$ , has the standard problems with infinities of any ground state. It has been discussed by Shifman, Vainshtein and Zakharov [7] in relation to the nonperturbative effects in QCD and the operator product expansion. In what follows we shall just use a bag type of model [8] as a means of stepping around these difficulties since we are only interested here in the thermal properties. The finite temperature part, which is zero at  $T = 0$ , is free of such problems. We shall see in the next section how the diagonal elements of  $\theta^{\mu\nu}(T)$  are calculated in a straightforward way on the lattice. The trace  $\theta_\mu^\mu(T)$  is connected to the thermodynamical contribution to the energy density  $\varepsilon(T)$  and pressure  $p(T)$  for relativistic fields [9] and relativistic hydrodynamics [10]

$$\theta_\mu^\mu(T) = \varepsilon(T) - 3p(T). \quad (2.2)$$

The gluon field strength tensor is denoted by  $G_a^{\mu\nu}$ , where  $a$  is the color index for  $SU(N)$ . The basic equation for the relationship between the gluon

condensate and the trace of the energy momentum tensor at finite temperature was written down by Leutwyler [6] using the trace anomaly in the form

$$\langle G^2 \rangle_T = \langle G^2 \rangle_0 - \langle \theta_\mu^\mu \rangle_T, \quad (2.3)$$

where the gluon field strength squared summed over the colors is

$$G^2 = \frac{-\beta(g)}{2g^3} G_a^{\mu\nu} G_{\mu\nu}^a, \quad (2.4)$$

for which the brackets with the subscript  $T$  mean thermal average. The renormalization group beta function  $\beta(g)$  in terms of the coupling may be written as

$$\beta(g) = \mu \frac{dg}{d\mu} = -\frac{1}{48\pi^2} (11N_c - 2N_f)g^3 + O(g^5). \quad (2.5)$$

He has calculated [6] for two massless quarks using the low temperature chiral expansion the trace of the energy momentum tensor at finite temperature in the following form:

$$\langle \theta_\mu^\mu \rangle_T = -\frac{\pi^2}{270} \frac{T^8}{F_\pi^4} \left\{ \ln \frac{A_p}{T} - \frac{1}{4} \right\} + O(T^{10}), \quad (2.6)$$

where the logarithmic scale factor  $A_p$  is about 0.275 GeV and the pion decay constant  $F_\pi$  has the value of 0.093 GeV. The value of the gluon condensate for the vacuum  $\langle G^2 \rangle_0$  was taken to be about 2 GeV/fm<sup>3</sup>, which is consistent with the previously calculated values [7]. The results sketched by Leutwyler at QUARK MATTER'96 in Heidelberg [6] show a long flat region for  $\langle G^2 \rangle_T$  as a function of the temperature until it arrives at values of at least 0.1 GeV where it begins to show a falloff from the vacuum value proportional to the power  $T^8$ .

### 3. Lattice data for the equation of state

The lattice calculation at finite temperature proceeds (talk by Krzysztof Redlich [11]) in the following way. From the action expectation value at zero temperature,  $P_0$ , as well as the spatial and temporal action expectation values at finite temperature,  $P_\sigma$  and  $P_\tau$ , respectively, and  $N_\tau$  the number of temporal steps, the dimensionless interaction measure  $\Delta(T)$  [12–14] is given by

$$\Delta(T) = -6N N_\tau^4 a \frac{dg^{-2}}{da} [2P_0 - (P_\sigma + P_\tau)]. \quad (3.1)$$

The crucial part of these recent calculations is the use of the full lattice beta function,  $\beta_{\text{fn}} = adg^{-2}/da$  in obtaining the lattice spacing  $a$ , or scale of the simulation, from the coupling  $g^2$ . Without this accurate information on the temperature scale in lattice units it would not be possible to make any claims about the behavior of the gluon condensate. The dimensionless interaction measure is equal to the thermal ensemble expectation value of  $(\varepsilon - 3p)/T^4$ . Thus by the equation 2.2 above is equal to the expectation value of the trace of the temperature dependent part of the energy momentum tensor, [4] which may be written (after suppressing the brackets) as follows:

$$\theta_{\mu}^{\mu}(T) = \Delta(T) \times T^4. \quad (3.2)$$

There are no other contributions to the trace for QCD on the lattice. The heat conductivity is zero. Since there are no non-zero conserved quantum numbers and, as well, no velocity gradient in the lattice computations, hence no contributions from the viscosity terms appear. For a scale invariant system, such as a gas of free massless particles, the trace of the energy momentum tensor, equation 3.2, is zero. A system that is scale variant, perhaps from a particle mass, has a finite trace, with the value of the trace measuring the magnitude of scale breaking. At zero temperature it has been well understood from Shifman *et al.* [7] how in the QCD vacuum the trace of the energy momentum tensor relates to the gluon field strength squared,  $G_0^2$ . Since the scale breaking in QCD occurs explicitly at all orders in a loop expansion, the thermal average of the trace of the energy momentum tensor should not go to zero above the deconfinement transition. So a finite temperature gluon condensate  $G^2(T)$  related to the degree of scale breaking at all temperatures, can be defined to be equal to the trace. We have used [5] the lattice simulations [2,3] in order to get the temperature dependent part of the trace and, thereby, the value of the condensate at finite temperature. In what follows we will use the assumptions of a bag type of model [8] which includes a bag pressure  $-B$  and an energy density of the vacuum of the same magnitude  $B$ . These assumptions mean that the vacuum contribution to the trace of the energy momentum tensor becomes simply  $T_{0\mu}^{\mu} = 4B$ . Using this form we may write the equation of state for the *total* energy density  $\varepsilon(T)$  and pressure  $p(T)$  as follows:

$$\varepsilon(T) - 3p(T) = T_{\mu}^{\mu}(T). \quad (3.3)$$

The trace of the energy momentum tensor as a function of the temperature is shown in Figure 1. We notice that for  $T < T_c$  it remains constant at  $4B$ . However, above  $T_c$  in both cases there is a rapid rise in  $T_{\mu}^{\mu}(T)$ . Accordingly, the vacuum gluon condensate  $G_0^2$  becomes just  $4B$ . Here we have assumed the value  $4B = 0.012 \text{ GeV}^4$  for both cases [7]. By taking the published

data [2,3] for  $\Delta(T)$ , and using equations 2.3 and 3.2 we obtained the gluon condensate  $G^2(T)$  as shown in Figure 1.

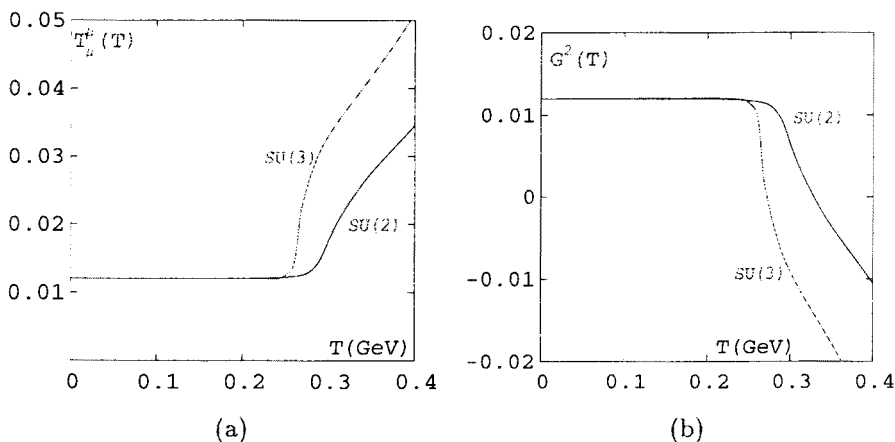


Fig. 1. The left plots show  $T_\mu^\mu(T)$  for the lattice gauge theories SU(2) and SU(3) as indicated. The right plots show the corresponding gluon condensates. The values for the critical temperature  $T_c = 0.290, 0.264$  GeV for SU(2) and SU(3), respectively.

#### 4. High temperature behavior of the equation of state

In this section we present a discussion of the properties of the equation of state at temperatures above  $2T_c$  up to around  $5T_c$ . If one were to consider a simple bag type of model with the bag constant  $B$  independent of the temperature, one finds immediately the simple equality

$$\varepsilon - 3p = 4B. \quad (4.1)$$

This form is in clear conflict with what we found above as shown in Fig. 1(a), for which there was a steady rise in its value. Some of the early work in this temperature range was done using perturbative estimates by Källman [15] and Gorenstein and Mogilevsky [16]. These authors found essentially a linear rise in the equation of state a function of the temperature. Also Montvay and Pietarinen [17] looked at the asymptotic properties of the gluon gas. The asymptotic behavior of the form of  $\Delta(T)$  assumed by Källman [15] and Gorenstein and Mogilevsky [16] are qualitatively similar to our plots in as far as the data extends. However, the latter [16] represent their data in terms of energy density and the pressure separately showing how the two curves converge to each other at small values of  $1/T^3$ . All these results are

quite similar to the ideal relativistic massive gas. The equation of state is then

$$\varepsilon - 3p = \frac{1}{2\pi^2} m^3 T K_1\left(\frac{m}{T}\right). \quad (4.2)$$

We can readily see from the asymptotic properties of the modified Bessel function  $K_\nu(x)$  in this equation that the equation of state is always positive and growing as a function of the temperature. In the high temperature limit the equation of state relates to  $(mT)^2$ .

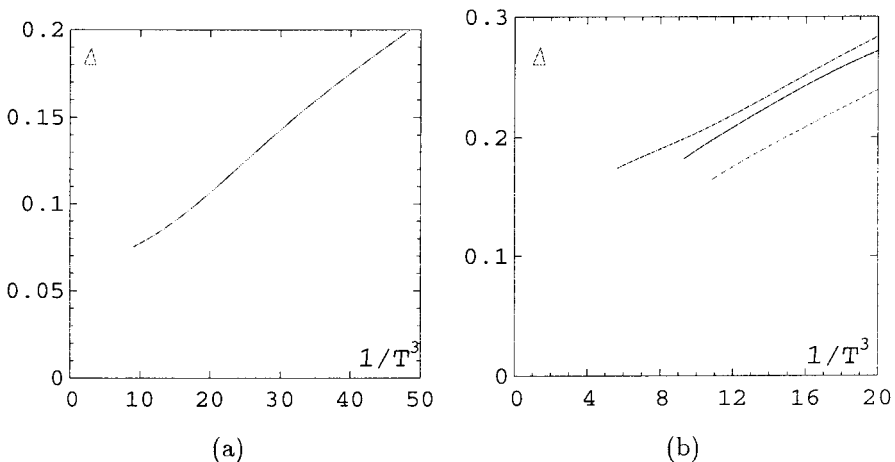


Fig. 2. Figure (a) shows  $\Delta(T)$  as a function of  $1/T^3$  for SU(2) lattice gauge theory [2], where the scale of the abscissa is  $T_c^{-3} \times 10^{-3}$ ; Figure (b) shows  $\Delta(T)$  for SU(3) with the three lattice sizes [3]  $16^3 \times 4$  (top),  $32^3 \times 6$  (middle),  $32^3 \times 8$  (bottom). The scale of the abscissa is the same as for SU(2).

We plot here for the sake of comparison with these authors the high temperature behavior of  $\Delta(T)$  at temperatures in the above range. The following two plots show the high temperature behavior of SU(2) and SU(3) explicitly as a function of  $1/T^3$ . We see from these plots that the general appearance is much the same in all the different cases. Roughly speaking we could interpret  $\Delta(T)$  as a linearly increasing function in the variable  $1/T^3$ . In Figure 2(b) we have provided three lattice sizes for the sake of comparison. We notice that there exists a considerable gap with no indication of  $\Delta(T)$  approaching zero at small  $1/T^3$ . Although we are not able to exactly determine the asymptotic form of the function at high temperature, we are able to conclude about certain behavior from given models in the range of temperature between  $2T_c$  and  $5T_c$ .

Finally we briefly mention the results of finite temperature perturbation theory [18]. From the expression for the free energy one can derive the

interaction measure in the form

$$\Delta = \frac{11}{6}C_A^2d_A\alpha^2\left(\frac{1}{36} + \frac{1}{3}\left(\frac{C_A\alpha}{3\pi}\right)^{1/2} + O(\alpha)\right), \quad (4.3)$$

where  $\alpha$  is related to the coupling by  $g^2/4\pi$ . The constants  $C_A$  and  $d_A$  relate to the number of colors as  $N_c$  and  $N_c^2 - 1$  respectively. The significance of the factor  $\frac{11}{6}$  was pointed out to us by Bo Andersson [19] as the value in the rapidity of the gluon splitting. Thus we see from 4.3 that the temperature dependence of the equation of state is very strong in comparison with our plots.

## 5. Gluon condensate with quarks

In this section we would like to discuss the changes due to the presence of dynamical quarks with a finite mass. There have been recent computations of the thermodynamical quantities in full QCD with two flavors of staggered quarks [20, 21], and with four flavors [22]. These calculations are not yet as accurate as those in pure gauge theory for two reasons. The first is the prohibitive cost of obtaining statistics similar to those obtained for pure QCD. So the error on the interaction measure is considerably larger. The second reason, perhaps more serious, lies in the effect of the quark masses currently simulated. They are relatively heavy, which increases the contribution of the quark condensate term to the interaction measure,  $\Delta_m(T)$  in the equation (5.3) below.

In the presence of massive quarks the trace of the energy-momentum tensor takes the form from the trace anomaly [27] as follows:

$$\langle\Theta^\mu_\mu\rangle = m_q\langle\bar{\psi}_q\psi_q\rangle + \langle G^2\rangle, \quad (5.1)$$

where  $m_q$  is the light (renormalized) quark mass and  $\psi_q, \bar{\psi}_q$  represent the quark and antiquark fields respectively. For the sake of simplicity we choose two light quarks of the same mass  $m_q = 6\text{MeV}$  and respecting isospin symmetry. We are now able to write down an equation for the temperature dependence of the gluon condensate including the effects of the light quarks in the trace anomaly in the following form:

$$\langle G^2\rangle_T = \langle G^2\rangle_0 + m_q\langle\bar{\psi}_q\psi_q\rangle_0 - m_q\langle\bar{\psi}_q\psi_q\rangle_T - \langle\Theta^\mu_\mu\rangle_T. \quad (5.2)$$

It is possible to see that at very low temperatures the additional contribution to the temperature dependence from the quarks is rather insignificant. However, in the range where the chiral symmetry is being restored there is an additional effect from the term  $\langle\bar{\psi}_q\psi_q\rangle_T$ . Well above  $T_c$  after the

chiral symmetry has been completely restored the only remaining effect of the quark condensate is that of the vacuum. This term would contribute negatively to the gluon condensate. Thus we expect [5] that for the light quarks the temperature dependence can be quite important around  $T_c$ . In fact we may rewrite the above equation using the known vacuum estimates for  $\langle G^2 \rangle_0$  and  $m_q \langle \bar{\psi} \psi \rangle_0$  in the simpler form

$$\langle G^2 \rangle_T = B_4 - m_q \langle \bar{\psi}_q \psi_q \rangle_T - \Delta_m(T) T^4, \quad (5.3)$$

where the constant  $B_4$  has the value  $0.01183 \text{ GeV}^4$  for the  $u$  and  $d$  quarks alone [7]. The use of this value should not imply the given accuracy—in reality the accuracy is still the known value of  $0.012 \text{ GeV}^4$ . However, it should imply that it could be possible directly in the neighborhood of  $T_c$  that the temperature dependence from the quarks is sufficiently strong so that the rise in this contribution from temperature compensates to a certain extent for the strongly negative tendency of the pure gluon condensate [5]. For the case of pure  $SU(N)$  we know that the values just below  $T_c$  of  $\Delta(T) \times T^4$  are very small—that is, about the same size as  $m_q \langle \bar{\psi}_q \psi_q \rangle_0$ . Thus, to the extent that the chiral symmetry has not been completely restored, its effect on  $\langle G^2 \rangle_T$  will be present below  $T_c$ .

As an end to this sketch of the gluon condensate in QCD we will mention a few obvious points. Where in simulations on pure  $SU(N_c)$  gauge theory we could depend on considerable precision in the determination of  $T_c$  and  $\Delta(T)$  as well as numerous other thermodynamical functions, this is not the case for the theory with dynamical quarks. The statistics for the measurements are generally smaller. The determination of the temperature scale is thereby hindered so that it is harder to clearly specify a given quantity in terms of  $T$ . Thus, in general, we may state that the accuracy for the full QCD is way down when compared to the computations of the pure lattice gauge theories. However, there is a point that arises from the effect that the temperatures in full QCD are generally lower, so that  $\Delta(T) \times T^4$  is much smaller [5]. Here we can only speculate with the present computations [20–22]. Nevertheless, there could be an indication of how the stability of the full QCD keeps  $\langle G^2 \rangle_T$  positive for  $T > T_c$ . The condensates in full QCD have also been considered by Koch and Brown [28]. However, the lattice measurements they used were not obtained using a non-perturbative method, nor was the temperature scale obtained from the full non-perturbative beta-function.

## 6. Dilatation current

We have noticed in the previous sections that the fact that the trace of the energy momentum tensor does not vanish for the strong interactions has important implications for the equation of state. Here we shall discuss



briefly some more theoretical results relating to  $T_\mu^\mu$ . This arises with respect to the dilatation current as well as the special conformal currents, which are not conserved. The dilatation current  $D^\mu$  may be defined in terms of the position four-vector  $x^\mu$  and the energy momentum tensor  $T^{\mu\nu}$  as simply  $x_\alpha T^{\mu\alpha}$ . In the case of general energy momentum one can find [24]

$$\partial_\mu D^\mu = T_\mu^\mu. \quad (6.1)$$

We now look into a volume in four dimensional space-time  $\mathcal{V}_4$  containing all the quarks and gluons at a fixed temperature  $T$  in equilibrium [25]. The above equation 6.1 holds when the energy momentum and all the (color) currents are conserved over the surface  $\partial\mathcal{V}_4$  of the properly oriented four-volume  $\mathcal{V}_4$ . Then from 6.1

$$\oint_{\partial\mathcal{V}_4} \mathcal{D}_\mu dS^\mu = \int_{\mathcal{V}_4} T_\mu^\mu dV_4, \quad (6.2)$$

where the *dyxle* three-form is  $\mathcal{D}_\mu dS^\mu$  on the three dimensional surface  $\partial\mathcal{V}_4$  dual to  $D_\mu dx^\mu$  the dilatation current one-form [26]. On the right hand side of 6.2 the integrated form  $\int_{\mathcal{V}_4} T_\mu^\mu dV_4$  is an action integral involving the equation of state. Since  $T_\mu^\mu > 0$ , the action integral is not zero. This action integral gets quantized with the fields.

## 7. Summary and conclusions

Absolutely central to our development of this discussion is the nontrivial equation of state (3.3) with  $T_\mu^\mu(T) > 0$  for all reachable temperatures. Thus a strongly interacting gas of quarks and gluons in equilibrium is *never* an ideal gas.

The lattice simulations for pure  $SU(N_c)$  provide not only an accurate computation of the interaction measure  $\Delta(T)$ , but also of the temperature scale due to the calculation of the beta function at finite temperature. These properties are all needed in the computation of  $\theta_\mu^\mu(T)$  in the previous sections. It is clear that the gluon condensate becomes negative in pure gauge theory, and keeps dropping with increasing temperature. We have also given a discussion of these quantities in relation to full QCD. In all cases considered above we may conclude  $T_\mu^\mu(T)$  always remains positive so that the divergence of both the dilatation and the conformal currents remain positive, and are thereby not conserved. Therefore, the scale and conformal symmetries remain broken at all temperatures for nonabelian lattice gauge theories after quantization.

An important point for the physical meaning of these results is the comparison of the form  $\Delta(T) \times T^4$  with the corresponding term in Bjorken's

equation of state  $p\Delta_1$  [29]. The definition of  $\Delta_1$  relates to the change of the effective number of degrees of freedom for which a first order phase transition may appear. We also notice the general similarity in shape of  $\Delta_1$  to the lattice interaction measure with the main difference being that the peak in the lattice simulations [2, 3] is slightly above  $T_c$ . Further comparison with Bjorken's model could lead to a better physical understanding of the equation of state gotten from lattice simulations.

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