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CONFINEMENT OF COLOUR BY DUAL SUPERCONDUCTIVITY*

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The evidence from lattice that colour is confined by dual superconductivity of QCD vacuum is reviewed. Open problems are discussed.

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1. Introduction

Most of the evidence that QCD is the correct theory of strong interactions comes from short distances via perturbation theory [1]. It is commonly accepted that in such regime perturbative expansion works, due to asymptotic freedom. It is known, however, that the renormalized perturbative series of QCD is not convergent, and cannot even be regarded as an asymptotic series because of bad infrared behaviour.

At large distances the elementary excitations of the theory, quarks and gluons, never appear as particles: only colour singlets are visible in asymptotic states, a phenomenon known as "colour confinement".

A non perturbative formulation of the theory is needed, *i.e.* an evaluation of the Feynman path integral which defines it, which does not rely on expansions around the free gaussian action.

The only known way to do that is the lattice formulation [2]. The Feynman integral is UV regularized by a discretization of space–time, and IR by periodic boundary conditions on a finite volume, and then computed by numerical Monte Carlo techniques.

Of course numerical results never have the transparency of mathematical arguments. They can, however, be used either as a direct test of the theory or to explore the way in which the theory works.

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We shall investigate by use of lattice simulations the mechanism by which colour is confined. More precisely we shall test the mechanism of colour confinement known as "type II dual superconductivity" of QCD vacuum.

The basic idea is simple and appealing [3–5]: the chromoelectric field produced by a $q\,\bar{q}$ pair is channeled by dual Meissner effect into an Abrikosov flux tube, so that the energy E is proportional to the distance R

$$E = \sigma R\,,$$

σ is the string tension.

"Dual" means that the role of electric and magnetic quantities is interchanged with respect to ordinary superconductors.

Ordinary superconductivity is produced by the condensation in the ground state of charged particles (Cooper pairs). The ground state has no definite charge: it is superposition of states with different charges. This results in a non zero expectation value for the charged field Φ describing Cooper pairs in the Ginzburg–Landau equation

$$\bar{\Phi} = \langle 0|\Phi|0\rangle \neq 0\,, \tag{1}$$

$\bar{\Phi}$ is the order parameter which signals the change of symmetry of the system in the transition to superconductivity. $\bar{\Phi} \neq 0$ characterizes a Higgs phenomenon. The photon acquires a mass,

$$m = \sqrt{2}q\bar{\Phi} \tag{2}$$

which results in a finite penetration depth of the magnetic field (Meissner effect): $\lambda_{\mathrm{p}} = 1/m$. The correlation function

$$\mathcal{D}(x) = \langle \Phi^\dagger(x)\Phi(0)\rangle \underset{|\vec{x}|\to\infty}{\simeq} Ae^{-M|\vec{x}|} + |\bar{\Phi}|^2 \tag{3}$$

at large distances determines the correlation length $\lambda_\Phi = 1/M$ and the order parameter $|\bar{\Phi}|$.

If $\lambda_{\mathrm{p}} \gg \lambda_\Phi$, *i.e.* if $M \gg m$, the superconductor is type II, and the formation of Abrikosov flux tubes is energetically favoured in the process of penetrating the system by a magnetic field [6].

In a dual superconductor the electric field acquires a finite penetration depth $1/m_{\mathrm{ph}}$ and magnetic $U(1)$ is spontaneously broken by condensation of magnetic charges (monopoles) in the vacuum.

The v.e.v. of a field Φ_M carrying magnetic charge, $\mu = |\Phi_M|$ will be the "disorder parameter" of the system, and the correlation function

$$\mathcal{D}_{\mathcal{M}}(x) = \langle \Phi_M^\dagger(x)\Phi_M(0)\rangle \underset{|\vec{x}|\to\infty}{\simeq} Ae^{-M_\Phi|\vec{x}|} + \mu^2 \tag{4}$$

at large distances will provide the mass of the Higgs field M_Φ and the disorder parameter μ. The ratio $M_\Phi/m_{\rm ph}$ determines the type of dual superconductor.

The name "disorder parameter" originates from the concept of duality [6]: going to the dual the coupling constant goes to its inverse (magnetic charge $M = n/2\pi e$) or, in the language of statistical mechanics, the role of the low temperature (ordered) phase, is interchanged with the higher temperature (disordered) phase [7, 8].

If the mechanism of colour confinement at work in QCD is dual superconductivity, monopoles should condense in the confined phase, and the v.e.v. $\bar{\Phi}_M$ of a magnetically charged field Φ_M should be the disorder parameter, with $\bar{\Phi}_M \neq 0$ in the confining phase and $\bar{\Phi}_M = 0$ in the deconfined one.

Measuring $\mathcal{D}_{\mathcal{M}}(x)$ at large distances (Eq. (4)) gives both the disorder parameter μ and the effective mass M_Φ. The photon mass $m_{\rm ph}$ can be determined from the penetration depth of the chromoelectric field, and the type of dual superconductivity can be established.

The problem with QCD is to identify the monopoles which condense in the vacuum, if any, and to verify that their condensation is related to confinement of colour.

Monopole species in non abelian gauge theories are identified by local fields Φ transforming in the adjoint representation of gauge group, or $\Phi = \sum_a \Phi^a \lambda^a$, where λ^a are the generators of the group in the fundamental representation (see Sec. 3).

Such monopoles are located in field configurations at the sites where two eigenvalues of Φ coincide. For SU(2) this means the sites were $\Phi = 0$.

Infinitely many candidates exist, actually a functional infinity, for the monopole species which can produce dual superconductivity by condensation. What monopoles are really relevant to confinement, or, equivalently, what field Φ identifies them, is a dynamical problem. A sensible guess is that pratically all choices for Φ are equivalent ('t Hooft) [12].

Monopoles defined by a given field Φ are exposed by a gauge transformation, which diagonalizes Φ. Sometimes the statement is made that QCD monopoles are gauge dependent objects, thus implying that phenomena related to them, like their condensation, could depend on the arbitrariness in the choice of the gauge and be unphysical. This is not true. Monopoles defined by some field Φ, could be more relevant to confinement than some other ones. This is a dynamical fact. The resulting physics is anyhow gauge independent.

The strategy that we shall adopt to investigate these issues will be to construct a disorder parameter for condensation of the monopoles defined by any field Φ, and to use it to detect (numerically) condensation in connection with confinement [10, 13].

We anticipate that there is an evidence of dual superconductivity for monopoles defined by different fields Φ, namely the Polyakov line, any component of the field strength tensor, or the so called maximal abelian projection.

In Sec. 2 we shall rapidly review the construction of the disorder parameter for dual superconductivity and the information that it can provide.

In Sec. 3 we shall revisit the so called abelian projection, *i.e.* the procedure to expose the monopoles associated to any field Φ.

The conclusions and the outlook will be presented in Sec. 4.

2. The disorder parameter

The basic idea to construct an operator which creates a monopole, or, more generally, a soliton with nontrivial topological charge, is translation.

In the same way as, for a particle,

$$\mathrm{e}^{ipa}\,|x\rangle = |x+a\rangle \tag{5}$$

for a field Φ

$$\mathrm{e}^{i\int d^3x \Pi_\Phi(\vec{x},t)\Phi_{\mathrm{cl}}(\vec{x}-\vec{y})}\,|\Phi(\vec{x},t)\rangle = |\Phi(\vec{x},t)+\Phi_{\mathrm{cl}}(\vec{x}-\vec{y})\rangle\,. \tag{6}$$

Here $|\Phi(\vec{x},t)\rangle$ is the state of the field in the Schrödinger representation, and Π_Φ is the conjugate momentum to the field Φ. $\Phi_{\mathrm{cl}}(\vec{x}-\vec{y})$ is the classical field of the soliton located at the site $\vec{y}$ in the point $\vec{x}$.

The effect of the operator

$$\mu(\vec{y},t) = \exp(i\int d^3x \Pi_\Phi(\vec{x},t)\Phi_{\mathrm{cl}}(\vec{x}-\vec{y})) \tag{7}$$

is to add a soliton to any field configuration.

For compact U(1) the field Φ is the vector potential $\vec{A}(\vec{x},t)$, the conjugate momentum is the electric field $\vec{\Pi}(\vec{x},t) = \vec{E}(\vec{x},t)$ and the Φ_{cl} is the field produced at $\vec{x}$ by a monopole located in $\vec{y}$.

We shall compute the correlator

$$\mathcal{D}(t) = \langle \bar{\mu}(\vec{0},t)\,\mu(\vec{0},0)\rangle \tag{8}$$

which propagates from time 0 to time t a monopole sitting at the origin $\vec{0}$ of space.

As $t\to\infty$, in the euclidean region by cluster property

$$\mathcal{D}(t) \simeq A\exp(-Mt) + \langle\mu\rangle^2\,, \tag{9}$$

where M is the lowest mass in the sector with quantum numbers of a monopole.

Since the operator μ carries monopole charge, $\langle\mu\rangle \neq 0$ in Eq. (9) signals spontaneous breaking of magnetic charge conservation, and hence dual superconductivity. M provides instead a lower limit to the mass of the effective dual Higgs field, and hence information on the type of superconductor, as discussed in Sec. 1.

Instead of the continuum version of Eq. (7) for μ, we use a compactified lattice version of it [11] which coincides with (7) in the limit $a \to 0$.

The corresponding definition of $\mathcal{D}(t)$ is

$$\mathcal{D}(t) = \frac{Z[S+\Delta S]}{Z[S]}\,. \tag{10}$$

$Z[S]$ is the partition function $\int \prod d\theta \exp(-\beta S)$ with S the action.

The action is a functional of the plaquettes

$$\Pi_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu})U_\mu^\dagger(n+\hat{\nu})U_\nu^\dagger(n)$$

$U_\mu(n) = \exp(i\theta_\mu(n))$ being the parallel transport along the link of the lattice exiting from site n in direction $\hat{\mu}$.

The group U(1) being abelian

$$\Pi_{\mu\nu} = \exp(i\theta_{\mu\nu}(n)) \tag{11}$$

with

$$\theta_{\mu\nu}(n) = \theta_\mu(n) + \theta_\nu(n+\hat{\mu}) - \theta_\mu(n-\hat{\nu}) - \theta_\nu(n) \simeq ea^2F_{\mu\nu}\,.$$

$S+\Delta S$ is obtained from S by the replacements

$$\theta^{i0}(\vec{n},0) \to \theta^{i0}(\vec{n},0) + b_i(\vec{n})\,, \tag{12}$$

$$\theta^{i0}(\vec{n},t) \to \theta^{i0}(\vec{n},t) - b_i(\vec{n})\,, \tag{13}$$

$b_i(\vec{n})$ is the vector potential which describes the field produced by a monopole sitting at $\vec{0}$.

Whatever form of the action as the lattice spacing $a \to 0$

$$S \to -\frac{a^2}{4}F_{\mu\nu}F_{\mu\nu} \tag{14}$$

and

$$S+\Delta S \to -\frac{a^2}{4}F_{\mu\nu}F_{\mu\nu} + \frac{a^3}{e}\sum_{\vec{n}}\left[F_{0i}(\vec{n},0)b_i(\vec{n}) - F_{0i}(\vec{n},t)b_i(\vec{n})\right] \tag{15}$$

which brings back to Eq. (7).

On the other hand

$$Z[S+\Delta S]=\int\left[\prod_{n,\mu}\right]\frac{d\theta_\mu(n)}{2\pi}\exp\{-\beta(S+\Delta S)\} \tag{16}$$

is invariant under change of variables $\theta_\mu(n)\to\theta_\mu(n)+\Delta_\mu(n)$ with arbitrary Δ_μ, because of compactness.

A change of variable

$$\theta_i(\vec{n},1)\to\theta_i(\vec{n},1)+b_i(\vec{n})$$

brings back

$$\theta^{i0}(\vec{n},0)+b_i(\vec{n})=\theta^0(\vec{n}+\hat{\imath},0)-\theta^0(\vec{n},0)+\theta^i(\vec{n},0)-\theta^i(\vec{n},1)+b_i(\vec{n})$$

to θ^{i0}, but changes $\theta^{ij}(\vec{n},1)$

$$\theta^{ij}(\vec{n},1)\to\theta^{ij}(\vec{n},1)+\Delta_i b_j-\Delta_j b_i$$

i.e. adds the field of a monopole at time $n_0=1$, and

$$\theta^{i0}(\vec{n},1)\to\theta^{i0}(\vec{n},1)+b_i(\vec{n})\,.$$

Again changing variables from $\theta_i(\vec{n},2)\to\theta_i(\vec{n},2)+b_i(\vec{n})$ sends back $\theta^{i0}(\vec{n},1)+b_i$ to $\theta^{i0}(\vec{n},1)$ but adds a monopole at $n_0=2$ and shifts $\theta^{i0}(\vec{n},2)\to\theta^{i0}(\vec{n},2)+b_i(\vec{n})$.

The game can be repeated till $n_0=t$ where b_i will cancel with $-b_i$ of Eq. (13).

The net effect of the construction is to add a monopole field $\Delta_i b_j-\Delta_j b_i$ at all times between 0 and t. The independence of $\mathcal{D}(t)$ on the choice of the classical gauge for b_i is explicit.

U(1) system is known to be a dual superconductor confining electric charges below $\beta_c=1.01$ (Wilson's action). $\langle\mu\rangle$ can be computed numerically and checked as a disorder parameter for monopole condensation [11]: in the limit of infinite volume one should have

$$\langle\mu\rangle\neq 0\quad\beta<\beta_c\qquad\text{and}\qquad\langle\mu\rangle=0\quad\beta>\beta_c\,. \tag{17}$$

To access the limit $V\to\infty$ it proves convenient to deal not with $\langle\mu\rangle$ itself, or $\mathcal{D}(t)$ but with

$$\rho(t)=\frac{d}{d\beta}\ln\mathcal{D}(t)\,. \tag{18}$$

In terms of ρ

$$\mathcal{D}(t) = \exp\left(\int_0^\beta \rho(t)dt\right) . \tag{19}$$

From definition of $\mathcal{D}(t)$ it follows

$$\rho(t) = \langle S \rangle_S - \langle S + \Delta S \rangle_{S+\Delta S} , \tag{20}$$

where, with obvious notation, the average of the quantity in the brackets is computed with the action in the subscript as weight.

At large t from Eq. (9)

$$\rho(t) \simeq C \exp(-Mt) + \rho_\infty \tag{21}$$

with $\rho_\infty = 2\frac{d}{d\beta} \ln\langle \mu \rangle$.

Fig. 1 shows a typical behaviour of $\rho(t)$. ρ_∞ is shown in Fig. 2.

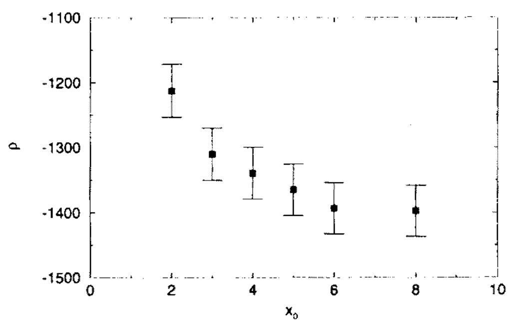

Fig. 1. $\rho(t)$ as a function of t (lattice $8^3 \times 16$).

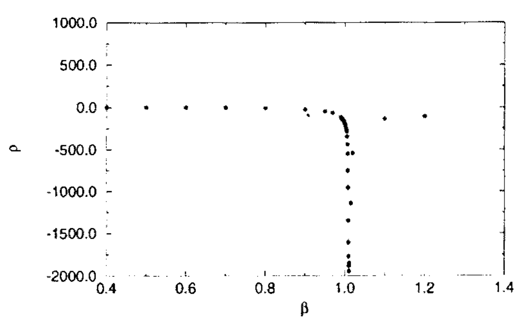

Fig. 2. ρ_∞ as a function of β. The negative peak signals the phase transition (lattice $8^3 \times 16$).

For $\beta > \beta_c$ the theory describes free photons, the path integral is gaussian and ρ_∞ can be computed explicitely. The result for a lattice $L^3 \times 2L$ is

$$\rho_\infty = -10.1 \cdot L + 9.542 \tag{22}$$

showing that $\rho_\infty \to -\infty$ or, by Eq. (19), $\mu \to 0$ for $\beta > \beta_c$.

As $L \to \infty$, $\rho \to$ finite limit for $\beta < \beta_c$, (Fig. 3), or $\langle\mu\rangle \neq 0$. $\langle\mu\rangle$ is a disorder parameter for the system in the thermodynamical limit. Indeed for any finite volume $\langle\mu\rangle$ cannot be identically zero for $\beta > \beta_c$, since it would be zero everywhere being an analytic function of β. Only when $L \to \infty$ Lee Yang singularities can develop.

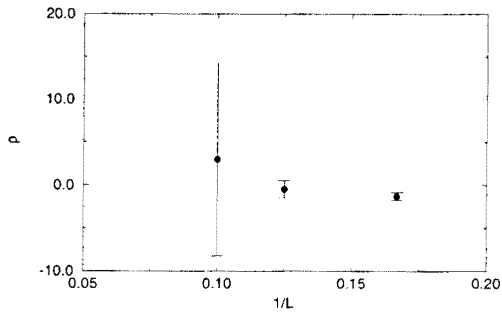

Fig. 3. ρ_∞ versus $1/L$ for $\beta = 1.009$.

Around β_c a finite size scaling analysis can be made of ρ. The transition is known to be weak first order, or second order: in any case an effective critical index ν can be defined. For dimensional reasons, if ξ is the correlation length

$$\mu = \mu\left(\frac{\xi}{L}, \frac{a}{\xi}\right) \underset{a/\xi \to 0}{\simeq} \mu\left(\frac{\xi}{L}, 0\right) \tag{23}$$

but ξ/L can be traded for $(\beta_c - \beta)L^{1/\nu}$ and the scaling law follows

$$\frac{\rho}{L^{1/\nu}} = f(L^{1/\nu}(\beta_c - \beta)) \,. \tag{24}$$

For the proper values of β_c and ν data coming from lattices of different size L should follow the scaling law (24). This is indeed what happens (Fig. 4). β_c and ν can be extracted and compared with values obtained by completely different approaches. We get

$$\beta_c = 1.01160(5) \qquad \nu = .29(2) \tag{25}$$

$$\text{As } \beta \to \beta_c \qquad \mu \simeq (\beta_c - \beta)^\delta \qquad \delta = 1.1 \pm .2 \tag{26}$$

β_c is in agreement with the commonly accepted value.

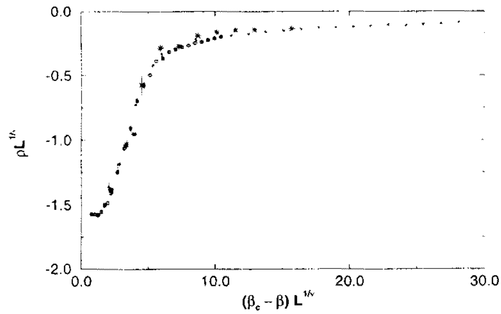

Fig. 4. Scaling.

A measure of the penetration depth of the electric field can also be made. The superconductor is at the border between type I and II.

As a conclusion we have a reliable disorder parameter for dual superconductivity, and the possibility of extracting critical indices from its behaviour at the transition.

A similar construction for the XY model in 3d, which describes superfluid liquid He_4 is a further test of this statement [14]. We will use it to test condensation of monopoles in QCD.

3. Revisiting the abelian projection

Let $\vec{\Phi} \cdot \vec{\lambda}$ be any gauge field in the adjoint representation of the gauge group: $\vec{\lambda}$ are the generators in the fundamental representation. To simplify the notation we shall refer to SU(2) in what follows, but the arguments will apply equally well to any gauge group, with some formal complication. Our field will then be $\vec{\Phi} \cdot \vec{\sigma}$, with $\vec{\sigma}$ the Pauli matrices.

Usually fixed colour axes are used, the same in all points of space–time. Let $\vec{\xi}_i^{\,0}$ $(i = 1, 2, 3)$ be the corresponding orthonormal unit vectors with

$$\vec{\xi}_i^{\,0} \cdot \vec{\xi}_j^{\,0} = \delta_{ij} \qquad \vec{\xi}_i^{\,0} \wedge \vec{\xi}_j^{\,0} = \vec{\xi}_k^{\,0} \,. \tag{27}$$

One can instead use a "body fixed frame", (BFF) [15], $\vec{\xi}_i(x)$, again with

$$\vec{\xi}_i \cdot \vec{\xi}_j = \delta_{ij} \qquad \vec{\xi}_i \wedge \vec{\xi}_j = \vec{\xi}_k \tag{28}$$

and

$$\vec{\xi}_3(x) = \hat{\Phi}(x) = \frac{\vec{\Phi}(x)}{|\vec{\Phi}(x)|} \,. \tag{29}$$

In general

$$\vec{\xi}_i(x) = R(x)\vec{\xi}_i^{\,0} \,. \tag{30}$$

The BFF is well defined except at the zeros of $\vec{\Phi}(x)$. $\xi_i(x)$ being unit vectors

$$\partial_\mu \vec{\xi}_i(x) = \vec{\omega}_\mu(x) \wedge \vec{\xi}_i(x) \tag{31}$$

or

$$D_\mu \vec{\xi}_i(x) \equiv (\partial_\mu - \vec{\omega}_\mu \wedge)\vec{\xi}_i(x) = (\partial_\mu - i\vec{\omega}_\mu \cdot \vec{T})\vec{\xi}_i(x) = 0\,. \tag{32}$$

Eq. (32) defines what in geometry is called a parallel transport. It follows from it that

$$[D_\mu, D_\nu]\,\vec{\xi}_i(x) = 0 \tag{33}$$

or, by completeness of $\vec{\xi}_i(x)$, $[D_\mu, D_\nu] = \vec{T}\vec{F}_{\mu\nu}(\omega) = 0$, *i.e.*

$$\vec{F}_{\mu\nu}(\omega) = \partial_\mu \vec{\omega}_\nu - \partial_\nu \vec{\omega}_\mu + \vec{\omega}_\mu \wedge \vec{\omega}_\nu = 0\,. \tag{34}$$

The solution of Eq. (31) is then

$$\vec{\xi}_i(x) = P \exp\left(i \int_C^x \vec{\omega}_\mu(x) \cdot \vec{T}\, dx^\mu\right) \vec{\xi}_i^0 \tag{35}$$

which is independent of the path C, as long as Eq. (34) is valid. By comparison with Eq. (30)

$$R(x) = P \exp\left(i \int_C^x \vec{\omega}_\mu(x) \cdot \vec{T}\, dx^\mu\right)\,, \tag{36}$$

In fact Eq. (34) is valid apart from singularities, which can make the connection non trivial. The singular fields are Dirac strings, with $F_{\mu\nu}(\omega)$ parallel to $\vec{\xi}_3(x)$, or $\hat{\Phi}(x)$, and occur in the sites where $\vec{\xi}_3 \parallel \vec{\xi}_3^0$, or $\vec{\Phi} \cdot \vec{\sigma}$ is diagonal. This can be seen by parametrizing $\vec{\xi}_i(x)$ in terms of polar coordinates with respect to $\vec{\xi}_i^{\,0}$, with $\vec{\xi}_3^{\,0}$ polar axis, and computing $\vec{\omega}_\mu(x)$. The result is

$$\vec{\omega}_\mu(x) = \begin{pmatrix} \sin\theta(x)\partial_\mu\psi(x) \\ -\partial_\mu\theta(x) \\ -\cos\theta(x)\partial_\mu\psi(x) \end{pmatrix} \tag{37}$$

which is singular where ψ is undefined, *i.e.* at $\theta = 0, \pi$. There an abelian field $\vec{\xi}_3 \cdot \vec{F}_{\mu\nu} = \pm(\partial_\mu\partial_\nu - \partial_\nu\partial_\mu)\psi_{\text{sing}}$ exists, parallel to the field $\hat{\Phi}(x)$. The line integral $\int \omega_\mu^3 dx_\mu$ around the singulariy is an integer multiple of 2π: the singularity is an abelian Dirac string. Monopoles exist at the sites where θ jumps from 0 to π, or where $\Phi(x)$ is zero.

A gauge transformation $R^{-1}(x)$, Eq. (30,36) will bring $\vec{\Phi}(x)$ along the third axis everywhere. This transformation is singular and is called abelian projection. As a consequence, when going to the fixed frame the singular field $F_{\mu\nu}(\omega)$ will add to $G_{\mu\nu}$, and the singularities will show up as pointlike U(1) Dirac monopoles. The U(1) field describing these monopoles is the abelian part of $F_{\mu\nu}$. Since in the abelian projected frame $\partial_\mu \hat{\Phi} = 0$ apart from singularities

$$D_\mu \hat{\Phi} = g\vec{W}_\mu \wedge \hat{\Phi} \tag{38}$$

and

$$\frac{1}{g}\hat{\Phi} \cdot (D_\mu \hat{\Phi} \wedge D_\nu \hat{\Phi}) = g(\vec{W}_\mu \wedge \vec{W}_\nu)\hat{\Phi}\,. \tag{39}$$

It follows that the abelian part of the field is

$$\hat{\Phi} \cdot \vec{G}_{\mu\nu} - \frac{1}{g}\hat{\Phi}(D_\mu \hat{\Phi} \wedge D_\nu \hat{\Phi}) \tag{40}$$

which is nothing but 't Hooft "electromagnetic field", and is a gauge invariant quantity [16].

Monopoles defined by the abelian projection on $\hat{\Phi}$ are gauge invariant objects.

Of course different choices for $\hat{\Phi}$ will bring to different definition of monopoles species.

Monopole are in any case U(1) Dirac monopoles. Their possible condensation in the vacuum is a dynamical fact and reflects the relevance of the field $\hat{\Phi}$ in describing the degrees of freedom responsible for confinement.

If monopoles defined by some field $\hat{\Phi}$ condense, the electric field correspondig to the U(1) identified by the abelian projection should form Abrikosov flux tubes and hence produce confinement. Chromoelectric field in flux tubes between heavy $Q\bar{Q}$ pairs observed on the lattice should be therefore oriented parallel to the field $\hat{\Phi}$ which identifies the monopoles.

This seems not to be the case, that field looking isotropically distributed in colour space [17, 18]. We will come back to this in Sec. 4.

A number of candidates for the field Φ identifyng monopoles have been proposed [12]:

- The Polyakov loop, *i.e.* the parallel transport along the time axis to infinity and back to the initial point via periodic boundary conditions.
- The field implicitly defined by maximizing with respect to gauge transformations the quantity

$$M = \sum_{\mu,n} \mathrm{Tr}\left\{\sigma_3 U_\mu(n) \sigma_3 U_\mu^\dagger(n)\right\}\,. \tag{41}$$

  The change $U_\mu(n) \to \Omega(n) U_\mu \Omega^\dagger(n+\mu)$ is performed and $\Omega(n)$ is chosen in such a way that M is maximum.

— Any component $G_{\mu\nu}$ of the field strength tensor.

Of course there is a functional infinity of possible choices: on the lattice any parallel transport coming back to the starting point is a candidate. The guess of 't Hooft is that all possibilities are physically equivalent [12].

There are claims that the choice corresponding to the maximal abelian gauge is better than others. The basis for this being that the corresponding abelian field is a large fraction of the total field (abelian dominance) and in addition the part do to monopoles almost saturates the abelian field (monopole dominance) [19].

Our strategy is to use the disorder parameter defined to describe U(1) monopoles for different fields Φ to detect dual superconductivity.

Preliminary evidence is in favour of 't Hooft's idea that all species of monopoles are physically equivalent [13]. Fig. 5 and Fig. 6 show the behaviour of the disorder parameter for SU(2) and SU(3), with Φ the Polyakov line.

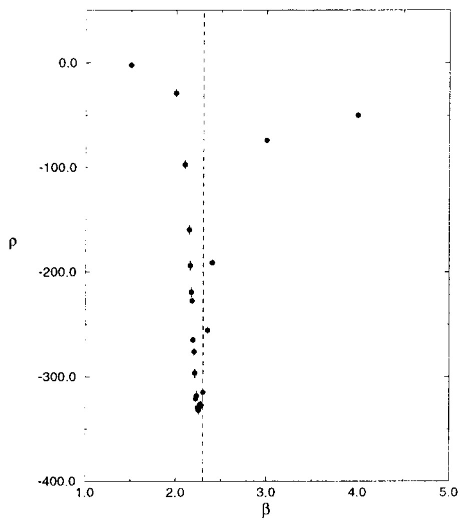

Fig. 5. ρ v.s. β SU(2) gauge theory (lattice $12^3 \times 4$).

Our program is to perform a careful finite size scaling analysis to determine the deconfining temperature and the critical indices of the deconfining phase transition [13].

4. Conclusion and outlook

The mechanism of colour confinement in dual superconductivity of vacuum is supported by many numerical experiments on lattice.

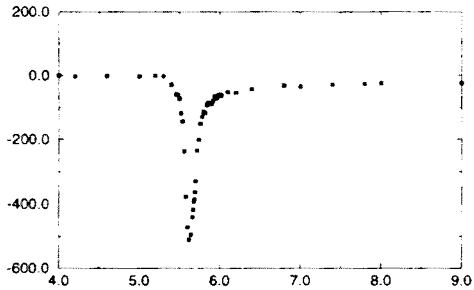

Fig. 6. ρ v.s. β SU(3) gauge theory (lattice $12^3 \times 4$).

1. Flux tubes are observed between propagating heavy quarks [18, 20].
2. Analysis in terms of a disorder parameter does show that a dual Higgs phenomenon takes place at the deconfinig transition [10, 13]. A more systematic analysis in this direction is on the way.

However

(*i*) Monopole species are defined by any field Φ in the adjoint representation and infinitely many choices are possible. The question whether some of them are better than others to produce confinement is an open question.

(*ii*) Maybe a more clever way exists to describe non abelian dual superconductivity, which could look like ordinary dual superconductivity for different species of monopoles defined by an abelian projections, and explain the observed colour content of flux tubes [17, 18].

In conclusion more work is needed to understand confinement.

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