STANDARD MODEL WITH DUALITY: THEORETICAL BASIS *

CHAN HONG-MO

Rutherford Appleton Laboratory
Chilton, Didcot, Oxon, OX11 0QX, United Kingdom
e-mail address: chanhm@v2.rl.ac.uk

AND TSOU SHEUNG TSUN

Mathematical Institute, University of Oxford
24-29 St. Giles', Oxford, OX1 3LB, United Kingdom
e-mail address: tsou@maths.ox.ac.uk

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The Dualized Standard Model which has a number of very interesting physical consequences is itself based on the concept of a nonabelian generalization to electric–magnetic duality. This paper explains first the reasons why the ordinary (Hodge) * does not give duality for the nonabelian theory and then reviews the steps by which these difficulties are surmounted, leading to a generalized duality transform formulated in loop space. The significance of this in relation to the Dualized Standard Model is explained, and possibly also to some other areas.

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1. Introduction

From the standpoint of our present understanding and observation, the Standard Model seems to encapsulate the major points of our knowledge in particle physics but yet leaves many of its own ingredients unexplained. Of these, the most striking are the origins of Higgs fields and fermion generations. Nor are details such as the fermion mass hierarchy or the CKM (Cabibbo–Kobayashi–Maskawa) mixing matrix [1] given any theoretical explanations. A way to further our understanding is perhaps to study more

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closely Yang–Mills theory itself, on which the Standard Model is based. Indeed, it was shown that by combining a recently derived generalized electric-magnetic duality for Yang–Mills theory with a well-known result of 't Hooft's on confinement [2] one obtains a scheme – the Dualized Standard Model — which purports to answer some of these puzzling questions [3].

This paper reviews the theoretical basis for the scheme, while our other paper [4] in the same volume reviews its physical consequences.

2. A first look at duality

It is well-known that electromagnetism is invariant under the interchange $E \to H$, $H \to -E$, which can be expressed equivalently as a symmetry under the Hodge star operation on the field tensor

$$^*F_{\mu\nu} = -\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \,. \tag{1}$$

When there are charges present, then this duality interchanges electric and magnetic charges.

Let us take one of the Maxwell equations:

$$\partial_\mu {}^*F^{\mu\nu} = 0 \,. \tag{2}$$

Using Gauss' theorem, it is easy to see that (2) is equivalent to the absence of magnetic monopoles. This is the physical content of (2). Using the Poincaré lemma [1], it is also easy to see that (2) is equivalent to the existence of a gauge potential A_μ such that $F_{\mu\nu}$ is its curl:

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu \,. \tag{3}$$

This is the geometric content of (2). Notice that both conditions are necessary and sufficient. This situation can be schematically represented as:

$$\underbrace{A_\mu \text{ exists}}_{\text{geometry}} \overset{\text{Poincaré}}{\Longleftrightarrow} \partial_\mu {}^*F^{\mu\nu} = 0 \overset{\text{Gauss}}{\Longleftrightarrow} \underbrace{\text{no magnetic monopoles}}_{\text{physics}} \,. \tag{4}$$

The dual of Eq. (2) is:

$$\partial_\mu F^{\mu\nu} = 0 \,, \tag{5}$$

which is satisfied where there are no electric sources. By the same line of argument as for (4) we deduce that where (5) is satisfied, there exists also a dual potential $\tilde{A}_\mu$ such that:

$$^*F_{\mu\nu} = \partial_\nu \tilde{A}_\mu - \partial_\mu \tilde{A}_\nu \,, \tag{6}$$

[1] This particularly simple example of Poincaré lemma can easily be seen by a direct construction of the gauge potential A_μ.

so that a symmetry is established under the * operation, that is

$$\underbrace{\tilde{A}_\mu \text{ exists}}_{\text{geometry}} \iff \partial_\mu F^{\mu\nu} = 0 \iff \underbrace{\text{no electric sources}}_{\text{physics}} . \tag{7}$$

This is the celebrated electric–magnetic duality.

For nonabelian gauge theories, however, the picture is totally different. Using the covariant derivative $D_\mu = \partial_\mu - ig[A_\mu, \,]$, we still have the analogue of (2):

$$D_\mu {}^*F^{\mu\nu} = 0 , \tag{8}$$

which is usually known as the Bianchi identity. However, since there is no nonabelian analogue to Gauss' theorem, *i.e.* in this case there is no satisfactory way of converting a volume integral into a surface integral, (8) has *nothing* to say about the existence or otherwise of the nonabelian analogue of the magnetic monopoles. In fact, even the concept of flux is lost so that one has to give an entirely new kind of definition to a nonabelian monopole. Furthermore, although (8) holds identically for any tensor $F_{\mu\nu}$ which is the covariant curl of a potential A_μ, the converse is false. In fact, on can hardly formulate the converse given that the covariant derivative D_μ has to involve the potential A_μ. This means that the above diagram (4), so significant in the abelian case, has hardly any content in the nonabelian case:

$$A_\mu \text{ exists} \Longrightarrow D_\mu {}^*F^{\mu\nu} = 0 \quad \overset{?}{\cdots} \quad ? \tag{9}$$

Further, there need not exist a dual potential related to ${}^*F_{\mu\nu}$ in the same way as A_μ is related to $F_{\mu\nu}$. In fact, Gu and Yang [5] constructed some explicit counter-examples of potentials A_μ which satisfy $D_\mu F^{\mu\nu} = 0$ (and of course $D_\mu {}^*F^{\mu\nu} = 0$) but no $\tilde{A}_\mu$ exists for which ${}^*F_{\mu\nu}$ is its covariant curl. So we have also the 'would-be' analogue of (7):

$$\tilde{A}_\mu \text{ exists} \quad \overset{\text{Gu–Yang}}{\not\Longleftarrow} \quad D_\mu F^{\mu\nu} = 0 \quad \overset{\text{Yang–Mills}}{\Longleftrightarrow} \quad \text{no electric sources} . \tag{10}$$

Hence we see clearly that the nonabelian theory is not symmetric under the Hodge star, as the abelian theory is.

However, this does not mean necessarily that there is no nonabelian generalization to duality. Indeed, it was shown in [6, 7] that there is a generalized dual transform under which nonabelian theory is invariant. This generalized transform (A) reduces to the usual star operation (1) in the abelian case, but (B) does not do so in general in the nonabelian case, as it must not because of the Gu-Yang counter-examples [5].

3. Nonabelian monopoles and loop space

Nonabelian duality is closely connected to the concept of nonabelian monopoles, which in turn is best expressed in the language of loop space. We shall therefore first recall, in this section, some old results [8, 9] on these topics, partly to introduce the notation.

Let us first recall the general definition of a monopole in a gauge theory whether abelian or not. Let G be the gauge group. Then a (magnetic) monopole is defined as the class of closed curves in G [10]. Two curves are in the same class if they can be continuously deformed into each other. For example, if G=U(1), then the monopole charge is given by an integer — this is the original magnetic case. If G=SO(3), then the monopole charge is a sign: ± 1. For the gauge group of the Standard Model, which is our main concern, and which is strictly speaking $SU(3) \times SU(2) \times U(1)/\mathbf{Z}_6$ and not $SU(3) \times SU(2) \times U(1)$ as usually written, the monopole charge is again an integer n. In this case, a monopole of charge n carries (a) a dual colour charge $\zeta = e^{2\pi n i/3}$, (b) a dual weak isospin charge $\eta = (-1)^n$, and (c) a dual weak hypercharge $\tilde{Y} = \frac{2n\pi}{3g_1}$, where g_1 is the weak hypercharge coupling [11].

The monopole charge thus defined is quantized and conserved. But how does one express it in an equation? We found that we can do so using Polyakov's loop space formulation of gauge theory [8, 12].

Let $\xi^\mu(s)$, $s = 0 \to 2\pi$, be a closed curve in spacetime beginning and ending in a fixed point $\xi^\mu(0) = \xi^\mu(2\pi) = x_0^\mu$. Then the phase factor or Wilson loop or holonomy [13] is the following loop-dependent but gauge-invariant element of the gauge group G:

$$\Phi[\xi] = P_s \exp\left(ig \int_0^{2\pi} A_\mu(\xi(s)) \dot{\xi}^\mu(s) ds \right) . \tag{11}$$

where P_s means path-ordering with respect to s. From this we can define the 'loop space connection'

$$F_\mu[\xi|s] = \tfrac{i}{g} \Phi^{-1}[\xi]\, \delta_\mu(s) \Phi[\xi] \tag{12}$$

and the corresponding 'loop space curvature'

$$G_{\mu\nu}[\xi|s] = \delta_\nu F_\mu[\xi|s] - \delta_\mu F_\nu[\xi|s] + ig[F_\mu[\xi|s], F_\nu[\xi|s]] , \tag{13}$$

where $\delta_\mu(s)$ denotes the loop derivative at the point s on the loop.

With this apparatus, one can first of all write down for example an $SO(3)$ monopole of charge -1:

$$G_{\mu\nu}[\xi|s] = \kappa, \quad \exp i\pi\kappa = -1 . \tag{14}$$

Secondly, what is more important, one can prove the so-called extended Poincaré lemma [8], which states that, apart from some minor technical conditions, the vanishing of the loop curvature is equivalent to the existence of a local gauge potential A_μ giving rise to $G_{\mu\nu}[\xi|s]$ in the above manner.

Thus we can now replace the contentless (9) with the true nonabelian analogue of (4):

$$\underbrace{A_\mu \text{ exists}}_{\text{geometry}} \iff G_{\mu\nu} = 0 \iff \underbrace{\text{no magnetic monopoles}}_{\text{physics}} \tag{15}$$

once again linking geometry to physics via a simple condition.

4. Nonabelian duality

Just as we sought a nonabelian version (15) of (4), we now seek to generalize the notion of duality suitable for the nonabelian case. We recall that the abelian duality transformation * satisfies the following two conditions:

(I) It is its own inverse apart from a sign: $*(*F_{\mu\nu}) = -F_{\mu\nu}$.

(II) It interchanges electricity and magnetism: $e \longleftrightarrow \tilde{e}$.

We thus look for a generalized duality transformation for a nonabelian gauge theory which satisfies (I) and (II), requiring that it (A) reduces to * in the abelian case but (B) does not do so in general in the nonabelian case.

First, we need to make clear what is meant by (II) in a nonabelian theory. We recall that for the abelian theory, in the 'electric' description in terms of A_μ, an electric charge is a *source* represented by a nonvanishing current on the right-hand side of (5), while a magnetic charge is a *monopole* which in terms of A_μ is topological in origin but also representable by a nonvanishing dual current on the right-hand side of (2). Hence, for a nonabelian theory, in the 'electric' description in terms of A_μ, an electric charge should also be a *source* represented by nonvanishing current on the right of:

$$D_\mu F^{\mu\nu} = j^\nu , \tag{16}$$

while a magnetic charge should be a *monopole* represented, by virtue of (15), by a nonvanishing loop curvature $G_{\mu\nu}$.

To write down the generalized duality transform, introduce the following set of variables [7]:

$$E_\mu[\xi|s] = \Phi_\xi(s,0) F_\mu[\xi|s] \Phi_\xi^{-1}(s,0) . \tag{17}$$

where

$$\Phi_\xi(s_1, s_2) = P_s \exp\left(ig \int_{s_1}^{s_2} A_\mu(\xi(s)) \dot{\xi}^\mu(s) ds \right) . \tag{18}$$

We see immediately that the E variables are the F variables parallely transported by (18). It is clear that $E_\mu[\xi|s]$ depends only on a segment of the loop $\xi(s)$ around s, and is therefore a 'segmental' variable rather than a full 'loop' variable. In the limit that the segment shrinks to a point, we have

$$E_\mu[\xi|s] \longrightarrow F_{\mu\nu}(\xi(s))\dot{\xi}^\nu(s)\,. \tag{19}$$

However, the limit (19) must not be taken before other loop operations such as loop differentiation are performed, as these loop operations do require at least a segment of loop on which to operate.

It is not too difficult to show that the variables $E_\mu[\xi|s]$ constitute an equivalent set of variables to $F_\mu[\xi|s]$. Using these, we can now define the duality transform by

$$\omega^{-1}(\eta(t))\tilde{E}_\mu[\eta|t]\omega(\eta(t)) = -\frac{2}{\bar{N}}\epsilon_{\mu\nu\rho\sigma}\dot{\eta}^\nu(t)\int \delta\xi ds E^\rho[\xi|s]\dot{\xi}^\sigma(s)\dot{\xi}^{-2}(s)\delta(\xi(s)-\eta(t))\,. \tag{20}$$

At first sight, this is quite unlike (1). However, if we regard the loop dependence of $E^\rho[\xi|s]$ as a continuous index, then the loop integral on the right is nothing but saturating indices, just like the summation on the right hand side of (1). By (19) we see that it is reasonable that the tangents $\dot{\xi}^\sigma(s)$ and $\dot{\eta}^\nu(t)$ should occur. The factor $\bar{N}$ is an (infinite) normalization constant inherent in doing the functional integral. One novel ingredient is the *local* quantities $\omega(x)$ on the left hand side. For concreteness, let us take G=SU(3). Then ω is a 3×3 unitary matrix which represents the change from the frame in internal colour space with respect to which E_μ is defined to the frame in internal dual colour space with respect to which $\tilde{E}_\mu$ is defined. Such a change in frame is necessary to balance the two sides of Eq. (20) since E_μ is 'electrically' charged but 'magnetically' neutral, transforming thus only under SU(3), not under its dual $\widetilde{\mathrm{SU}}(3)$ (see the last section for a discussion of dual gauge symmetries), while for $\tilde{E}_\mu$, the reverse holds. In the abelian case, the factors ω^{-1} and ω commute through and cancel, so that there we do not see this feature. Moreover, we do *not* always have the freedom by gauge transformations to set $\omega = 1$ everywhere, because in the presence of charges either E or $\tilde{E}$ (or both) has to be patched[2], so that ω may have to be patched also. It thus takes on some dynamical properties and, as can be seen in our companion paper [4], the rows or columns of the matrix ω can even be interpreted as the vacuum expectation values of Higgs fields. As such, they play a crucial role in the Dualized Standard Model.

[2] This is similar to the case of the electric potential A_μ in the presence of a magnetic monopole, requiring either patching or equivalently the Dirac string.

Coming back to the duality transform (20), we note that it has been constructed specifically in such a way as to satisfy the condition (I) above and (A) to reduce to the Hodge * in the abelian case yet (B) without doing so for the general nonabelian case [7]. Furthermore, it was shown there that it satisfies also the condition (II) by the following chain of arguments. As known already to Polyakov [12], a *source* in the 'electric' description in terms of A_μ can be represented in his loop notation of (12) as nonvanishing loop divergence $\delta^\mu(s)F_\mu[\xi|s] \neq 0$, which by the relation (17) can also be expressed as nonvanishing loop divergence of $E_\mu[\xi|s]$, namely $\delta^\mu E_\mu[\xi|s] \neq 0$. The duality transform (20), however, is so constructed that a nonvanishing loop divergence for E_μ gives a nonvanishing 'loop curl' for the dual variable $\tilde{E}_\mu$, *i.e.* $\delta_\nu(t)\tilde{E}_\mu[\eta|t] - \delta_\mu(t)\tilde{E}_\nu[\eta|t] \neq 0$. Further, using (17) again, but now for $\tilde{E}_\mu$, it is seen that a nonvanishing 'curl' for $\tilde{E}_\mu$ means nonvanishing loop curvature $\tilde{G}_{\mu\nu}$, or in other words, by the dual of (15), a *monopole* in the 'magnetic' description. Hence, we have that a *source* in the 'electric' description is a *monopole* in the magnetic description. Moreover, because of (I), the converse is also true, namely that a 'magnetic' *source* is the same as an 'electric' *monopole*. This then is the nonabelian generalization of (II) as desired.

For a pure Yang–Mills theory with neither sources nor monopoles, then it follows by (15) that both the potential A_μ and the dual potential $\tilde{A}_\mu$ exist, substantiating thus the claim that the pure theory is symmetric under the dual transform (20). For the situation with sources and monopoles around, however, some more tools are needed.

5. Dynamics and the Wu–Yang criterion

In abelian theory, the equations of motion governing the dynamics of a charge, whether electric or magnetic, can be derived from its topological definition as a monopole by the Wu–Yang criterion [14]. For concreteness, consider first a magnetic charge regarded as a monopole in the electric description in terms of A_μ. Instead of the usual minimally coupled action, one starts with the free field plus free particle action, which one varies under the constraint that there exists a magnetic monopole. Introducing a Lagrange multiplier λ_μ for the constraint, we have

$$\mathcal{A} = -\tfrac{1}{4}\int F_{\mu\nu}F^{\mu\nu} - \int \bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi + \int \lambda_\mu(\partial_\nu {}^*F^{\mu\nu} + 4\pi j^\mu)\,, \qquad (21)$$

where the magnetic current j^μ is given by

$$j^\mu = \tilde{e}\bar{\psi}\gamma^\mu\psi\,, \qquad (22)$$

and $\tilde{e}$ is the magnetic coupling related to the usual electric coupling e by the Dirac quantization condition

$$e\tilde{e} = 2\pi . \tag{23}$$

Here we have assumed the monopole to be a Dirac particle, but we could equally have formulated the procedure classically. Varying with respect to $F_{\mu\nu}$ we get

$$\partial_\mu F^{\mu\nu} = 0 , \tag{24}$$

which is equivalent by (4) and duality to the existence of a dual potential $\tilde{A}_\mu$. In fact we have

$$\tilde{A}_\mu = 4\pi\lambda_\mu , \tag{25}$$

with

$${}^*F_{\mu\nu} = \partial_\nu \tilde{A}_\mu - \partial_\mu \tilde{A}_\nu . \tag{26}$$

Varying with respect to ψ we get

$$(i\partial_\mu \gamma^\mu - m)\psi = -\tilde{e}\tilde{A}_\mu \gamma^\mu \psi . \tag{27}$$

Together with the constraint

$$\partial_\mu {}^*F^{\mu\nu} = -4\pi j^\nu \tag{28}$$

equations (24) and (27) constitute the equations of motion for the field-monopole system [14].

The argument can be repeated for electric charges by regarding them as monopoles in the magnetic description in terms of $\tilde{A}_\mu$. The constraint is then given by

$$\partial_\mu F^{\mu\nu} = -4\pi j^\mu , \tag{29}$$

yielding instead the usual Maxwell and Dirac equations, *i.e.* exactly the duals of (24) and (27). One concludes therefore that electromagnetism is dual symmetric even in the presence of charges.

6. Dynamics of nonabelian charges

We wish next to extend the argument to nonabelian Yang-Mills theory using the formalism developed above. Again we shall use the Wu Yang criterion to study the dynamics of nonabelian charges, regarding them as monopoles. In loop variables, the free field action is

$$\mathcal{A}_F = -\frac{1}{4\pi N}\int \delta\xi ds \mathrm{Tr}(E_\mu E^\mu)\dot{\xi}^{-2} . \tag{30}$$

The free (Dirac) particle action is as before

$$\mathcal{A}_M = \int \bar{\psi}(i\partial_\mu \gamma^\mu - m)\psi . \tag{31}$$

The constraint that there is a monopole is

$$\partial_\nu E_\mu - \partial_\mu E_\nu = -4\pi J_{\mu\nu} , \tag{32}$$

where the magnetic current $J_{\mu\nu}$ has the form

$$J_{\mu\nu}[\xi|s] = \tilde{g}\epsilon_{\mu\nu\rho\sigma}(\dot{v}\omega\gamma^\rho t^i \dot{\xi}^\sigma \omega^{-1}\psi)t_i , \tag{33}$$

and t_i is a generator in the relevant representation of G. The monopole charge is originally given as a nonvanishing loop curvature $G_{\mu\nu}$, which is the loop covariant curl of F_μ. However, as already mentioned above, it can be shown that by going over to the variables E_μ by (17), the loop covariant curl becomes simply the loop curl; hence the constraint (32).

The full action

$$\mathcal{A} = \mathcal{A}_F + \mathcal{A}_M + \int \delta\xi ds \mathrm{Tr}(W^{\mu\nu}(\partial_\nu E_\mu - \partial_\mu E_\nu + 4\pi J_{\mu\nu})) \tag{34}$$

is then varied with respect to the variables $E_\mu[\xi|s]$ and $v(x)$, giving respectively

$$\delta^\mu(s)E_\mu[\xi|s] = 0 , \tag{35}$$

$$(i\partial_\mu \gamma^\mu - m)v(x) = -\tilde{g}\tilde{A}_\mu \gamma^\mu v(x) , \tag{36}$$

where the dual potential $\tilde{A}_\mu$ is given by the Lagrange multiplier $W^{\mu\nu}$, in analogy to (25):

$$\tilde{A}_\mu(x) = 4\pi\epsilon_{\mu\nu\rho\sigma}\int \delta\xi ds \omega(\xi(s))W^{\rho\sigma}[\xi|s]\omega^{-1}(\xi(s))\dot{\xi}^\nu \dot{\xi}^{-2}\delta(\xi(s) - x) . \tag{37}$$

As already noted above, (35) is equivalent to the usual Yang–Mills source-free equation

$$D^\mu F_{\mu\nu} = 0 . \tag{38}$$

To study the dynamics of nonabelian electric charges, we start from Yang–Mills equation (38) with a nonvanishing right hand side. This implies that

$$\delta^\mu(s)E_\mu[\xi|s] \neq 0 , \tag{39}$$

which in turn implies

$$\delta_\nu \tilde{E}_\mu - \delta_\mu \tilde{E}_\nu \neq 0 . \tag{40}$$

But this is the condition that signals the occurrence of a monopole of the $\tilde{E}_\mu$ field (*cf.* (32)). Since the free field action (30) can equally be expressed in terms of the dual variables:

$$\mathcal{A}_F = \frac{1}{4\pi\bar{N}} \int \delta\xi ds \mathrm{Tr}(\tilde{E}_\mu \tilde{E}^\mu)\dot{\xi}^{-2}\,, \tag{41}$$

we can easily derive, by imposing the appropriate constraint

$$\delta_\nu \tilde{E}_\mu - \delta_\mu \tilde{E}_\nu = -4\pi J_{\mu\nu} \tag{42}$$

with an expression for the current similar to (33), the equations of motion of nonabelian electric charges as:

$$\delta^\mu(s)\tilde{E}_\mu[\xi|s] = 0\,, \tag{43}$$

$$(i\partial_\mu\gamma^\mu - m)\psi(x) = -gA_\mu\gamma^\mu\psi(x)\,. \tag{44}$$

We see that the equations of motion for the nonabelian electric charge are exactly the duals of those given above for the nonabelian magnetic charge, namely (35) and (36). Hence we conclude that, as claimed, the dynamics is indeed symmetric under the generalized duality transform (20) even in the presence of charges, just as in the abelian case.

7. Remarks and conclusion

We have presented the salient features of nonabelian duality, without supplying many details. A few remarks, therefore, are in order.

Firstly, since both the potentials $A_\mu(x)$ and $\tilde{A}_\mu(x)$ are local spacetime variables, it may be tempting to speculate that the duality transform (20) itself could perhaps be formulated entirely in terms of local spacetime variables rather than loop variables. At present, we certainly do not know a way of doing that. Suppose we start with the variables $A_\mu(x)$, then by (15) we deduce that in the presence of monopoles $A_\mu(x)$ cannot be everywhere defined. If at the same time there are no sources, then $\tilde{A}_\mu(x)$ is everywhere defined. By duality the existence of sources, while allowing $A_\mu(x)$ to be everywhere defined, forces $\tilde{A}_\mu(x)$ to be undefined in certain regions of spacetime. This means that if there is only one type of charges present (whether monopole or source), we may, by choosing our variables, stick to spacetime variables only. However, if both charges, or dyons, are present, then it seems that loop space variables are inevitable. Unfortunately, the rigorous mathematics of loop space analysis remains largely unexplored [15]. For the work reported above, we have devised certain operational rules which seem to us consistent at least for the use we put them to, but the lack for a general loop calculus

is often acutely felt. For more details, we refer the interested reader to [7] and earlier work cited therein. Nevertheless, the existing operational rules already allow one to explore Feynman diagram techniques using loop space variables [16], which can be a first step towards building a full quantum field theory in these variables.

Secondly, because of dual symmetry, a nonabelian gauge theory is not invariant just under the usual gauge group G but rather two copies of it: G $\times\tilde{\mathrm{G}}$. Here we denote the group under which $\tilde{A}_\mu$ transforms as $\tilde{\mathrm{G}}$, although as a group it is identical to G. This makes it easier notationally and also underlines the fact that $\tilde{\mathrm{G}}$ has parity opposite to that of G, because of the ϵ-symbol in the transform (20). This extra symmetry is a direct consequence of duality, which in turn is inherent in any gauge theory. That this symmetry has interesting physical consequences will be shown in detail in our companion article [4]. We note further that although the gauge symmetry is found to be doubled, the number of degrees of freedom remains the same. In a way not yet fully explored, the potentials $A_\mu(x)$ and $\tilde{A}_\mu(x)$ represent the same degrees of freedom, since the duality transform (20) is an equation relating $E_\mu[\xi|s]$ and $\tilde{E}_\mu[\xi|s]$. The situation is even more immediately evident in the abelian case. Under a U(1) transformation $\lambda(x)$,

$$\begin{aligned} A_\mu(x) &\mapsto A_\mu(x) + \partial_\mu\lambda(x)\,, \\ \tilde{A}_\mu(x) &\mapsto \tilde{A}_\mu(x)\,; \end{aligned} \tag{45}$$

while under a $\widetilde{\mathrm{U}(1)}$ transformation $\tilde{\lambda}(x)$,

$$\begin{aligned} A_\mu(x) &\mapsto A_\mu(x)\,, \\ \tilde{A}_\mu(x) &\mapsto \tilde{A}_\mu(x) + \partial_\mu\tilde{\lambda}(x)\,. \end{aligned} \tag{46}$$

The two phases $\lambda(x)$ and $\tilde{\lambda}(x)$ are entirely independent. Similarly the wave function $\psi(x)$ of an electric charge and the wave function $\tilde{\psi}(x)$ of a magnetic monopole will transform under $\lambda(x)$

$$\begin{aligned} \psi(x) &\mapsto \mathrm{e}^{i\lambda(x)}\,\psi(x)\,, \\ \tilde{\psi}(x) &\mapsto \tilde{\psi}(x)\,; \end{aligned} \tag{47}$$

and under $\tilde{\lambda}(x)$

$$\begin{aligned} \psi(x) &\mapsto \psi(x)\,, \\ \tilde{\psi}(x) &\mapsto \mathrm{e}^{i\tilde{\lambda}(x)}\,\tilde{\psi}(x)\,. \end{aligned} \tag{48}$$

However, the variables $A_\mu(x)$ and $\tilde{A}_\mu(x)$ clearly do not represent different degrees of freedom, because their field tensors $F_{\mu\nu}(x)$ and ${}^*F_{\mu\nu}(x)$ are related

by the following *algebraic* equation

$$^{*}F_{\mu\nu}(x) = -\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}(x)\,. \tag{49}$$

That A_μ and $\tilde{A}_\mu$ should correspond to two gauge symmetries but yet represent the same physical degree of freedom can have very interesting physical consequences [3, 4, 21, 23].

Thirdly, since $\tilde{A}_\mu(x)$ is a local field, we can construct *its* phase factor:

$$\tilde{\Phi}[\xi] = P_s \exp\left(i\tilde{g}\int_0^{2\pi}\tilde{A}_\mu(\xi(s))\dot{\xi}^\mu(s)ds\right) \tag{50}$$

in complete analogy to the familiar $\Phi[\xi]$ in (11). Now, in the famous work of 't Hooft [2] on confinement the trace of $\Phi[\xi]$ has a very important role to play as an order parameter which he called $A(C)$, depending on the loop C. Hence, by the duality discussed above, one expects that the trace of $\tilde{\Phi}[\xi]$ in (50) will play the role of 't Hooft's disorder parameters $B(C)$ [2]. This turns out to be indeed the case. Using Dirac's quantization condition

$$g\tilde{g} = 4\pi \tag{51}$$

it was shown [17] that the traces of Φ and $\tilde{\Phi}$ do indeed satisfy the commutation relation

$$A(C)B(C') = B(C')A(C)\exp\left(\frac{2\pi i n}{N}\right) \tag{52}$$

for G=SU(N), as required by 't Hooft for his order-disorder parameters [2]. It follows then that we can apply 't Hooft's confinement result [2] to our situation, namely that if the G symmetry is confined then the $\tilde{G}$ symmetry as defined above is broken and Higgsed, and vice versa. As can be seen in our companion paper [4], this plays a crucial role in the Dualized Standard Model [3]. When applied, for example, to the confined colour group SU(3), it implies a completely broken dual colour symmetry $\widetilde{SU}(3)$ which may be identified with generations. The explicit form (50) for the 't Hooft disorder parameter $B(C)$, which up to quite recently was known only by a somewhat abstract definition, is likely to be useful also in the problem of confinement [18].

Apart from giving rise to the physical consequences reviewed in [4], ranging from masses of fermions and their mixing [19] to flavour-changing neutral current decays [20–22] and very high energy cosmic rays [20, 21, 23] the considerations above raise also some intriguing theoretical questions that are

beginning to be asked. For example, throughout this lecture so far we have been concerned only with the nonabelian generalization of electric–magnetic duality in a strictly non-supersymmetric context and in exactly 4 spacetime dimensions. We have not touched upon the possible extension to supersymmetry and/or higher spacetime dimensions. This could be interesting, given the vast amount of exciting work [24] which has been done in recent years following the seminal papers of Seiberg and Witten [25] on supersymmetric duality. In a completely different direction, the doubling of the symmetry is reminiscent of complexification in geometry and particularly general relativity. One would like to know how this generalized duality relates to the vast literature of self-dual fields, both in geometric Yang–Mills theory and in general relativity, especially in the twistor description [26,27]. The vistas that are being opened up are truly fascinating.

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