STANDARD MODEL WITH DUALITY: PHYSICAL CONSEQUENCES *

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The Dualized Standard Model offers a natural place both to Higgs fields and to fermion generations with Higgs fields appearing as frame vectors in internal symmetry space and generation appearing as dual colour. If they are assigned those niches, it follows that there are exactly 3 generations of fermions, and that at the tree-level, only one generation has a mass (fermion mass hierarchy) while the CKM matrix is the identity. However, loop corrections lift this degeneracy giving nonzero CKM mixing and masses to fermions of the two lower generations. A recent calculation to 1-loop level, with just a few parameters, yields a very good fit to the empirical CKM matrix and sensible values also to the lower generation masses. In addition, predictions are obtained, in areas ranging from low energy flavour-changing neutral current decays to extremely high energy cosmic rays, which are testable in experiments now being planned.

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As is well-known, the Standard Model works very well in reproducing experimental results. However, it contains within itself also a number of widely-recognized shortcomings. For instance, at the more fundamental level, Higgs fields and fermion generations are introduced into the model

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as phenomenological requirements to fit experimental facts without any theoretical reason being given for why they should be there in the first place. At the more practical level, this basic lack of understanding is reflected in the large number of empirical parameters. And even more disturbingly, the values of these parameters are seen to reveal some startling patterns of the greatest physical significance, and yet no theoretical explanation is given for their existence. For example, there is first the so-called fermion mass hierarchy puzzle. The masses of the U-type quarks, for example, are quoted in the latest databook [1] as $m_t = 176 \pm 5$ GeV, $m_c = 4.1 - 4.5$ GeV, $m_u = 3 - 8$ MeV, dropping by more than two orders of magnitude from generation to generation. The mass patterns of the D-type quarks and charged leptons are similar, although the drop from generation to generation is there a little less dramatic. Then secondly, there is the peculiarity in the mixing pattern, say for example between the U-type and D-types quarks as parametrized by the CKM matrix [2]. This matrix is found experimentally to be close to unity but yet differs significantly from it, with off-diagonal elements ranging in magnitude from about 22 percent to a few parts permille. Now these, the fermion masses and CKM matrix elements, are all parameters on which the properties of our whole physical world crucially depend. It would surely be disappointing if we can find no understanding at all why they should take the values they take or fall into the noted patterns as they do.

Answers to these questions are often sought for from beyond the Standard Model but with, to our minds, no obvious great success. The difficulty is that in going beyond the Standard Model, one opens up a wide range of freedom for theoretical constructions and so often ends up by putting in more than one gets out. In this paper, we wish to describe an approach we have recently suggested [3], in which we attempt to find answers for these same questions from within the framework of the Standard Model itself. This would have at least the advantage of economy and restraint, if it proves at all possible. The Standard Model, however, has already been closely studied, and it is not obvious that one can still find room enough in it to accommodate the structures of present interest. On the other hand, gauge theory is extremely rich in structure, and there are areas in it which are still largely unexplored.

Our new proposal, indeed, is based on a nonabelian generalization of electric-magnetic duality which was discovered only recently [4]. When combined with 't Hooft's famous result [5] on confinement of 1978, which we shall refer to here as the 't Hooft Theorem, this generalized electric-magnetic duality becomes extremely powerful, and when interpreted in a certain way, leads to the following results. First, it gives rise to a new quantum number which shares some properties with the fermion generation index, and to certain scalar fields which can play the role of Higgs fields. Secondly, if these two abstract quantities occurring naturally in the theory are identified respectively with the physical objects that they resemble, then one predicts straightforwardly (*i*) that there exist 3 and only 3 generations of fermions of each type, (*ii*) that there is a fermion mass hierarchy with one generation of fermions much heavier than the other 2, and (*iii*) that the CKM matrix is close to the identity matrix. All these features are qualitatively as experimentally observed.

At the tree-level, the 2 lower generation fermions of each type have vanishing masses and the CKM matrix equals the identity matrix. Loop corrections, however, lift this tree-level degeneracy and give small but nonzero values both to the masses of the 2 lower generation fermions and to the off-diagonal CKM matrix elements. What is more, these corrections are found to be calculable perturbatively. A calculation to 1-loop has already been carried out. It is found that with just a few parameters one is able to obtain a very good fit to the empirical CKM matrix as well as sensible values to the fermion masses, a result which we shall summarize later.

Further, as a consequence to the approach, new Higgs and gauge bosons carrying generation indices are predicted at masses in the 100 TeV range. Their direct detection is probably out of reach even for LHC, but their exchange will lead to new effects which may be detectable by some experiments now being planned. Of these effects, we shall consider 2, namely (a) flavour-changing neutral current decays and (b) cosmic ray air showers at ultra-high energies. As we shall see, interesting results are obtained for both.

In this paper, we shall deal only with the general framework and the physical consequences. For the theoretical basis of the approach, the reader is referred to our companion paper [6] in the same volume.

To set up the framework for our discussion, let us first recall the basic tenets of electric-magnetic duality. As is well-known, electromagnetism is dual symmetric in the sense that in addition to the Maxwell potential $A_\mu(x)$ with which the theory is usually described there exists also (under certain conditions) a dual potential $\tilde{A}_\mu(x)$ which is to magnetism what $A_\mu(x)$ is to electricity. Thus, for example, while in the description in terms of $A_\mu(x)$, the electric charge e is a source and the magnetic charge $\tilde{e}$ is a monopole, so in the description in terms of $\tilde{A}_\mu(x)$, the electric charge will appear as a monopole while the magnetic charge will appear as a source. Further, since the theory possesses a local $U(1)$ gauge symmetry, it will possess also a local gauge symmetry under the dual gauge group $\tilde{U}(1)$, so that the theory is invariant in all under the doubled gauge symmetry $U(1) \times \tilde{U}(1)$, where $\tilde{U}(1)$ is the same group as $U(1)$ but has the opposite parity. Notice, however, that although the gauge symmetry is doubled so that there seems to be twice the usually acknowledged number of gauge degrees of freedom, the number of physical degrees of freedom remains the same. This can be seen as follows.

In electromagnetism, the dual transform is an operation by the usual Hodge star:

$$*F_{\mu\nu}(x) = -\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}(x)\,, \tag{1}$$

which means in electromagnetism essentially just interchanging E and H. Eq. (1) is an algebraic relation giving $*F$ explicitly in terms of F so that obviously no new physical degrees of freedom have been introduced. Nevertheless, with F derivable from a potential A, thus:

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) \tag{2}$$

and $*F$ given by a similar expression in terms of $\tilde{A}$, one sees that one can perform independent gauge transformations on A and $\tilde{A}$ without changing either the physically measurable quantities F and $*F$ or the dual relation between them. In spite of the doubled gauge degree of freedom, therefore, there is only one photon, not two.

Given the importance of nonabelian gauge theories to particle physics, a natural question to ask is whether the above dual symmetry known in electromagnetism extends to Yang–Mills fields. If duality is defined by the Hodge star in (1) as in the abelian theory, then the answer has long been known to be no [7]. There is in general no guarantee in the nonabelian theory for the existence of the dual potential $\tilde{A}_\mu(x)$ having the same relation to $*F_{\mu\nu}(x)$ as $A_\mu(x)$ has to $F_{\mu\nu}(x)$. However, it was shown in [4] that if one defines the dual transform differently, thus:

$$\omega^{-1}\tilde{E}_\mu\omega = -\frac{2}{\bar{N}}\epsilon_{\mu\nu\rho\sigma}\dot{\eta}^\nu \int \delta\xi ds E^\rho \dot{\xi}^\sigma \dot{\xi}^{-2}\delta(\xi - \eta), \tag{3}$$

which reduces to (1) for the abelian but not for the nonabelian theory, then the dual symmetry is restored, meaning that there is a dual potential $\tilde{A}_\mu(x)$ bearing the same relationship to $\tilde{E}_\mu$ as the ordinary Yang–Mills potential $A_\mu(x)$ to E_μ. Here the field is given in terms of some loop-dependent quantity E_μ instead of the usual local field tensor $F_{\mu\nu}(x)$, and the integral in (3) is also a loop integral, so that the generalized dual transform is not as simple as it looks. However, for the discussion in this paper of the physical consequences of the approach, we shall not need to know much of the details involved. In the companion paper [6], we shall summarize for the theoretically-minded reader the rather intricate arguments leading to the above conclusion. Here, we need to note only in (3) the quantity $\omega(x)$ which is a local transformation matrix transforming from the local frame in which E_μ is measured to the dual frame in which $\tilde{E}_\mu$ is measured. For what follows, this ω will play an important role.

As a consequence of the generalized dual symmetry derived in [4], one recovers the analogy with the abelian theory desired. Take for example the standard chromodynamics with an SU(3) gauge symmetry. In the description in terms of $A_\mu(x)$, a colour electric charge g is a source while a colour magnetic charge $\tilde{g}$ is a monopole, but in a description in terms of $\tilde{A}_\mu(x)$, g will appear as a monopole while $\tilde{g}$ will appear as a source. And the theory will have in all the parity-doubled local gauge symmetry SU(3)×$\widetilde{\text{SU}}$(3), in close parallel to the U(1)×$\tilde{\text{U}}$(1) symmetry of electromagnetism noted above. Again, however, as in electromagnetism, this doubling of the gauge symmetry implies no increase in the number of physical degrees of freedom.

Next, let us turn to what we call here the 't Hooft Theorem [5]. This says that in a theory with gauge symmetry SU(N) (which we call here generically "colour"), if colour is confined, then dual colour is Higgsed, and conversely, if colour is Higgsed, then dual colour is confined. By duality here, however, one means a certain commutation relationship between two loop-dependent operators, called by 't Hooft the order–disorder parameters $A(C)$ and $B(C)$, the former being the Wilson phase factor:

$$A(C) = \text{Tr} P \exp\{ig \int_C A_\mu(x) dx^\mu\} . \tag{4}$$

A priori, this need not mean the same thing as the duality discussed in the preceding paragraph as defined in (3). However, we were able to show in a recent paper [8] that the operator $B(C)$ constructed in the same way as (4) above, but with instead of $A_\mu(x)$ the dual potential $\tilde{A}_\mu(x)$ as given by (3), and instead of g a dual coupling $\tilde{g}$ related to g by the Dirac quantization condition:

$$g\tilde{g} = 4\pi , \tag{5}$$

does satisfy the 't Hooft commutation relation, so that the two usages of the term duality are in fact interchangeable and that the results can be combined.

If that is the case, then the combination would be extremely powerful, leading to some very interesting consequences. In particular, suppose we apply it to the Standard Model with the gauge symmetry SU(3)×SU(2)×U(1). Then according to [4], the theory would also have a dual gauge symmetry $\widetilde{\text{SU}}(3) \times \widetilde{\text{SU}}(2) \times \tilde{\text{U}}(1)$. Further, since in the usual interpretation of the Standard Model SU(3) colour is confined while the electroweak SU(2) symmetry is broken and Higgsed, the 't Hooft theorem [5] implies that the dual colour symmetry $\widetilde{\text{SU}}(3)$ would be broken while dual weak isospin, corresponding to the symmetry $\widetilde{\text{SU}}(2)$, would be confined. In other words, the roles of the two dual groups would be interchanged.

How would these dual symmetries, if they exist, manifest themselves in the physical world? Consider first dual colour. Presumably, as in other symmetries, particles will form representations of this dual group. In particular, we expect that there will be fermions in the fundamental representation of $\widetilde{SU}(3)$ forming triplets of dual colour. However, the symmetry being broken, the members of a triplet will behave somewhat differently, having for example possibly different masses. They would thus be rather like the members of different generations of any particular fermion type, say the U- or D-type quarks or the charged leptons or neutrinos. In other words, dual colour, necessarily broken, would seem to offer a natural niche for the generation index to fit into. The beauty is that if the generation index is assigned that niche, then it follows that there will naturally be 3 and only 3 generations of fermions, a fact which seems to be strongly supported by present experiment.

Would such an assignment work? To answer this question, we have first to understand a little about the symmetry-breaking pattern of dual colour. As for other gauge symmetries, such as weak isospin, we expect the dual colour group also to be broken spontaneously via the Higgs mechanism. But are there scalar fields around in the theory to play the role of Higgs fields for breaking this symmetry, or do they have to be introduced *ad hoc* as in the usual formulation of the electroweak theory? An interesting feature of the dual framework is that there indeed are scalar fields occurring naturally in the theory which have the potential for being Higgs fields. Recall the transformation matrix ω introduced in the dual transform (3) above. For colour, this is a 3×3 space-time dependent unitary matrix transforming from the colour frame to the dual colour frame. Its columns therefore transform as a 3 of dual colour, *i.e.* as the fundamental representation of $\widetilde{SU}(3)$, while its rows transform as a 3 of colour $SU(3)$. Under Lorentz transformations, however, they are space-time scalars. Moreover, the matrix ω being unitary, its rows and columns have unit (nonvanishing) lengths. They share therefore many properties that one would want for the vacuum expectation values of Higgs fields. Indeed, if one repeats the same arguments for the electroweak theory, one finds that they would work very well as the Higgs fields normally required for symmetry-breaking in that theory.

At first sight, it might seem rather revolutionary to consider the rows and columns of ω as Higgs fields, but at second look, the move is not so new as it appears. The rows and columns of ω are basically just the frame vectors in internal symmetry space. Their geometrical significance is therefore not very different from the vierbeins in General Relativity, and in that theory one is used to regarding the vierbeins as dynamical variables. In making the frame vectors in internal space into Higgs fields, one is in a sense copying what is standard procedure in relativity, and at the same time giving to Higgs fields a geometrical significance which they otherwise lack. Conceptually, of course, a geometrical significance for Higgs fields would be most welcome in the gauge theory framework where all the other (gauge and fermion) fields are known to have deep geometrical significance.

Suppose then we accept this proposal and apply it to $\widetilde{SU}(3)$ of dual colour. One obtains first 3 dual colour triplets of Higgs fields and, if one makes what seems the simplest assumption about the dual hypercharges they carry, the further result that $\widetilde{SU}(3)$ is completely broken with no residual symmetry [3].

Let us denote these Higgs fields by $\tilde{\phi}^{(a)}_{\tilde{a}}$, where $(a) = 1, 2, 3$ are just labels for the 3 triplets while $\tilde{a} = 1, 2, 3$ denotes the dual colour or the generation of their components. How would they couple to the fermions? We recall that the couplings of Higgs fields to fermions (Yukawa couplings) are what give us the tree-level fermion mass matrices. Now fermions are known to exist in 3 generations so that it will be natural to assign them in the present scheme to dual colour triplets, thus: $\psi_{\tilde{a}}, \tilde{a} = 1, 2, 3$. However, not both the left- and right-handed fermions can be assigned to dual colour triplets or otherwise one cannot construct a Yukawa coupling for them. Taking then a clue from the Weinberg–Salam electroweak theory, one proposes to make the left-handed fermions ψ_L dual colour triplets and the right-handed fermions ψ_R dual colour singlets. In that case, one can write a Yukawa coupling in the form:

$$\sum_{(a)} \sum_{[b]} Y_{[b]} \bar{\psi}^{\tilde{a}}_L \tilde{\phi}^{(a)}_{\tilde{a}} \psi^{[b]}_R \,. \tag{6}$$

To obtain the tree-level fermion mass matrix, one just substitutes as usual for the Higgs fields their vacuum expectation values, which we may take as:

$$\tilde{\phi}^{(1)} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}; \quad \tilde{\phi}^{(2)} = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}; \quad \tilde{\phi}^{(3)} = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}, \tag{7}$$

giving:

$$m = \begin{pmatrix} xa & xb & xc \\ ya & yb & yc \\ za & zb & zc \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} (a, b, c) \,. \tag{8}$$

with x, y, z real and $a = Y_{[1]}, b = Y_{[2]}, c = Y_{[3]}$ in general complex. One notes that the mass matrix is factorizable as indicated.

Now a factorizable mass matrix as that in (8) leads to 2 important immediate consequences. First, as noted already by many others but particularly by Fritzsch [9], a rank 1 matrix such as (8) has only one nonzero eigenvalue which may be interpreted as one generation having a much larger mass than the other two generations and hence a reasonable zeroth-order description of the observed fermion mass hierarchy mentioned at the beginning. Second, the first factor (x, y, z) in (8) depends only on the vacuum expectation values of the Higgs fields but not on the fermion type, so that the CKM matrix, which depends only on the first factor and not on the second, will automatically be the identity matrix at tree-level, again a reasonable zeroth-order description of the empirical CKM matrix as already mentioned. The interesting thing is that both these features which are much desired by phenomenology here follow spontaneously as a natural consequence of the theoretical framework.

The above scenario, though desirable as a zeroth-order approximation, is obviously not good enough as a realistic description of nature, for the 2 lower generation fermions, though light compared with the highest generation, are not actually massless, and the CKM matrix does in fact differ considerably from the identity. Hence, the scheme can only be considered acceptable if it is capable also of explaining nonzero lower generation masses and the deviations of the CKM matrix from the identity. Our contention, to be supported by the results summarized below, is that it is indeed capable of doing so via loop corrections.

Within the scheme, there are of course many different types of possible loop diagrams exchanging gluons or dual gluons, Higgses or dual Higgses. However, because of the unusual properties built into the scheme, these loop diagrams share a common property, namely that they leave the mass matrix factorizable. That this is so can be seen as follows. First, ordinary colour gluons and the standard electroweak Higgses do not affect the generation (*i.e.* dual colour) index since they themselves carry no dual colour, and therefore corrections due to loops exchanging these bosons will leave a factorizable mass matrix still factorizable. Secondly, although the dual gluons do affect the generation or dual colour index they can couple only to the left-handed fermions. This means that they will only alter the left-handed factor of the factorized mass matrix but will leave it still factorized. Lastly, the dual colour Higgses both affect the dual colour index and couple to both left- and right-handed fermions and so are a potential danger to factorizability, but their couplings to the fermions, being closely related to the mass matrix, is itself factorizable. Because of this, it is not difficult to see that loops of dual colour Higgses will also leave the mass matrix factorizable. Thus, by examining each type of exchanges in turn, one easily convinces oneself that no 1-loop diagram of any type will modify the factorizable nature of the mass matrix. Indeed, we are of the opinion, though cannot claim to have rigorously demonstrated the fact, that even higher loops of any order will still leave the factorizability of the mass matrix intact.

Although the mass matrix remains factorized under loop corrections, this does not mean necessarily that the lower generation masses must remain zero or that the CKM matrix must remain the identity. For the CKM matrix, this is readily seen. For example, the dual gluon loop, as explained above, though leaving factorizability intact, modifies the left-hand factor of the mass matrix, and this modification can depend on whether the fermion addressed is the U-type or the D-type quark. Hence, the CKM matrix which basically measures the relative orientation of these left-hand factors of respectively the U-type and D-type quarks need no longer be the identity when the dual gluon loop correction is taken into account.

That the lower generation fermion masses also need not remain zero after loop corrections is not so obvious and is in fact quite intriguing. Given that the mass matrix is still factorizable after loop corrections and hence still of rank 1, it follows that it has always only one nonzero eigenvalue. However, it is not obvious that the 2 remaining zero eigenvalues ought to be interpreted as the masses of the 2 lower generations. The point is that, though still factorizable, the mass matrix can be rotated by the loop corrections and this rotation depends on the renormalization scale. And once a mass matrix has an orientation which is scale-dependent, it is not so obvious what ought to be defined as the masses and the state vectors of the physical states. Indeed, we believe that this question would have arisen already in the usual (*i.e.* nondualized) standard model, had the effect there not been so negligibly small. The ambiguity comes about as follows. Consider first a mass matrix with a scale-independent orientation. Then once it is diagonalized at some scale it will remain diagonal at any other scale. And if it is a hermitian matrix, then its eigenvectors will be orthogonal as physical state vectors ought to be. It would then be appropriate, as is usually done, to define the mass of each physical state as the appropriate eigenvalue evaluated at the scale equal to its value, as one would as if only one state is involved. However, if the matrix rotates as the scale changes, then the eigenvector of one eigenvalue at the scale equal to its value will not usually be orthogonal to the eigenvetor of another eigenvalue evaluated at the other scale equal to its own value. Hence, these two vectors can no longer be associated with two independent physical states. In fact, we do not know a valid criterion for defining the masses and physical states from a general mass matrix which rotates with changing scales.

However, for the special case we have here of a factorizable mass matrix, it is possible to define the masses and the states in such a way that each mass is evaluated at the scale equal to its value and still have all the physical state vectors mutually orthogonal. Let us illustrate the problem with the U-type quarks. At any scale, the factorizable mass matrix has only one nonzero eigenvalue. If one evaluates the mass matrix at the scale equal to this eigenvalue, then one can unambiguously define this value as the mass of the top quark, and the corresponding eigenvector as the physical top state vector. The other 2 eigenvalues at this scale are zero but they ought not for consistency to be identified as the masses of the lower states c and u

for they would be evaluated at the wrong scale. Also, at this stage, we do not know which are the physical state vectors corresponding to respectively the c and u states. However, we do know that the state vectors of c and u, being by definition orthogonal to the state vector of the top, have to lie in the subspace spanned by the zero eigenvectors of the mass matrix evaluated at the top-mass scale. Suppose now we run the scale to a lower energy. Since the mass matrix rotates with changing scales, the 2 zero eigenvectors at the top mass will no longer be zero eigenvectors at the new scale, so that the 2×2 mass submatrix in the subspace spanned by the 2 originally zero eigenvectors need no longer be zero. However, it will still be factorizable and of rank 1, and has therefore again only one nonzero eigenvalue, a situation exactly the same as that we started with for the full mass matrix. For logical consistency, therefore, one should again evaluate this nonzero eigenvalue of the submatrix at a scale equal to its value and define this value as the mass of the second generation, namely that of the charm quark c. It follows also that the corresponding eigenvector at this scale should be defined as the c physical state vector which, by definition, will be automatically orthogonal to the top state vector already defined. Having then defined both the t and c physical state vectors, one can unambiguously define (up to a sign) the physical u state vector as the vector orthogonal to both. Furthermore, by repeated the procedure once more running the scale even lower in energy, one can clearly define also the u mass. In this way, all masses and state vectors are uniquely defined, each mass is evaluated at the scale equal to its value, and the 3 physical state vectors are mutually orthogonal, as is appropriate.

Though described above only in words, this procedure for evaluating the CKM matrix and the lower generation fermion masses is not merely a theoretical prescription but one that can be put into actual practice. Indeed, a calculation in this direction to 1-loop level has already been done [10]. The calculation being somewhat complicated and containing a number of quite intriguing details, we have space here only to give a bare outline of the main steps involved and to summarize the main results.

First, among the many 1-loop diagrams calculated, some are found to be large and not calculable perturbatively. They affect, however, only the normalization of the mass matrix, not its orientation which, as explained above was of the most interest. It pays, therefore, at present to abandon the calculation of the normalization and focus one's attention just on the orientation. This reduction in objective then removes the necessity for evaluating several of the diagrams. Secondly, on putting in the estimate for the dual gluon mass obtained from the stringent empirical bounds on flavour-changing neutral current decays, one finds that most of the remaining diagrams affect the orientation of the mass matrix only to a negligible degree, leaving in the end only one diagram that really matters, namely the dual colour Higgs loop. Thirdly, by a rather fortunate accident, the effect of this last diagram is to a very good approximation independent of some parameters, such as the Higgs boson masses, on which it formally depends. Fourthly, by adjusting, among the remaining parameters, the strength of the Yukawa coupling ρ, one finds one can indeed account for the fermion mass of the second generation as a 'leakage' from the highest by the procedure detailed above. Then fixing the mass scales and Yukawa couplings, one each for each fermion type, by fitting the masses of the two higher generations, one is finally left with only 2 real parameters, namely the 2 ratios between the 3 vacuum expectation values x, y, z of the dual colour Higgs fields, with which to evaluate the CKM matrix and all the fermion masses of the lowest generation.

Emphasis was put on fitting the CKM matrix, which depends only on the orientation, rather than on the lowest generation masses which depend also on the as yet incalculable normalization of the mass matrices. The following is a sample of the sort of fits obtained:

$$|V| = \begin{pmatrix} 0.9752 & 0.2215 & 0.0048 \\ 0.2211 & 0.9744 & 0.0401 \\ 0.0136 & 0.0381 & 0.9992 \end{pmatrix}. \tag{9}$$

which is to be compared with the following experimental values entered in [1]:

$$|V| = \begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993 \end{pmatrix}. \tag{10}$$

The agreement is seen to be good. This we find encouraging since it is not at all obvious that the CKM matrix can be so fitted. In particular we note the large value of V_{cd} and V_{us}, *i.e.* the Cabibbo angle, compared with the other elements. This comes about directly from the special way described above of how the lower generation states are defined by 'running' and can thus be considered as some confirmation of its validity. We note, however, that all CKM matrix elements in the calculation are real, so that at least at the 1-loop level we have worked with so far, there is no possibility of a CP-violating phase.

There are two rather astounding features common to all the fits we have found thus far: (a) The approximate equality, to a few percent accuracy, of all the Yukawa coupling strengths ρ for the 3 fermion types that we have fitted, namely the U- and D-type quarks and the charged leptons. (b) The proximity, to within around 1 part in ten thousand, of the normalized vector (x, y, z) representing the vacuum expectation values of the dual colour Higgs fields to one of its fixed points $(1, 0, 0)$. These seem to us possibly indicative of a deeper symmetry that we do not yet understand. In particular, the fitted values of the ρ's are so close that one could easily obtain as good a fit as the best by requiring all ρ's to be identical. In other words, had we known a theoretical reason why the ρ's should be the same, we could have fitted very well all CKM matrix elements and all masses of the second generation with only 3 parameters, namely one common ρ and the 2 ratios of (x, y, z).

Further, an attempt may also be made to estimate the fermion masses of the lowest generation by the method outlined above. However, in contrast to the calculation of the CKM matrix which depends only on the orientation of the mass matrix, the estimates for the lowest generation fermion masses depend also on the change with scales of the normalization, not only of the mass matrix but also of the Yukawa coupling strength ρ. If we naively just assume that both the normalizations of the mass matrix and ρ are constants independent of scale changes, then, using the same parameters (9) as those determined above in fitting the CKM matrix, one obtains:

$$m_u = 235 \text{ MeV}, \quad m_d = 17 \text{ MeV}, \quad m_e = 7 \text{ MeV}, \tag{11}$$

which are to be compared with the experimental values quoted in [1]:

$$m_u = 2 - 8 \text{ MeV}, \quad m_d = 5 - 15 \text{ MeV}, \quad m_e = 0.5 \text{ MeV}. \tag{12}$$

Considering that in obtaining the values in (11) scale-dependences of normalizations have been neglected over several orders of magnitude in energy, we regard the estimates as quite sensible except perhaps for the u, for which a change in scale over 4 decades of energy is involved.

Suppose we tentatively accept these results as reasonable confirmation that the dualized standard model is capable of explaining the mass and mixing patterns of fermions, our next question should be whether the scheme leads also to new predictions which can be tested against experiment. The most obvious to examine first is the predicted existence of a new batch of particles, namely the dual colour gauge and Higgs bosons. At first sight, it appeared that the calculation of 1-loop effects summarized above might give an estimate of these bosons' masses, but unfortunately for this purpose, though fortunately for the calculation itself as already noted, the result is to a good approximation independent of these mass parameters. On the other hand, the stringent experimental bounds on flavour-changing neutral current decay give a lower limit for the dual gluon mass of around several 100 TeV, corresponding to a lower limit on the dual Higgs mass of several 10 TeV. If these limits are accepted, then it is unlikely that these particles can be produced even by the LHC. However, the exchange of these particles can lead to effects detectable in experiment. Flavour-changing neutral current decays, as already noted, are one such example.

Another prediction of this sort, which at first sight looks quite alarming, is that of a strong interaction for neutrinos at high energy. This arises as follows. Identifying generation with dual colour implies that neutrinos also carry dual colour and hence will interact via dual gluon exchange. Now the coupling of the dual gluon is related to the coupling of the gluon by the Dirac quantization condition (5). Substituting the experimental value of $\alpha_s = (g^2/4\pi) \sim 0.120$ gives a value for $\tilde{g}$ of order 10, which is very large. There is thus predicted a very strong interaction between neutrinos due to the dual gluon exchange. However, because the dual gluon is very heavy, this interaction will not be effective at energies available to present or near future laboratories. To search for this effect we shall have to look to cosmic rays.

Now it so happens that there is indeed a long-term puzzle in cosmic ray physics which seems explainable as a manifestation of this phenomenon. Over the last 30 years or so, a small number of very high energy air shower events with primary energy $E > 10^{20}$ eV have been observed [11]. They are a mystery because in theory they should not exist. High energy air showers are thought to be mostly due to protons, but protons at the energy beyond 5×10^{19} eV will quickly lose it via its interaction with the 2.7 K microwave background. Indeed, it was shown by Greisen [12], and by Zatsepin and Kuźmin [13], that protons with energy in excess of that cut-off cannot reach us from a distance of more than 50 Mpc. At the same time, protons at such energies will hardly be deflected by the magnetic fields either in the galaxy or in intergalactic space so that it should be easy to identify any candidate sources within a radius of 50 Mpc. However, searches along the direction of the observed events fail to reveal any likely source within that sort of distance. The conclusion would seem thus to be either that there are some rather exotic sources nearby without us knowing about them or that these air showers are not due to protons at all but to some other particles.

An interesting possibility for the present scheme is that they are due to neutrinos having acquired strong interactions at high energy as predicted above. This explanation not only seems feasible but appears even capable of overcoming several difficulties plaguing the proton explanation. First, neutrinos, being neutral in charge, would not interact with the 2.7 K microwave background as the protons would and can therefore reach us with energy above the Greisen–Zatsepin–Kuźmin cut-off even if they have originated from a distant source much beyond 50 Mpc. Secondly, since they are strongly interacting at high energy, they can be produced copiously in high energy collisions, say of protons in an active galactic nucleus but, in contrast to protons, can escape from the strong radiation fields which are thought to surround active galactic nuclei. Thirdly, when they arrive on earth, their strong interactions with the air nuclei together with the fact that dual gluons and gluons, as mentioned already in the beginning, represent basically the same physical degree of freedom [14], can give them a sufficiently large cross section to initiate air showers as observed. (On this point, we disagree with the conclusion of a recent paper by Burdman, Halzen and Gandhi [15] which claims the opposite. See *e.g.* also [16].) Indeed, working in this direction, it is even possible to make a rough estimate for the high energy neutrino-air nucleus cross section and hence suggest direct experimental tests for the hypothesis that air showers above the Greisen–Zatsepin–Kuźmin cut-off are due to neutrinos rather than protons [14]. It seems thus that the prediction of a strong interaction for neutrinos at high energy not only may not prove to be an embarrassment but may even help to resolve a long-term puzzle in cosmic rays physics.

A neutrino at 10^{20} eV primary energy impinging on a proton at rest in the atmosphere corresponds to a CM energy of 400 TeV, which we note is just above the lower bound imposed on the dual gluon mass by the present experimental bounds on flavour-changing neutral current decays. If we take seriously the proposal in the preceding paragraph, it would appear then that the mere existence of these extremely high energy air showers would imply also an upper bound on the mass of the dual gluon. That being the case, we could turn the argument around and use this estimate for the dual gluon mass to predict the branching ratios of various flavour-changing neutral decays which can be tested against experiment. An attempt in that direction has already been made [14], which gives branching ratios seemingly within reach of some future experiment now being planned. These predictions can be further sharpened [17] using the recent results from the CKM matrix calculation [10] described above.

So far, we have dealt in this review only with the dual colour symmetry $\widetilde{SU}(3)$ and its interpretation as generation. However, as already mentioned above, within the Dualized Standard Model framework, there is also a dual weak isospin symmetry $\widetilde{SU}(2)$ which is confined and might lead to further physical consequences. Analogy with colour confinement would suggest that at low energy, only $\widetilde{SU}(2)$ singlets can exist which are analogues to hadrons of SU(3) colour, but deep inelastic experiments at high energy could reveal a substructure to these $\widetilde{SU}(2)$ singlet states analogous to the parton substructure of hadrons in colour SU(3). However, an estimate of the gauge coupling $\tilde{g}_2$ of $\widetilde{SU}(2)$ via the Dirac quantization condition (5) from the empirical value of the ordinary weak isospin coupling g_2 gives a value many times larger than the coupling g_3 responsible for colour confinement, suggesting thus that the energy scale required to reveal the $\widetilde{SU}(2)$ substructure may be much higher than that required for the SU(3) case. This might mean therefore that deep inelastic experiments, provided that the energy is high enough, could reveal a substructure to what we believe at present to be elementary particles, such as quarks and leptons, or the Higgs and gauge bosons.

In any case, the Dualized Standard Model seems not only to offer tentatively viable explanations to some long-term puzzles in particle as well as in astroparticle physics, but can give rise to some interesting new predictions which may one day be testable against experiment.

We shall conclude this brief review by a general suggestion on terminology. In the literature, the term 'generation' has been used interchangeably with the term 'family', with perhaps a slight preference for the latter because, in the words of Jarlskog [18], there was no known "mother–daughter relationship between the copies". However, if the explanation for fermion masses offered above by the Dualized Standard Model is accepted, then there is now a known mother–daughter relationship between the 3 copies, with the higher mass copies indeed giving birth, in a sense, to the masses of the lower copies, via the mechanism due to the rotating mass matrix. It would seem thus that the term 'generation' is most appropriate. We suggest therefore that one keeps the term 'generation' in the sense it has been used throughout this paper, but adopt the term 'family' to denote, in accordance with the biological usage of the term, the collection of members related by this "mother–daughter" relationship. In other words, the fermion types would then be labelled as the U-family, the D-family *etc.*, while the t, c and u would be labelled as the members of the 1st, 2nd, and 3rd generations of the U-family.

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