VISUALIZATION OF THE TOPOLOGICAL STRUCTURE OF THE VACUUM IN LATTICE QCD * **

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We analyze the topological structure of the pure gluonic SU(2) vacuum at finite temperature in both phases of the theory by computing correlation functions between the topological charge density and the monopole density in maximum abelian projection. On gauge average we find a nontrivial spatial correlation between both topological objects. We show that the coexistence of monopoles and instantons also holds per gauge configuration.

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1. Introduction and theory

There are two different kinds of topological objects which seem to be important candidates for the confinement mechanism: color magnetic monopoles and instantons. In lattice calculations we demonstrated that color magnetic monopoles and instantons are correlated on realistic gauge field configurations [1]. Similar phenomena were discussed by other groups on semiclassical configurations [2]. This might indicate that both confinement mechanisms have the same topological origin and that both approaches can be united. It is believed that instantons and also monopoles can explain chiral symmetry breaking [3, 4]. In this contribution we study the origin of the relation between the topological objects by analyzing the correlation functions per gauge configuration and by visualizing the topological structure by means of 3D graphics.

To investigate monopole currents we project SU(N) onto its abelian degrees of freedom, such that an abelian $U(1)^{N-1}$ theory remains [5]. We

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employ the so-called maximum abelian gauge being most favorable for our purposes. For the definition of the monopole currents $m_i(x,\mu)$, $i=1,...,N$, we use the standard method [6]. From the monopole currents we define the local monopole density as

$$\rho(x) = \frac{1}{4NV_4}\sum_{\mu,i}|m_i(x,\mu)| \,. \tag{1}$$

There exist several definitions of the topological charge on the lattice. The field theoretic prescriptions are a straightforward discretization of the continuum expression. To get rid of the renormalization constants we apply the "Cabbibo-Marinari cooling method" which smooths the quantum fluctuations of a gauge field. Other topological charge operators can be obtained from the geometric definitions. The discrete set of link variables is interpolated to the continuum and then the topological charge is calculated directly. Concerning the correlation between monopoles and instantons it was shown in [7] that the geometric Lüscher charge definition yields qualitatively the same results as the field theoretic prescriptions. Therefore we employ in these studies the field theoretic plaquette and hypercube prescription of the topological charge density [8]

$$q^{(P,H)}(x) = -\frac{1}{2^4 32\pi^2}\sum_{\mu,\ldots=\pm 1}^{\pm 4}\tilde{\epsilon}_{\mu\nu\rho\sigma}\,\mathrm{Tr}\;O^{(P,H)}_{\mu\nu\rho\sigma}\,, \tag{2}$$

with $O^{(P,H)}_{\mu\nu\rho\sigma}$ being the Wilson operator for the closed path along a plaquette and hypercube, respectively.

To measure correlations between topological quantities we calculate functions of the type

$$\langle q(0)q(r)\rangle\;,\;\langle\rho(0)|q(r)|\rangle\;. \tag{3}$$

They are normalized after subtracting the corresponding cluster values.

2. Results

Our simulations were performed for pure SU(2) gauge theory on a $12^3\times 4$ lattice with periodic boundary conditions using the Metropolis algorithm. In the case of the Wilson plaquette action the observables were studied both in the confinement and the deconfinement phase at inverse gluon coupling $\beta = 4/g^2 = 2.25$ and 2.4, respectively. For each run we made 100 measurements, separated by 100 iterations.

The normalized auto-correlation functions $\langle q(0)q(r)\rangle$ of the topological charge density with $q(x)$ in the hypercube definition are displayed in Fig. 1

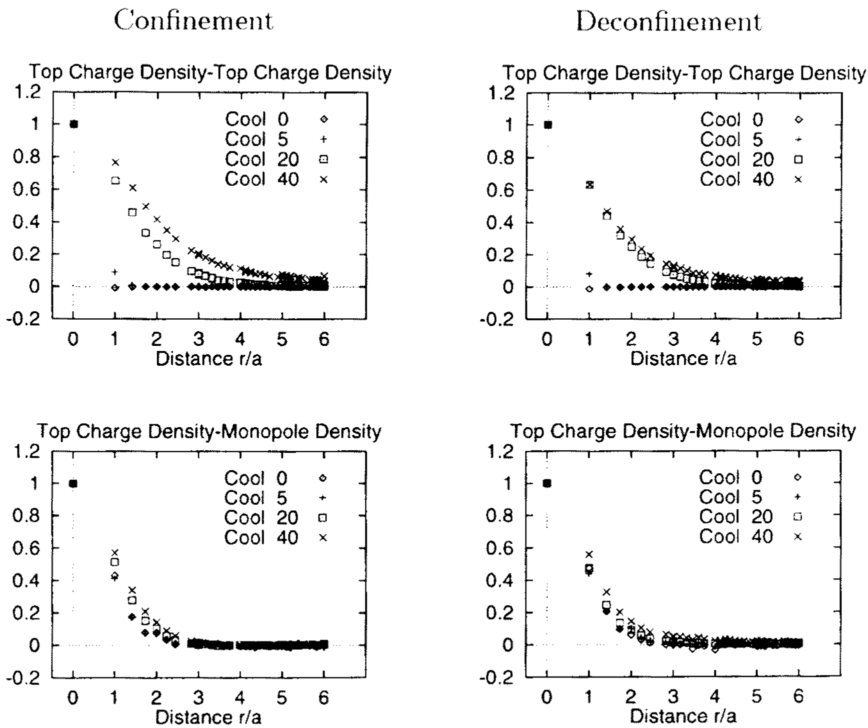

Fig. 1. Auto-correlation functions of the topological charge density for 0, 5, 20, 40 cooling steps in both phases of the theory (top). After cooling the existence of extended instantons becomes visible. Correlation functions between the monopole density and the absolute value of the topological charge density after 0, 5, 20, 40 cooling steps (bottom). These normalized correlation functions are hardly affected by cooling And are similar in both phases.

(top). They are presented for 0, 5, 20, 40 cooling steps in both phases. Without cooling the auto-correlation function is δ-peaked due to the dominance of quantum fluctuations. It becomes broader with cooling reflecting the existence of extended instantons. As a measure for the local relation between abelian monopoles and instantons, we calculate the correlation functions $\langle\rho(0)|q(r)|\rangle$ between the monopole density and the absolute value of the topological charge density. They are displayed in Fig. 1 (bottom) both in the confinement and the deconfinement phase for several cooling steps. The shape of these correlations hardly changes under the influence of cooling and is essentially unaffected by the phase transition. They extend over approximately two lattice units. This indicates that there exists a nontrivial local correlation between these topological objects and that the probability for finding monopoles around instantons is clearly enhanced.

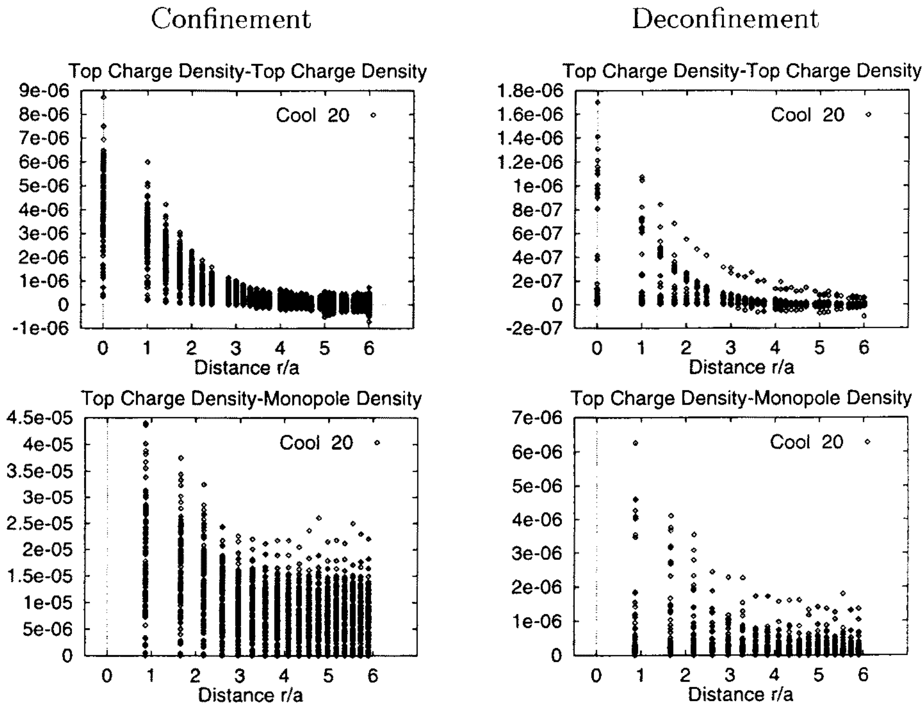

Fig. 2. Auto-correlation functions of the topological charge density after 20 cooling steps for 100 independent configurations in both phases (top). In contrast to the confinement phase only 15 % of the configurations carry a topological charge in the deconfinement phase. The corresponding $\rho|q|$-correlations are displayed in both phases (bottom). All configurations with nonvanishing qq-auto-correlation give rise to a nontrivial $\rho|q|$-correlation.

Next we analyze the origin of the nontrivial correlation between monopoles and instantons. In particular we are interested in the reason for the similarity in both phases. Fig. 2 displays the auto-correlations of the topological charge density in the plaquette definition and the $\rho|q|$-correlations after 20 cooling steps for 100 independent configurations. In the confinement phase the auto-correlation functions have many different amplitudes indicating a variety of topologically nontrivial configurations. Also the corresponding monopole-instanton correlations show many different amplitudes. In the deconfinement phase only about 15 % of the auto-correlation functions are nontrivial. All of these configurations give rise to a nontrivial $\rho|q|$-correlation. This indicates that the relation between monopoles and instantons found on gauge average also holds for single configurations.

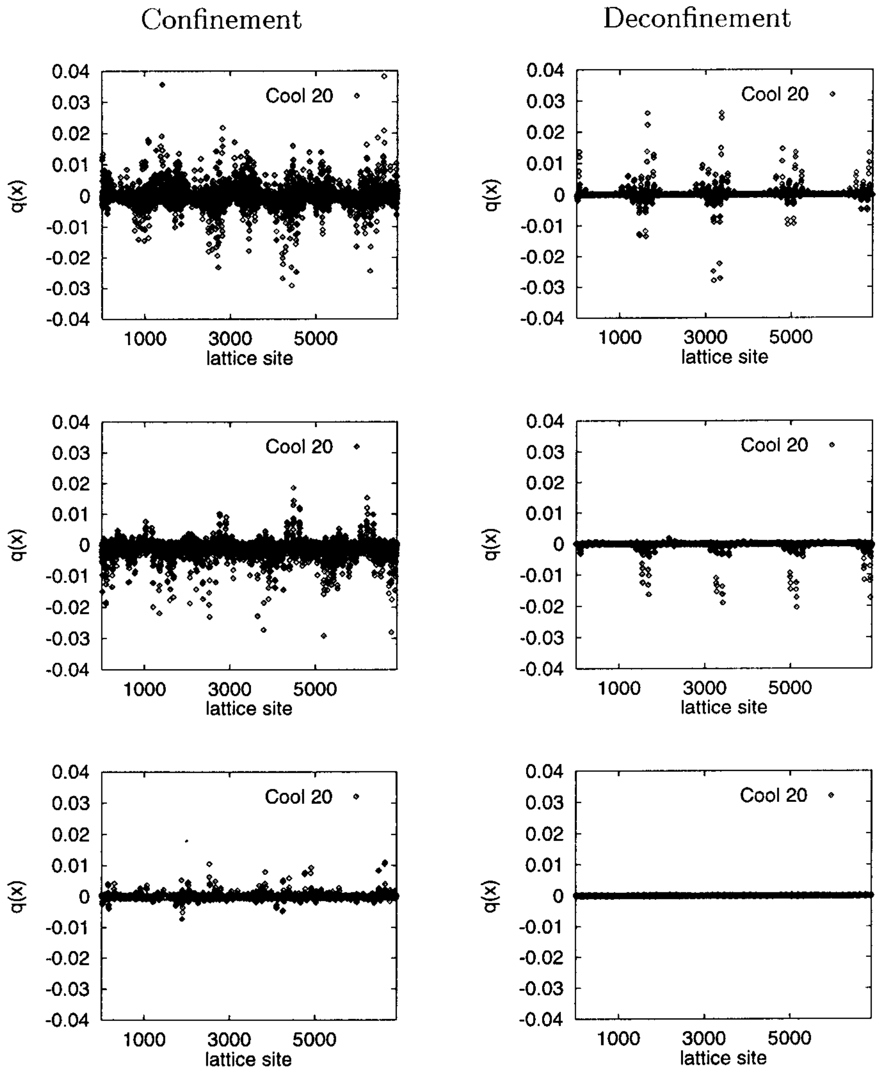

Fig. 3. Topological charge density $q(x)$ as a function of the linearized lattice site x for configurations with the largest (top), a medium (middle), and a small (bottom) amplitude of the auto-correlation function.

Fig. 3 displays topological charge densities $q(x)$ as a function of the linearized lattice site x for 20 cooling steps in both phases. On top we chose the configuration with the largest amplitude of the auto-correlation function, in the middle (bottom) one with a medium (low) value of correlation strength, respectively. These simple one-dimensional representations give a first impression of the topological content of the four-dimensional configurations. They become topologically more and more trivial for a decreasing strength of the auto-correlation function. Therefore the auto-correlation of the topological charge can be used to select interesting configurations for three-dimensional visualization.

Confinement Deconfinement

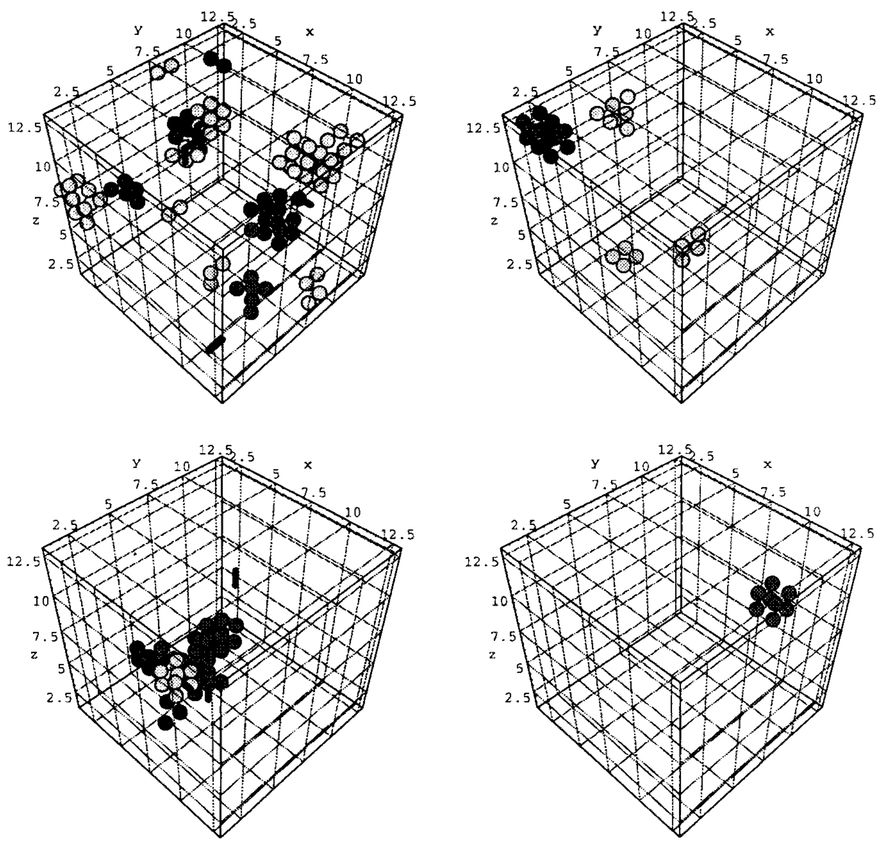

Fig. 4. The four topologically nontrivial configurations of Fig. 3 for fixed time slice. Light (dark) dots represent the topological charge density with $q(x) > 0.005$ ($q(x) < -0.005$). Lines correspond to monopole loops.

In Fig. 4 we visualize the relation between instantons and monopoles by directly displaying clusters of topological charge and by drawing monopole loops in fixed time slices for specific configurations. For any value of the topological charge density $q(x) > 0.005$ a light dot and for $q(x) < -0.005$ a dark dot is plotted. Monopole currents are represented by lines. Fig. 4 presents the four topologically nontrivial configurations of Fig. 3. It can be seen that clusters of topological charge are accompanied by monopole loops.

Next we study the influence of the type of the gluonic action on the stability of topologically nontrivial configurations. Beside the standard Wilson action we employed a simplified fixed-point action [9] with $\beta = 1.50$ ($T/T_c = 0.83$) and $\beta = 1.65$ ($T/T_c = 1.20$). The speed of cooling was chosen such that the decrease of the action under cooling is approximately the same for both actions.

Fig. 5 displays cooling histories of the action and the topological charge for 100 independent configurations in the deconfinement phase for the Wilson action and the fixed-point action. Except for a single configuration with topological charge $Q = 2$ no clear plateau of the action can be found for the

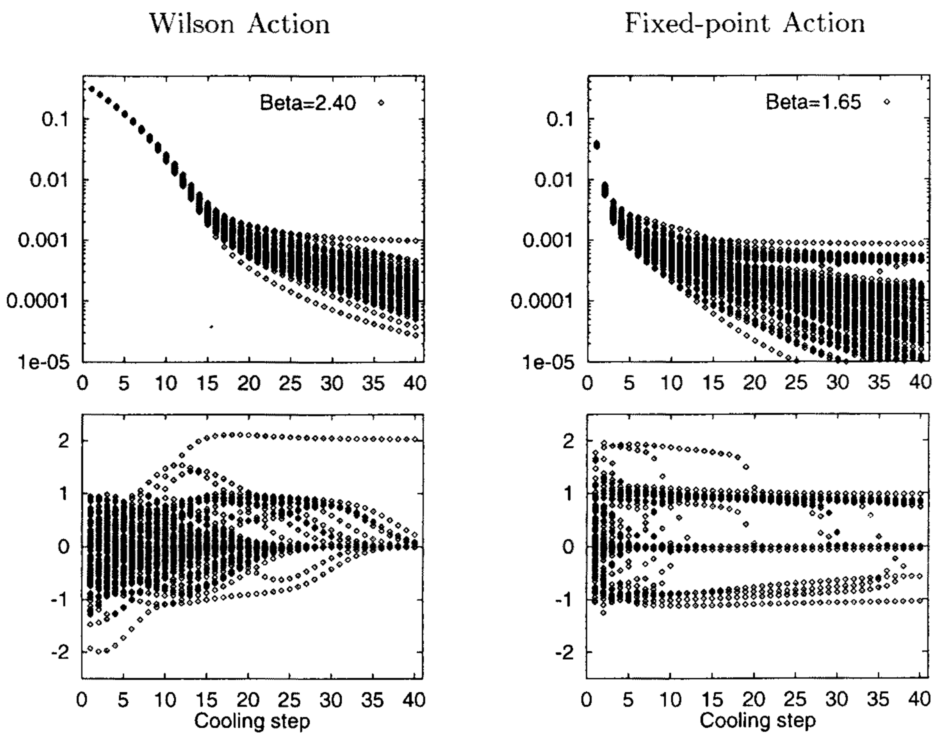

Fig. 5. The action (top) and the topological charge (bottom) for the Wilson action and a fixed-point action as a function of cooling in the deconfinement phase. In contrast to the Wilson action clear plateaus can be observed for the fixed-point action. The action of the stable configurations is exactly a multiple of the action of a single instanton.

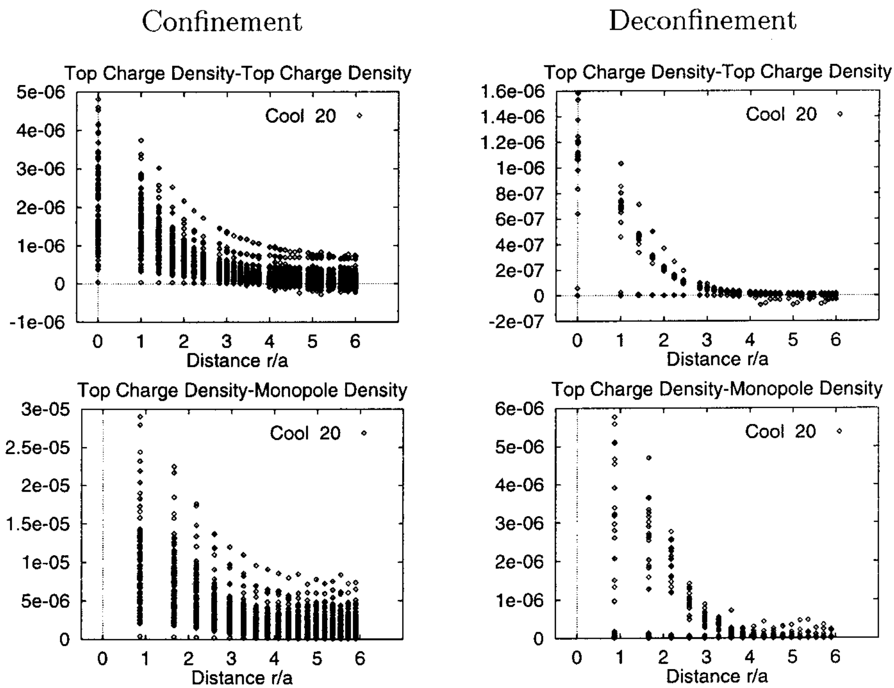

Fig. 6. Auto-correlation functions of the topological charge density (top) and $\rho|q|$-correlations (bottom) for a fixed-point action after 20 cooling steps for 100 independent configurations in both phases. Like in the Wilson case the configurations with a nontrivial auto-correlation function are those with a nontrivial $\rho|q|$-correlation.

Wilson action. After 30 cooling steps the configurations become unstable and the instantons vanish. The performance of the fixed-point action is much better. After 20 cooling steps two plateaus form whose action is exactly the action of one instanton and an anti-instantons, respectively.

In Fig. 6 the auto-correlation functions of the topological charge and the $\rho|q|$-correlations are shown for the fixed-point action in the confinement and deconfinement phase after 20 cooling steps. They yield qualitatively the same result as those produced by the Wilson action.

3. Conclusion

We analyzed the topological structure of the SU(2) vacuum at finite temperature in both phases of the theory. The auto-correlation functions of the topological charge density are nontrivial after 20 cooling steps reflecting the existence of instantons. Correlation functions between the topological charge density and the monopole density are hardly affected by cooling indicating a close spatial relation between instantons and monopoles. This observation also holds in the deconfinement phase. Studying the origin of the coexistence between monopoles and instantons in more detail we computed correlation functions per configuration. In the deconfinement phase only approximately 15 % of the configurations carry a topological charge and all of these configurations give rise to nonvanishing $\rho|q|$-correlations. Besides the standard Wilson action we employed a fixed-point action to investigate the impact of the action on the stability of topologically nontrivial configurations under cooling. In the deconfinement phase no strict plateau of the action can be seen for the Wilson action and instantons vanish after 30-40 cooling steps. With the fixed-point action however there exists a clear plateau of the action and the topological charge after 20 cooling steps. The action of these configurations is a multiple of the action of a single instanton.

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