ANALYTICAL CALCULATION OF SMALL ANGLE BHABHA CROSS-SECTION AT LEP1*

N.P. Merenkov[1], A.B. Arbuzov, V.S. Fadin, E.A. Kuraev, L.N. Lipatov, and L. Trentadue

[1]Institute of Physics and Technology
Kharkov, 310108, Ukraine

(Received January 14, 1997)

The analytical approach is applied for description of small angle Bhabha scattering at LEP1. The QED correction to the Born cross-section is calculated with leading and next-to-leading accuracy in the second order of the perturbation theory and with leading one in the third order. All contributions due to photon emission and pair production in the second order are calculated starting from essential Feynman diagrams. The third order corrections are computed by means of the electron structure function method. Inclusive and calorimeter event selections for asymmetrical wide-narrow circular detectors are investigated.

PACS numbers: 12.20. Ds

The small angle Bhabha scattering (SABS) process is used to measure luminosity of electron-positron colliders. At LEP1 an experimental accuracy on the luminosity better then one per mille has been reached [1] and to obtain the total accuracy a systematic theoretical error must be added. That is why in recent years a considerable attention has been devoted to theoretical investigation of SABS cross-section [2-11].

The theoretical calculation of SABS cross-section at LEP1 has to cope with two problems. The first one is the description of the experimental restrictions used for event selection in terms of final particles phase space. The second concerns the computation of matrix element squared with the required accuracy. There are two approaches for the theoretical study of SABS at LEP1: the one based on Monte Carlo programs [3-6] and the analytical approach [7-11].

* Presented at the Cracow International Symposium on Radiative Corrections to the Standard Model, Cracow, Poland, August 1–5, 1996.

The advantage of the MC approach is the possibility to model different types of detectors and event selection [3]. But at this approach some problems with exact matrix element squared exist. Contrary, the advantage of the analytical one is the possibility to use exact matrix element squared and its defect is low mobility relative the change of an experimental conditions for event selection. Nevertheless, the analytical calculations have a great importance because allow to check numerous MC calculations for different types of *ideal* detectors.

In this talk we list some analytical results for SABS cross-section at LEP1 suitable for inclusive and calorimeter event selection in the case of asymmetrical wide-narrow circular detectors.

1. First order correction

Let us introduce the dimentionless quantity $\Sigma = \frac{1}{4\pi\alpha^2}Q_1^2\sigma_{obs}$, where $Q_1^2 = \epsilon^2\theta_1^2$ (ϵ is the beam energy and θ_1 is the minimal angle of wide detector). The *experimentally* measurable cross section σ_{obs} is defined as follows

$$\sigma_{obs} = \int dx_1 dx_2 \Theta d^2q_1^\perp d^2q_2^\perp \Theta_1^c\Theta_2^c \frac{d\sigma(e^+ + e^- \to e^+ + e^- + X)}{dx_1 dx_2 d^2q_1^\perp d^2q_2^\perp}, \quad (1)$$

where X denotes the particles created in the final state, $x_1(x_2)$, $q_1^\perp(q_2^\perp)$ are, respectively, the energy fraction and the transverse component of the momentum of the electron (positron) in the final state. Functions Θ_i^c do take into account angular cuts while function Θ-cutoff on invariant mass of detected electron and positron:

$$\Theta_1^c = \theta(\theta_3 - \theta_-)\theta(\theta_- - \theta_1), \quad \Theta_2^c = \theta(\theta_4 - \theta_+)\theta(\theta_+ - \theta_2), \quad \Theta = \theta(x_1x_2 - x_c)\,.$$

$$\theta_- = \frac{|\vec{q}_1^{\,\perp}|}{x_1\epsilon}, \quad \theta_+ = \frac{|\vec{q}_2^{\,\perp}|}{x_2\epsilon}\,. \quad (2)$$

For wide-narrow angular acceptance

$$\theta_3 > \theta_4 > \theta_2 > \theta_1, \qquad \rho_i = \frac{\theta_i}{\theta_1} > 1\,.$$

The first order correction includes the contributions due to virtual and real (soft and hard) photon emission

$$\Sigma_1 = \Sigma_{V+S} + \Sigma_H + \Sigma^H\,. \quad (3)$$

For the case of inclusive event selection (IES) contribution due to virtual and soft photon (with the energy less than $\Delta\epsilon, \Delta \ll 1$) reads

$$\Sigma_{V+S} = 2\frac{\alpha}{\pi}\int\limits_{\rho_2^2}^{\rho_4^2}\frac{dz}{z^2}[2(L-1)\ln\Delta + \frac{3}{2}L - 2], \quad L = \ln\frac{\epsilon^2\theta_1^2 z}{m^2}, \tag{4}$$

where $z = (\vec{q}_2^{\perp})^2/Q_1^2$, m is electron mass.

The second (third) term in r.h.s. of Eq. (3) is responsible for the correction due to hard photon emission by positron (electron). It can be derived by the integration of the differential cross-section of the single photon emission (see [10] Eq. (A.16)) over the region

$$1 < z < \rho_3^2, \quad x^2\rho_2^2 < z_1 = \frac{\vec{q}_1^{\perp 2}}{Q_1^2} < x^2\rho_4^2, \quad x_c < x_2 < 1 - \Delta, \quad x_1 = 1 \,,$$

$$(\rho_2^2 < z < \rho_4^2, \quad x^2 < z_1 < x^2\rho_3^2, \quad x_c < x_1 < 1 - \Delta, \quad x_2 = 1) \,. \tag{5}$$

The result may be written as follows:

$$\Sigma_H = \frac{\alpha}{2\pi}\int\limits_1^{\rho_3^2}\frac{dz}{z^2}\int\limits_{x_c}^{1-\Delta}\frac{1+x^2}{1-x}dx\left[(L-1)\left(\Delta_{42} + \Delta_{42}^{(x)}\right) + \widetilde{K}(x,z;\rho_4,\rho_2)\right],$$

$$\Sigma^H = \frac{\alpha}{2\pi}\int\limits_{\rho_2^2}^{\rho_4^2}\frac{dz}{z^2}\int\limits_{x_c}^{1-\Delta}\frac{1+x^2}{1-x}\left[\left(1+\theta_3^{(x)}\right)(L-1) + K(x,z;\rho_3,1)\right]dx \,, \tag{6}$$

$$\widetilde{K} = \frac{(1-x)^2}{1+x^2}\left(\Delta_{42} + \Delta_{42}^{(x)}\right) + \Delta_{42}\widetilde{L}_1 + \Delta_{42}^{(x)}\widetilde{L}_2$$
$$+\left(\overline{\theta}_4^{(x)} - \theta_2^{(x)}\right)\widetilde{L}_3 + (\overline{\theta}_4 - \theta_2)\widetilde{L}_4 \,,$$
$$K(x,z;\rho_3,1) = \frac{(1-x)^2}{1+x^2}\left(1+\theta_3^{(x)}\right) + L_1 + \theta_3^{(x)}L_2 + \overline{\theta}_3^{(x)}L_3 \,. \tag{7}$$

When writing Eqs.(6) and (7) we used the following notations for θ-functions

$$\theta_i^{(x)} = \theta(x^2\rho_i^2 - z), \quad \theta_i = \theta(\rho_i^2 - z), \quad \overline{\theta}i^{(x)} = 1 - \theta_i^{(x)}, \quad \overline{\theta}_i = 1 - \theta_i \,,$$

$$\Delta_{42} = \theta_4 - \theta_2, \quad \Delta_{42}^{(x)} = \theta_4^{(x)} - \theta_2^{(x)} \,,$$

and for $\widetilde{L}_i$

$$\widetilde{L}_1 = \ln\left|\frac{(z-\rho_2^2)(\rho_4^2 - z)x^2}{(x\rho_4^2 - z)(x\rho_2^2 - z)}\right|, \quad \widetilde{L}_2 = \ln\left|\frac{(z - x^2\rho_2^2)(x^2\rho_4^2 - z)}{x^2(x\rho_4^2 - z)(x\rho_2^2 - z)}\right| \,,$$

$$\widetilde{L}_3 = \ln\left|\frac{(z-x^2\rho_2^2)(x\rho_4^2-z)}{(x^2\rho_4^2-z)(x\rho_2^2-z)}\right|, \quad \widetilde{L}_4 = \ln\left|\frac{(z-\rho_2^2)(x\rho_4^2-z)}{(\rho_4^2-z)(x\rho_2^2-z)}\right| .$$

The quantities L_i may be obtained from $\widetilde{L}_i$ by substitution $\rho_4 \to \rho_3$, $\rho_2 \to 1$.

The sum in r.h.s. of Eq. (3) does not depend on auxiliary parameter Δ and reads as

$$\Sigma_1 = \frac{\alpha}{2\pi}\Bigg\{\int\limits_1^{\rho_3^2}\frac{dz}{z^2}\Bigg[-\Delta_{42} + \int\limits_{x_c}^{1}\Bigg((L-1)P_1(x)(\Delta_{42}+\Delta_{42}^{(x)}) + \frac{1+x^2}{1-x}\widetilde{K}\Bigg)dx\Bigg]$$

$$+\int\limits_{\rho_2^2}^{\rho_4^2}\frac{dz}{z^2}\Bigg[-1 + \int\limits_{x_c}^{1}\Bigg((L-1)P_1(x)(1+\theta_3^{(x)}) + \frac{1+x^2}{1-x}K\Bigg)dx\Bigg]\Bigg\} , \qquad (8)$$

where function $P_1(x)$ defines the iterative form of nonsinglet electron structure function (see for example [5]). The first (second) line in the r.h.s. of Eq. (8) is the contribution due to photon emission by positron (electron). The terms accompanied with x-dependent θ-functions under integral sign correspond to initial-state correction while the rest – to final-state one.

Formula (8) is suitable for IES. Let us investigate the calorimeter event selection (CES) labeled in [3] as CALO1. The CALO1 cluster is the cone with angular radius $\delta = 0.01$ around final electron (or positron) momentum direction. If photon belongs to the cluster the whole cluster energy is measured, and electron can have any possible energy. Therefore, the limits of x-integration for Σ_{obs} extend from 0 to 1 in this case. If photon escapes the cluster the event looks the same as for IES. The above restrictions on x-integration limits can be written symbolically as follows:

$$\int\limits_{x_c}^{1} dx + \int\limits_{0}^{x_c}(if|\vec{r}| < \delta)dx = \int\limits_{0}^{1} dx - \int\limits_{0}^{x_c}(if|\vec{r}| > \delta)dx , \qquad (9)$$

where $\vec{r} = \frac{\vec{k}}{\omega} - \frac{\vec{q}_1^{\perp}}{\epsilon_1}$ and $\omega(\vec{k})$ is the energy (transverse momentum) of hard photon.

As we saw for IES it is necessary to differ the contributions into Σ_1 due to electron and positron radiation

$$\Sigma_1 = \Sigma^{\gamma} + \Sigma_{\gamma} , \qquad (10)$$

and according to (9) we have

$$\Sigma^{\gamma} = \Sigma_i + \Sigma_f + \Sigma_i^c + \Sigma_f^c, \qquad \Sigma_{\gamma} = \widetilde{\Sigma}_i + \widetilde{\Sigma}_f + \widetilde{\Sigma}_i^c + \widetilde{\Sigma}_f^c , \qquad (11)$$

where index $i(f)$ labels initial (final) state and c shows on a cluster form dependence.

The quantities Σ_i and $\widetilde{\Sigma}_i$ coincide with corresponding initial-state correction for IES (see comments after Eq. (8)). For Σ_f and $\widetilde{\Sigma}_f$ we can use the IES form of differential cross-section with enlarged x-integration limits. The result reads

$$
\begin{aligned}
\Sigma_f &= \frac{\alpha}{2\pi}\int\limits_{\rho_2^2}^{\rho_4^2}\frac{dz}{z^2}\left[-\frac{1}{2}+\int\limits_0^1\Big(1-x+\frac{1+x^2}{1-x}L_1\Big)dx\right],\\
\widetilde{\Sigma}_f &= \frac{\alpha}{2\pi}\int\limits_1^{\rho_3^2}\frac{dz}{z^2}\left[-\frac{1}{2}\Delta_{42}+\int\limits_0^1\left[\Big(1-x+\frac{1+x^2}{1-x}\widetilde{L}_1\Big)\Delta_{42}\right.\right.\\
&\left.\left.+\frac{1+x^2}{1-x}(\bar{\theta}_4-\theta_2)\widetilde{L}_4\right]dx\right], \qquad (12)
\end{aligned}
$$

In order to find the additional contributions into Σ_1, which depend on the cluster form, it is enough to use the simplified differential cross-section of single photon radiation, neglecting electron mass, and take into account the restrictions $|\vec{r}| < \delta$ (for initial state) and $|\vec{r}| > \delta$ (for final state). The contribution due to initial-state electron emission may be written as follows:

$$
\Sigma_i^c = \frac{\alpha}{2\pi}\int\limits_0^{x_c}\frac{1+x^2}{1-x}dx\int\frac{dz}{z^2}\int dz_1\Psi\Phi(z_1,z;x,\lambda), \quad \lambda=\frac{\delta}{\theta_1}, \qquad (13)
$$

where Ψ defines the limits of z (in the straight brackets) and z_1 (in the curve brackets) integration:

$$
\begin{aligned}
\Psi &= [a^2,a_0^2](x^2z_+,x^2)+[b^2,a^2](x^2z_+,x^2z_-)+[b_0^2,b^2](x^2\rho_3^2,x^2z_-),\\
a_0 &= \rho_2,\quad b_0=\rho_4,\quad a=\max(\rho_2,1+\lambda(1-x)),\\
b &= \min(\rho_4,\rho_3-\lambda(1-x)),\quad z_\pm=(\sqrt{z}\pm+\lambda(1-x))^2,
\end{aligned}
$$

and function Φ reads

$$
\begin{aligned}
\Phi &= \frac{2}{\pi}\Big(\frac{1}{z_1-xz}+\frac{1}{z-z_1}\Big)\arctan\Big[\frac{z-z_1}{(\sqrt{z}-\sqrt{z_1})^2}Q\Big],\\
Q &= \sqrt{\frac{\lambda^2x^2(1-x)^2-(\sqrt{z_1}-x\sqrt{z})^2}{(\sqrt{z_1}+x\sqrt{z})^2-\lambda^2x^2(1-x)^2}}. \qquad (14)
\end{aligned}
$$

The cluster depndent contribution due to final-state electron emission reads:

$$
\begin{aligned}
\Sigma_f^c &= \frac{\alpha}{2\pi}\int_0^{x_c}\frac{1+x^2}{1-x}dx[\int\frac{dz}{z^2}\int dz_1\Psi F(z_1,z;x,\lambda) \\
&+\int_{a_0^2}^{b^2}\frac{dz}{z^2}(\ln\left|\frac{x\rho_3^2-z}{\rho_3^2-z}\right|+l_+)+\int_{a^2}^{b_0^2}\frac{dz}{z^2}(\ln\left|\frac{1x-z}{1-z}\right|+l_-)], \\
F &= \frac{2}{\pi}(\frac{1}{z_1-xz}-\frac{1}{z_1-x^2z})\arctan[\frac{(\sqrt{z_1}-x\sqrt{z})^2}{(z_1-x^2z)Q}], \\
l_\pm &= \ln\frac{\lambda(2\sqrt{z}\mp\lambda(1-x))}{z\pm 2x\lambda\sqrt{z}-\lambda^2x(1-x)} \,. \qquad (15)
\end{aligned}
$$

In order to obtain $\widetilde{\Sigma}_i^c$ it is enough to substitude $\tilde{a}, \tilde{b}, \tilde{a}_0$ and $\tilde{b}_0$ instead of a, b, a_0 and b_0, respectively, in the expression for Ψ, where

$$
\begin{aligned}
\tilde{a} &= \rho_2+\lambda(1-x), \quad \tilde{a}_0 = \max(1,\rho_2-\lambda(1-x)\,, \\
\tilde{b} &= \rho_4-\lambda(1-x), \quad \tilde{b}_0 = \min(\rho_4+\lambda(1-x),\rho_3)\,.
\end{aligned}
$$

Finally, the cluster dependent contribution due to final-state positron emission may be written as follows:

$$
\begin{aligned}
\widetilde{\Sigma}_f^c &= \frac{\alpha}{2\pi}\int_0^{x_c}\frac{1+x^2}{1-x}dx\int_1^{\rho_3^2}\frac{dz}{z^2}\Big[\theta(\tilde{a}_0^2-z)-\theta(z-\tilde{b}_0^2)\Big]\widetilde{L}_4 \\
&+\Sigma_f^c(a,b,a_0,b_0\to\tilde{a},\tilde{b},\tilde{a}_0,\tilde{b}_0;\quad \rho_3,1\to\rho_4,\rho_2)\,. \qquad (16)
\end{aligned}
$$

CALO2 event selection differs from CALO1 one with the form of the cluster (see [3]). Only cluster dependent contributions into Σ_1 will be changed in this case. Analytical formulae are very cumbersome, and we give the result for symmetrical wide-wide case only ($\Sigma^\gamma = \Sigma_\gamma$)

$$
\begin{aligned}
\Sigma_i^c &= \frac{\alpha}{2\pi}\int_0^{x_c}\frac{1+x^2}{1-x}dx\int\frac{dz}{z^2}\int dz_1\frac{2}{\pi}\left(\frac{1}{z_1-xz}+\frac{1}{z-z_1}\right) \\
&\times[\Psi_1\Phi_1+\Psi_2\Phi_2+\Psi_3\Phi_3]\,, \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
\Phi_1 &= \arctan Q_i^{(-)} - \arctan \eta, \quad \Phi_2 = \arctan \eta^{-1}, \quad \Phi_3 = \arctan \frac{1}{Q_i^{(+)}}, \\
\eta &= r_i \cot \frac{\Phi - \delta}{2}, \quad r_i = \frac{(\sqrt{z} - \sqrt{z_1})^2}{z - z_1}, \\
Q_i^{\pm} &= r_i \sqrt{\frac{x^2(\sqrt{z} + \sqrt{z_1})^2 - (1-x)^2(\sqrt{z_1} \pm x\bar{\lambda})^2}{(1-x)^2(\sqrt{z_1} \pm x\bar{\lambda})^2 - x^2(\sqrt{z} - \sqrt{z_1})^2}}, \\
\Psi_1 &= [z_3^{(-)}, 1](x^2 J_+^2, x^2 z_+) + [(\rho_3 - (1-x)\bar{\lambda})^2, z_3^{(-)}](x^2\rho_3^2, x^2 z_+), \\
\Psi_2 &= [z_1^{(+)}, 1](x^2 z_+, x^2) + [(\rho_3 - (1-x)\bar{\lambda})^2, z_1^{(+)}](x^2 z_+, x^2 J_-^2) \\
&\quad + [\rho_3^2, (\rho_3 - (1-x)\bar{\lambda})^2](x^2\rho_3^2, x^2 J_-^2), \\
\Psi_3 &= [z_1^{(+)}, (1 + (1-x)\bar{\lambda})^2](x^2 J_+^2, x^2) + [\rho_3^2, (1 + (1-x)\bar{\lambda})^2](x^2 J_-^2, x^2 z_-).
\end{aligned} \tag{18}
$$

The corresponding formula due to final electron emission reads:

$$
\begin{aligned}
\Sigma_f^c &= \frac{\alpha}{2\pi} \int_0^{x_c} \frac{1+x^2}{1-x} dx \Big[\int \frac{dz}{z^2} \int dz_1 \frac{2}{\pi} \Big(\frac{1}{z_1 - xz} - \frac{1}{z_1 - x^2 z} \Big) \\
&\quad \times [\Psi_1 F_1 + \bar{\Psi}_2 F_2 + \Psi_3 F_3] \\
&\quad + \int_1^{z_3^{(-)}} \frac{dz}{z^2} \ln \left| \frac{(x\rho_3^2 - z)(J_+^2 - z)}{(\rho_3^2 - z)(xJ_+^2 - z)} \right| + \int_{(1+(1-x)\bar{\lambda})^2}^{\rho_3^2} \frac{dz}{z^2} \Big(\ln \left| \frac{x - z}{1 - z} \right| + \bar{l}_- \Big) \Big],
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
F_1 &= \arctan \frac{1}{Q_f^{(-)}}, \quad F_2 = \arctan \zeta, \quad F_3 = \arctan Q_f^{(+)}, \\
\zeta &= r_f \cot \frac{\Phi - \delta}{2}, \quad r_f = \frac{(\sqrt{z_1} - x\sqrt{z})^2}{z_1 - x^2 z}, \quad \bar{l}_- = l_-(\lambda \to \bar{\lambda}), \\
Q_f^{(\pm)} &= \frac{r_f}{r_i} Q_i^{(\pm)}, \qquad \sin\delta = \sqrt{\frac{z_1}{z}} \sin\Phi, \\
\bar{\Psi}_2 &= \left[z_1^{(+)}, 1\right](x^2 J_+ x^2) + \left[z_3^{(-)}, z_1^{(+)}\right](x^2 J_+^2, x^2 J_-^2) \\
&\quad + \left[\rho_3^2, z_3^{(-)}\right](x^2 \rho_3^2, x^2 J_-^2).
\end{aligned} \tag{20}
$$

The quantities Φ and $\bar{\lambda}$ which enter into Eqs.(17)-(20) define the form and the size of CALO2 cluster, namely

$$\Phi = \frac{3\pi}{32}, \qquad \bar{\lambda} = \frac{\theta_0}{\theta_1}, \qquad \theta_0 = \frac{0.051}{16}.$$

Finally, the functions $J_{\pm}$ and $z_i^{(\pm)}$ are defined as follows

$$\begin{aligned} J_{(\pm)} &= \frac{1}{\beta}\left[\sqrt{z\beta - x^2(1-x)^2\bar{\lambda}^2\sin^2\Phi} \pm (1-x)\bar{\lambda}(1-2x\sin^2\frac{\Phi}{2})\right], \\ \beta &= 1-4x(1-x)\sin^2\frac{\Phi}{2}, \ z_i^{(\pm)} \\ &= (\rho_i \pm (1-x)\bar{\lambda})^2 - 4x(1-x)\rho_i(\rho_i \pm \bar{\lambda})\sin^2\frac{\Phi}{2}. \end{aligned}$$

2. Second order correction

In this Section we want to give the second order correction to SABS cross-section with next-to-leading accuracy. It contains the contributions due to pair production and two photon emission. The result for pair production have published yet for symmetrical angular acceptance [11]. That is why we will deal with photonic correction only

$$\Sigma_2^{ph} = \Sigma^{\gamma\gamma} + \Sigma_{\gamma\gamma} + \Sigma_\gamma^\gamma . \tag{21}$$

The first term in r.h.s. of Eq. (21) is resposible for two photon (real and virtual) emission by the electron, the second one – by the positron, and the third one describes the situation when both, the electron and the positron, radiate.

The leading contributions in the case of IES read

$$\begin{aligned} \Sigma^{\gamma\gamma L} &= \frac{\alpha^2}{4\pi^2}\int\limits_{\rho_2^2}^{\rho_4^2}\frac{dz}{z^2}L^2\int\limits_{x_c}^{1}dx\left[\frac{1}{2}(1+\theta_3^{(x)})P_2(x) + \int\limits_{x}^{1}\frac{dt}{t}P_1(t)P_1\left(\frac{x}{t}\right)\theta_3^{(t)}\right], \\ \Sigma_{\gamma\gamma}^{L} &= \frac{\alpha^2}{4\pi^2}\int\limits_{1}^{\rho_3^2}\frac{dz}{z^2}L^2\int\limits_{x_c}^{1}dx\left[\frac{1}{2}(\Delta_{42}+\Delta_{42}^{(x)})P_2(x) + \int\limits_{x}^{1}\frac{dt}{t}P_1(t)P_1\left(\frac{x}{t}\right)\Delta_{42}^{(t)}\right], \\ \Sigma_{\gamma}^{\gamma L} &= \frac{\alpha^2}{4\pi^2}\int\limits_{0}^{\infty}\frac{dz}{z^2}L^2\int\limits_{x_c}^{1}dx_1\int\limits_{\frac{x_c}{x_1}}^{1}dx_2P_1(x_1)P_1(x_2) \\ &\times\left(\Delta_{31}+\Delta_{31}^{(x_1)}\right)\left(\Delta_{42}+\Delta_{42}^{(x_2)}\right), \end{aligned} \tag{22}$$

where

$$P_2(x) = \int\limits_x^1 \frac{dt}{t} P_1(t) P_1\left(\frac{x}{t}\right), \quad \int\limits_0^1 P_2(x) dx = 0 .$$

In the case of CES we have to take in the r.h.s. of Eqs. (22) the terms accompanied with x-dependent θ-functions only.

As concerns next-to-leading contribution into Σ_2^{ph} we have result for symmetrical angular acceptance ($\Sigma^{\gamma\gamma} = \Sigma_{\gamma\gamma}$) and asymmetrical wide-narrow one for IES only

$$\Sigma^{\gamma\gamma} = \frac{1}{4}\left(\frac{\alpha}{\pi}\right)^2 \int\limits_1^{\rho^2} \frac{dz}{z^2} L\, Y , \tag{23}$$

$$\begin{aligned}
Y &= y + \int\limits_{x_c}^1 dx \left\{ A + \int\limits_0^{1-x} dx_1 \left[\frac{1}{x_1} 4\, \frac{1+x^2}{1-x} (\theta_\rho^{(x)} l_1 + l_2) \right.\right. \\
&+ \left(-1 - \frac{1+x}{1-x_1} \frac{x}{(1-x_1)^2} \right) (l_4 + \theta_\rho^{(x)} l_3 + 2\theta_\rho^{(1-x_1)} l_5) + \left. \frac{2(1+x)}{1-x_1} \theta_\rho^{(1-x_1)} \right] \\
&- 4\, \frac{1+x^2}{1-x}\, \overline{\theta}_\rho^{(x)} \left[\int\limits_{1-\sqrt{z}/\rho}^{1-x} dx_1 \left(\frac{1}{x_1} l_5 + \frac{2}{x_2} \ln \frac{x}{1-x_1} \right) + \left.\left. \int\limits_0^{\sqrt{z}/\rho - x} \frac{dx_1}{x_1} l_6 \right] \right\} ,
\end{aligned}$$

$$y = 12\zeta_3 + 10\zeta_2 - \frac{45}{4} - 16 \ln^2(1-x_c) - 28 \ln(1-x_c) ,$$

$$\begin{aligned}
A &= (1 + \theta_\rho^{(x)}) \left[2(5+2x) + 4(x+3)\ln(1-x) + 4\, \frac{1+x^2}{1-x} \ln x \right] \\
&+ 2\, \frac{1+x^2}{1-x} \left[\left(\frac{3}{2} - \ln x \right) K(x, z; \rho, 1) - \frac{1}{2} \ln^2 x - \frac{(1-x)^2}{2(1+x^2)} \right. \\
&+ \left. 2\ln(1-x) \left(\theta_\rho^{(x)} \ln \left| \frac{x^2\rho^2 - z}{x\rho^2 - z} \right| + \ln \left| \frac{(z-1)(z-x^2)(\rho^2 - z)}{(z-x)^2 (x\rho^2 - z)} \right| \right) \right] \\
&+ \overline{\theta}_\rho^{(x)} \left[\frac{16}{1-x} \ln(1-x) + \frac{14}{1-x} - (1-x)\ln x \right. \\
&+ 2\, \frac{1+x^2}{1-x} \left(-\frac{3}{2} \ln^2 x + 3 \ln x \ln(1-x) - L_{i2}(1-x) - \frac{x(1-x) + 4x\ln x}{2(1+x^2)} \right. \\
&+ \left.\left. \frac{(1+x)^2}{1+x^2} \ln \left| \frac{(\sqrt{z} - x\rho)}{\rho - \sqrt{z}} \right| + 2 \ln \left| \frac{\sqrt{z} - x\rho}{\rho} \right| \ln \left| \frac{x(x\rho^2 - z)}{x^2\rho^2 - z} \right| \right) \right] ,
\end{aligned}$$

$$l_1 = \ln \left| \frac{(x^2\rho^2 - z)(x\rho^2 - z)}{(x(1-x_1)\rho^2 - z)(x(x+x_1)\rho^2 - z)} \right| ,$$

$$l_3 = \ln\left|\frac{(1-x_1)^2(1-x-x_1)(x^2\rho^2-z)^2}{x^3x_1(x(1-x_1)\rho^2-z)^2}\right| ,$$

$$l_2 = \ln\left|\frac{(z-x)^2(z-(1-x_1)^2)(z-(x+x_1)^2)}{(z-(1-x_1))(z-x(1-x_1))((x+x_1)-z)(x(x+x_1)-z)}\right|$$

$$+\ln\left|\frac{((1-x_1)^2\rho^2-z)((x+x_1)^2\rho^2-z)(x\rho^2-z)}{((x+x_1)\rho^2-z)((1-x_1)\rho^2-z)(x^2\rho^2-z)}\right| ,$$

$$l_4 = \ln\left|\frac{(1-x_1)^2xx_1(z-1)(z-x^2)(z-(1-x_1)^2)^2}{x_2(z-(1-x_1))^2(z-x(1-x_1))^2}\right|$$

$$+\ln\left|\frac{(\rho^2-z)(x(1-x_1)\rho^2-z)^2}{(x^2\rho^2-z)((1-x_1)^2\rho^2-z)^2}\right| ,$$

$$l_5 = \ln\left|\frac{x((1-x_1)^2\rho^2-z)^2}{(1-x_1)^2(x(1-x_1)\rho^2-z)((1-x_1)\rho^2-z)^2}\right| ,$$

$$l_6 = \ln\left|\frac{(x\rho^2-z)((x+x_1)^2\rho^2-z)^2}{(x^2\rho^2-z)(x(x+x_1)\rho^2-z)((x+x_1)\rho^2-z)}\right| .$$

For wide-narrow angular acceptance it needs to consider only the case of positron emission $\Sigma_{\gamma\gamma}$, because the corresponding expression for electron emission $\Sigma^{\gamma\gamma}$ is just Eq. (23) with (ρ_4^2, ρ_2^2) as the limits of z-integration and ρ_3 instead ρ under the integral sign.

The analytical expression for $\Sigma_{\gamma\gamma}$ in this case has the following form:

$$\Sigma_{\gamma\gamma} = \frac{1}{4}\left(\frac{\alpha}{\pi}\right)^2 \int\limits_1^{\rho_3^2} \frac{dz}{z^2} L\, A_N^W , \tag{24}$$

$$A_N^W = y\Delta_{42} + \int\limits_{x_c}^1 dx\Big\{\Delta_{42}\Big[4(4+3x)+6(x+3)\ln(1-x)$$

$$+\left(x-1+4\,\frac{1+x^2}{1-x}\right)\ln x\Big] + \Delta_{42}^{(x)}\Big[(1-x)(3+\ln x)$$

$$+2(x+3)\ln(1-x)+4\,\frac{1+x^2}{1-x}\ln x\Big] + \overline{\Delta}_{42}^{(x)}\frac{2}{1-x}(4$$

$$+(1+x)^2)\ln(1-x)+2\,\frac{(1+x)^2}{1-x}\left(\theta_4\overline{\theta}_4^{(x)}\ln\left|\frac{\sqrt{z}-x\rho_4}{\rho_4-\sqrt{z}}\right|\right.$$

$$\left.-\theta_2\overline{\theta}_2^{(x)}\ln\left|\frac{\sqrt{z}-x\rho_2}{\rho_2-\sqrt{z}}\right|\right) + \frac{1+x^2}{1-x}B + \int\limits_0^{1-x} dx_1\Big[2\,\frac{1+x^2}{(1-x)x_1}$$

$$
\times\Big(\Delta_{42}^{(x)} l_{1+} + \Delta_{42} l_{2+} + (\overline{\theta}_4^{(x)} - \theta_2^{(x)}) l_{1-} + (\overline{\theta}_4 - \theta_2) l_{2-}\Big) + \Big(-1 - \frac{1+x}{1-x_1}
$$

$$
- \frac{x}{(1-x_1)^2}\Big)\Big(\Delta_{42}^{(x)}\Big(\ln\frac{(1-x_1)^2 x_2}{x^3 x_1} + l_{3+}\Big) + \Delta_{42}\Big(\ln\frac{(1-x_1)^2 x x_1}{x_2} + l_{4+}\Big)
$$

$$
+\Delta_{42}^{(1-x_1)}\Big(2\ln\frac{x}{(1-x_1)^2} + l_{5+}\Big) + (\overline{\theta}_4^{(x)} - \theta_2^{(x)}) l_{3-} + (\overline{\theta}_4 - \theta_2) l_{4-}
$$

$$
+(\overline{\theta}_4^{(1-x_1)} - \theta_2^{(1-x_1)}) l_{5-}\Big) + 2\,\frac{1+x}{1-x_1}\Delta_{42}^{(1-x_1)}\Big] + 2\,\frac{1+x^2}{1-x}\theta_4\overline{\theta}_4^{(x)}\Big[\int\limits_{1-\sqrt{z}/\rho_4}^{1-x} dx_1
$$

$$
\times\Big(\frac{1}{x_1}\bar{l}_6 - \frac{4}{x_2}\ln\frac{x}{1-x_1}\Big) + \int\limits_0^{\sqrt{z}/\rho_4 - x}\frac{dx_1}{x_1}\bar{l}_7\Big] + 2\,\frac{1+x^2}{1-x}\theta_2\overline{\theta}_2^{(x)}
$$

$$
\times\Big[\int\limits_{1-\sqrt{z}/\rho_2}^{1-x} dx_1\Big(\frac{1}{x_1}\tilde{l}_6 + \frac{4}{x_2}\ln\frac{x}{1-x_1}\Big) + \int\limits_0^{\sqrt{z}/\rho_2 - x}\frac{dx_1}{x_1}\tilde{l}_7\Big]\Big\},
$$

$$
B = \Delta_{42}\Big(4\ln(1-x)\ln\left|\frac{(z-\rho_2^2)(z-x^2\rho_2^2)(x^2\rho_4^2-z)(\rho_4^2-z)}{(z-x\rho_2^2)^2(x\rho_4^2-z)^2}\right| - \ln^2 x\Big)
$$

$$
+2\overline{\Delta}_{42}^{(x)}\ln(1-x)\ln\left|\frac{(z-x^2\rho_2^2)(x^2\rho_4^2-z)}{x^4(z-x\rho_2^2)(x\rho_4^2-z)}\right| + (3-2\ln x)\widetilde{K}(x,z;\rho_4,\rho_2)
$$

$$
+\overline{\Delta}_{42}^{(x)}\Big(7 - 2\ln x\ln(1-x) - 3\ln^2 x - 2L_{i2}(1-x) - \frac{x(1-x)+4x\ln x}{1+x^2}\Big)
$$

$$
+2(\overline{\theta}_4 - \theta_2)\ln(1-x)\ln\left|\frac{(x\rho_4^2-z)^3(z-\rho_2^2)^2(z-x^2\rho_2^2)}{(\rho_4^2-z)^2(x^2\rho_4^2-z)(z-x\rho_2^2)^3}\right|
$$

$$
+2(\overline{\theta}_4^{(x)} - \theta_2^{(x)})\ln(1-x)\ln\left|\frac{(z-x^2\rho_2^2)(x\rho_4^2-z)}{(x^2\rho_4^2-z)(x\rho_2^2-z)}\right|
$$

$$
+4\theta_4\overline{\theta}_4^{(x)}\ln\left|\frac{x\rho_4-\sqrt{z}}{\rho_4}\right|\ln\left|\frac{x(x\rho_4^2-z)}{x^2\rho_4^2-z}\right|
$$

$$
+4\theta_2\overline{\theta}_2^{(x)}\ln\left|\frac{\sqrt{z}-x\rho_2}{\rho_2}\right|\ln\left|\frac{z-x^2\rho_2^2}{x(z-x\rho_2^2)}\right|,
$$

$$
l_{1\pm} = (1\pm\hat{c})\ln\left|\frac{(z-x^2\rho_2^2)(z-x\rho_2^2)}{(z-x(1-x_1)\rho_2^2)(z-x(x+x_1)\rho_2^2)}\right|,
$$

$$
l_{2\pm} = (1\pm\hat{c})\ln\left|\frac{(z-x\rho_2^2)^3(z-(1-x_1)^2\rho_2^2)^2(z-(x+x_1)^2\rho_2^2)^2}{(z-x^2\rho_2^2)(z-x(1-x_1)\rho_2^2)(z-x(x+x_1)\rho_2^2)}\right|
$$

$$
-(1\pm\hat{c})\ln[(z-(1-x_1)\rho_2^2)^2(z-(x+x_1)\rho_2^2)^2],
$$

$$l_{3\pm} = (1 \pm \hat{c}) \ln \left| \frac{z - x^2\rho_2^2}{z - x(1-x_1)\rho_2^2} \right|, \quad l_{4\pm} = (1 \pm \hat{c}) \ln \left| \frac{z - \rho_2^2}{z - (1-x_1)\rho_2^2} \right|,$$
$$l_{5\pm} = (1 \pm \hat{c}) \ln \left| \frac{(z - (1-x_1)^2\rho_2^2)^2}{(z - x(1-x_1)\rho_2^2)(z - (1-x_1)\rho_2^2)} \right|,$$
$$\tilde{l}_6 = \ln \left| \frac{x^2(z - (1-x_1)^2\rho_2^2)^4}{(1-x_1)^4(z - x(1-x_1)\rho_2^2)^2(z - (1-x_1)\rho_2^2)^2} \right|,$$
$$\tilde{l}_7 = \ln \left| \frac{(z - x\rho_2^2)^2(z - (x+x_1)^2\rho_2^2)^4}{(z - x^2\rho_2^2)^2(z - x(x+x_1)\rho_2^2)^2(z - (x+x_1)\rho_2^2)^2} \right|,$$

where $x_2 = 1 - x - x_1$, and $\hat{c}$ is the operator of the substitution

$$\hat{c}f(\rho_2) = f(\rho_4), \quad \bar{l}_6 = -\hat{c}\tilde{l}_6, \quad \bar{l}_7 = -\hat{c}\tilde{l}_7 .$$

One can verify that in the symmetrical limit formula (24) coincides with Eq. (23).

For opposite side emission the next-to-leading contribution into Σ_2^{ph} reads

$$\Sigma_\gamma^\gamma = \left(\frac{\alpha}{\pi}\right)^2 L \int\limits_0^\infty \frac{dz}{z^2}\, T, \tag{25}$$

$$T = A\theta_\rho\overline{\theta}_1 - \int\limits_{x_c}^1 dx \left[\frac{1+x^2}{2(1-x)}N(x,z;\rho,1) + \Xi(x) + \frac{\overline{\Xi}(x)}{1-x}\right]$$
$$\times \int\limits_{x_c/x_1}^1 dx_1 \left[(1+x_1)\Xi(x_1) + \frac{2\overline{\Xi}(x_1)}{1-x_1}\right],$$

where

$$A = -6 - 14\ln(1-x_c) - 8\ln^2(1-x_c) + \int\limits_{x_c}^1 dx \Big\{ 7(1+x)$$
$$+ \frac{1+x^2}{2(1-x)}[3K(x,z;\rho,1) + 7\overline{\theta}_\rho^{(x)}] + 2\ln\frac{x-x_c}{x}\Big[(3+x)(1+\theta_\rho^{(x)})$$
$$+ \frac{4}{1-x}\overline{\theta}_\rho^{(x)} + \frac{1+x^2}{1-x}N(x,z;\rho,1)\Big] + \frac{8}{1-x}\ln\frac{x(1-x_c)}{x-x_c}\Big\}.$$

We introduce the following reduced notation for θ-functions:

$$\Xi(x) = \theta_\rho\overline{\theta}_1 + \theta_\rho^{(x)}\overline{\theta}_1^{(x)}, \quad \overline{\Xi}(x) = \theta_\rho\overline{\theta}_\rho^{(x)} - \theta_1\overline{\theta}_1^{(x)} .$$

The quantity $K(x, z; \rho, 1)$ entering into espression for A is the K- factor for single photon emission, and the quantity $N(x, z; \rho, 1)$ may be derived by the help of Eq. (7) in the following way:

$$N(x, z; \rho, 1) = \left(\widetilde{K}(x, z; \rho_4, \rho_2) - \frac{(1-x)^2}{1+x^2}(\Delta_{42} + \Delta_{42}^{(x)}) \right) \Big|_{\rho_4 = \rho, \ \rho_2 = 1}$$

In the wide-narrow case the corresponding formula for Σ_γ^γ may be written as follows:

$$\Sigma_\gamma^\gamma = \frac{\alpha^2}{\pi^2} L \int\limits_0^\infty \frac{dz}{z^2} T_N^W . \tag{26}$$

where

$$T_N^W = \tilde{A} - \frac{1}{2} \left\{ \int\limits_{x_c}^1 dx \left[\frac{1+x^2}{2(1-x)} N(x, z; \rho_3, 1) + \Xi_{31}(x) + \frac{1}{1-x} \overline{\Delta}_{31}^{(x)} \right] \right.$$
$$\times \int\limits_{x_c/x}^1 dx_1 \left[(1+x_1) \Xi_{42}(x) + \frac{2}{1-x_1} \overline{\Delta}_{42}^{(x)} \right]$$
$$+ \int\limits_{x_c}^1 dx \left[\frac{1+x^2}{2(1-x)} N(x, z; \rho_4, \rho_2) + \Xi_{42}(x) + \frac{1}{1-x} \overline{\Delta}_{42}^{(x)} \right]$$
$$\left. \times \int\limits_{x_c/x}^1 dx_1 \left[(1+x_1) \Xi_{31}(x) + \frac{2}{1-x_1} \overline{\Delta}_{31}^{(x)} \right] \right\},$$

and

$$\tilde{A} = (-6 - 14 \ln(1-x_c) - 8 \ln^2(1-x_c)) \Delta_{42}$$
$$+ \int\limits_{x_c}^1 dx \left\{ \Delta_{42} \left[7(1+x) + \frac{8}{1-x} \ln \frac{x(1-x_c)}{x - x_c} \right] \right.$$
$$+ \frac{1+x^2}{2(1-x)} \left[\frac{3}{2} \Delta_{42} \overline{K}(x, z; \rho_3, 1) + \frac{3}{2} \Delta_{31} \widetilde{K}(x, z; \rho_4, \rho_2) \right.$$
$$\left. + \frac{7}{2} (\Delta_{42} \overline{\Delta}_{31}^{(x)} + \Delta_{31} \overline{\Delta}_{42}^{(x)}) \right] + \ln \frac{x - x_c}{x} \left[(3+x)(\Delta_{31} \Xi_{42}(x) + \Delta_{42} \Xi_{31}(x)) \right.$$
$$+ \frac{4}{1-x} (\overline{\Delta}_{42}^{(x)} \Delta_{31} + \overline{\Delta}_{31}^{(x)} \Delta_{42}) + \frac{1+x^2}{1-x} (\Delta_{42} N(x, z; \rho_3, 1)$$
$$\left. \left. + \Delta_{31} N(x, z; \rho_4, \rho_2)) \right] \right\},$$

where

$$\Xi_{42}(x) = \theta_4\overline{\theta}_2 + \theta_4^{(x)}\overline{\theta}_2^{(x)} = \Delta_{42} + \Delta_{42}^{(x)},$$
$$\Xi_{31}(x) = \Delta_{31} + \Delta_{31}^{(x)}, \quad \overline{\Delta}_{31}^{(x)} = \Delta_{31} - \Delta_{31}^{(x)}.$$

It is obvious that in symmetrical limit formula (26) coincides with (25) one.

3. Numerical results

In this Section we will use the standard abbreviation for inclusive event selection, namely **barel** instead of IES. The numerical calculations carried out for the beam energy and limited angles of detectors are as given in [3]. The Born cross-section

$$\Sigma_B = \frac{4\pi\alpha^2}{Q_1^2}\int\limits_{\rho_2^2}^{\rho_4^2}\frac{dz}{z^2}\left(1-\frac{\theta_1^2}{2z}\right)$$

(in symmetrical wide-wide case the limits of integration are 1 and ρ_3^2) equals 175.922 nb for $w-w$ **barel** and **calo1**, 139.971 nb for $w-w$ **calo2**, $n-n$ **barel** and **calo1**, 103.299 nb for $n-n$ **calo2**. The values of the Born cross-section for $n-w$ and $n-n$ cases are the same.

The results of our calculations of QED correction with the switched off vacuum polarization are shown in the Tables I–III. For comparsion we give also the corresponding numbers derived by the help of Monte Carlo program **bhlumi** [3].

As one can see from the Table I there is an approximately constant difference on the level of 0.3 *per mille* between our analytical and MC results inside first order correction. The possible reason of this effect is the following. In our calculation we systematiccaly ignore terms accompanied with $\theta^2 \simeq |t|/s$ as compared with unit. It is well known that such kind of terms have double logarithmic asymptotic behaviour and parametrically equal to $(\alpha|t|/\pi s)\ln^2\frac{|t|}{s}$ which is just 0.1 *per mille* for LEP1 conditions. As we know MC **bhlumi** program takes into account all first order contributions [12].

TABLE I

The SABS cross-section at LEP1 with first order QED correction. Vacuum polarization is switched off.

x_c	**bhlumi ww**	**ww**	**nn**	**wn**
		barel		
0.1	166.046	166.008	130.813	134.504
0.3	164.740	164.702	129.797	133.416
0.5	162.241	162.203	128.001	131.428
0.7	155.431	155.390	122.922	125.809
0.9	134.390	134.334	106.478	107.945
		calo1		
0.1	166.329	166.285	131.032	134.270
0.3	166.049	166.006	130.833	134.036
0.5	165.287	165.244	130.416	133.466
0.7	161.794	161.749	128.044	130.542
0.9	149.934	149.866	118.822	120.038
		calo2		
0.1	131.032	130.997	94.666	98.354
0.3	130.739	130.705	94.491	98.127
0.5	130.198	130.141	94.177	97.720
0.7	127.549	127.491	92.981	95.874
0.9	117.553	117.491	86.303	87.696

In the Table II we give the values of the SABS cross-section taking into account the second order photonic correction and compare our result suitable for **barel** event selection with exponentiated version of MC **bhlumi** program. The second and leading third order corrections are shown in the Table III. As concerns the second order we give contributions due to pair production and two photon emission.

TABLE II

The SABS cross-section at LEP1 with second order photonic correction.

		barel		
x_c	**bhlumi ww**	**ww**	**nn**	**nw**
0.1	166.892	166.958	131.674	134.808
0.3	165.374	165.447	130.524	133.583
0.5	162.530	162.574	128.474	131.127
0.7	155.668	155.597	123.206	125.255
0.9	137.342	137.153	108.820	109.677

TABLE III

The higher order corrections to SABS cross- section at LEP1.

barel			
pair production correction			
x_c	**ww**	**nn**	**nw**
0.1	0.007	- 0.004	0.015
0.3	- 0.033	- 0.033	- 0.020
0.5	- 0.058	- 0.050	- 0.041
0.7	- 0.090	- 0.074	- 0.069
0.9	- 0.142	- 0.115	-0.115
second order photonic correction			
0.1	0.742 + 0.208	0.679 + 0.182	0.249 + 0.091
0.3	0.546 + 0.199	0.556 + 0.171	0.069 + 0.098
0.5	0.140 + 0.231	0.292 + 0.182	- 0.314 + 0.013
0.7	- 0.027 + 0.234	0.117 + 0.187	- 0.571 + 0.170
0.9	2.961 + 0.048	2.458 - 0.116	1.822 - 0.090
leading third order correction			
0.1	- 0.055	- 0.047	- 0.006
0.3	- 0.065	- 0.053	- 0.018
0.5	- 0.038	- 0.040	0.004
0.7	0.089	0.058	0.124
0.9	0.291	0.220	0.331

Beside this we divide the photonic correction by leading and next-to-leading parts. Third order correction include both, pair production accompanied with single photon emission and three photon emission.

As we can see for intermediate values of x_c the next-to-leading photonic correction may be more than leading one but exponentiated version of **bhlumi** program absorbs its main part.

This work sponcored by INTAS Grant 93–1867. The speaker and other participants of CRAD'96 (A.B.A.,E.A.K. and L.N.L.) thank the Local Organizing Committe for financial support.

REFERENCES

[1] The LEP Collaboration: ALEPH, DELPI, L3 and OPAL and the LEP Electroweak Working Group, CERN–PPE/95; B.Pietrzyk, preprint LAPP-Exp-94.18. Invited talk at the Conference on "Radiative Corrections: Status and Outlook". Galtinburg, TN, USA, 1994; I.C.Brock *et al.* Preprint CERN-PPE/96-89' CMU-HEP/96-04 .

[2] Small Angle Bhabha Scattering. Yell.Rep. CERN 95-03, Part III.

[3] Events Generator for Bhabha Scattering, H.Anlauf *et al.* Conveners: S. Jadach, O. Nicrosini. Yell.Rep.CERN 96-01, v.2 p.229–298.

[4] S. Jadach, E. Richter-Was, B.F.L. Ward, Z. Was, *Comput. Phys. Commun.* **70**, 305 (1992).

[5] Montagna *et al.*, *Comput. Phys. Commun.* **76**, 328 (1993); M. Cacciori, G. Montagna, F. Piccinini, *Comput. Phys. Commun.* **90**, 301 (1995) , CERN-TH/95-169; G. Montagna *et al.*, *Nucl. Phys.* bf B401, 3 (1993); G. Montagna, O. Nicrosini, F. Piccinini, Preprint FNT/T-96/8.

[6] S. Jadach, E. Richter-Was, B.F.L. Ward, Z. Was, *Phys.Lett.* **B353**, 349, 362 (1995); S. Jadach, M. Melles, B.F.L. Ward, S.A. Yost, *Phys. Lett.* **B377**, 168 (1996).

[7] S. Jadach, M. Skrzypek, B.F.L. Ward, *Phys.Rev.* **D47**, 3733 (1993); S. Jadach, E. Richter-Was, B.F.L. Ward, Z. Was, *Phys. Lett.* **B260**, 438 (1991).

[8] W. Beenakker, F.A. Berends, S.C. van der Marck, *Nucl.Phys.***B355**, 281 (1991); W. Beenakker, B. Pietrzyk, *Phys. Lett.* **B304**, 366 (1993).

[9] M. Gaffo, H. Czyz, E. Remiddi, *Nuovo Cim.* **105A**, 271 (1992); *Int. J. Mod. Phys.* **4**, 591 (1993); *Phys.Lett.* **B327**, 369 (1994).

[10] A.B. Arbuzov *et al.*, Yell. Rep. CERN 95-03, p.369; preprint CERN-TH/95-313, UPRF-95-438 (to be published in *Nucl.Phys.*).

[11] A.B. Arbuzov, E. Kuraev, N.P. Merenkov, L. Trentadue, JETP **81**, 638 (1995); Preprint CERN-TH/95-241, JINR-E2-95-110.

[12] S. Jadach, B.F.L. Ward, *Phys.Rev.* **D40**, 3582 (1989).