POLRAD 2.0 CODE FOR THE RADIATIVE CORRECTIONS TO DIS OF POLARIZED PARTICLES*

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Physics of the radiative corrections to deep inelastic inclusive and semi-inclusive experiments of polarized particles is discussed. The new version of FORTRAN code POLRAD 2.0 is presented. Current status is described.

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1. Introduction

Our program POLRAD 2.0 based theoretically on the original approach proposed firstly in Ref. [1] and developed in the Ref. [2] was created to suit the demands of the present and future experiments with fixed polarized nuclear targets and at collider. Along with the possibilities of the previous versions of POLRAD [3], which calculated the QED lowest order radiative corrections (RC) to DIS of polarized leptons by polarized nuclei, the current version gives an opportunity to take into account both electroweak and higher order effects and to calculate the RC for semi-inclusive polarized experiments.

In Section 2 we present the short review of the basic formulae for the lowest order QED correction for each of the radiative tails: elastic, quasielastic and inelastic. Also the results of the ultrarelativistic approximation, the contribution of α^2 order correction and upgrade of the RC iteration procedure are discussed. The expressions for one-loop electroweak correction within the framework of standard theory and QCD improved parton model are given in Section 3. The POLRAD 2.0 part which calculates RC in semi-inclusive case is the modification of the code SIRAD [4] and is described in Section 4. Section 5 is devoted to testing and application of POLRAD 2.0.

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2. QED formulae for inclusive case

The model independent RC of the lowest order can be written as the sum of bremsstrahlung and loop effects:

$$\sigma = \sigma_{\rm in} + \sigma_{\rm el} + \sigma_{\rm q} + \sigma_{\rm v}\,. \tag{1}$$

Each σ denotes the double differential cross section $d^2\sigma/dxdy$, and x, y are usual scaling nucleon variables. $\sigma_{\rm in,el,q}$ are the contributions of radiative tails from continuous spectrum from elastic and quasielastic peaks respectively. The quantity σ_v is the contribution of virtual photon radiation and vacuum polarization effects.

The explicit form for these contributions was obtained in Ref. [2]. For the infrared free sum of σ_v and $\sigma_{\rm in}$ we have

$$\sigma_v + \sigma_{\rm in} = \frac{\alpha}{\pi}(\delta_R^{IR} + \delta_{\rm vert} + \delta_{\rm vac}^l + \delta_{\rm vac}^h)\sigma_o + \sigma_{\rm in}^F. \tag{2}$$

$\sigma_{\rm in}^F$ is the infrared free part of the IRT cross section

$$\sigma_{\rm in}^F = -\alpha^3 y \int\limits_{\tau_{\rm min}}^{\tau_{\rm max}} d\tau \sum_{i=1}^{8} \Bigg\{ \Theta_{i1}(\tau) \int\limits_0^{R_{\rm max}} \frac{dR}{R}\left[\frac{\Im_i(R,\tau)}{(Q^2+R\tau)^2} - \frac{\Im_i(0,0)}{Q^4}\right] + \sum_{j=2}^{k_i} \Theta_{ij}(\tau) \int\limits_0^{R_{\rm max}} dR \frac{R^{j-2}}{(Q^2+R\tau)^2}\Im_i(R,\tau) \Bigg\}. \tag{3}$$

Invariants, limits of integration and functions $\Theta_{ij}(\tau)$ are defined in (A.3), (14) and Appendix B of Ref. [2]. The quantities $\Im_i$ are defined as some combinations of inelastic structure functions (SF) and for spin 0, 1/2 and 1 could be found in (13) of Ref. [2].

The quantity δ_R^{IR} appears when the infrared divergence is extracted in accordance with the Bardin and Shumeiko method [5] from $\sigma_{\rm in}$. The virtual photon contribution consists of the lepton vertex correction $\delta_{\rm vert}$ and the vacuum polarization by leptons $\delta_{\rm vac}^l$ and by hadrons $\delta_{\rm vac}^h$. These corrections are given by formulae (20-25) of Ref. [2].

In the case of elastic scattering the nucleus remains in the ground state, so we have an additional relation $R = R_{\rm el} = (S_{xA} - Q^2)/(1+\tau_A)$ resulting in

$$\sigma_{\rm el} = -\frac{\alpha^3 y}{A^2} \int\limits_{\tau_{A\rm min}}^{\tau_{A\rm max}} d\tau_A \sum_{i=1}^{8}\sum_{j=1}^{k_i} \Theta_{ij}(\tau_A) \frac{2M_A^2 R_{\rm el}^{j-2}}{(1+\tau_A)(Q^2+R_{\rm el}\tau_A)^2} \Im_i^{\rm el}(R_{\rm el},\tau_A)\,. \tag{4}$$

Here invariants with the index "A" contain the nucleus momentum p_A instead of p ($p_A^2 = M_A^2$, M_A is nucleus mass). The quantities $\Im_i^{el}$ are given by formulae (27)–(29) of Ref. [2].

Quasielastic scattering corresponds to direct collisions of leptons with nucleons inside nucleus. We have to integrate numerically both over R and τ due to self movement of nucleons:

$$\sigma_q = -\frac{\alpha^3 y}{A} \int\limits_{\tau_{\rm min}}^{\tau_{\rm max}} d\tau \sum_{i=1}^{8} \sum_{j=1}^{k_i} \Theta_{ij}(\tau) \int\limits_{R_{\rm min}^{\rm q}}^{R_{\rm max}^{\rm q}} dR \frac{R^{j-2}}{(Q^2 + R\tau)^2} \Im_i^{\rm q}(R,\tau) \,. \tag{5}$$

The quantities $\Im_i^{\rm q}$ can be obtained in the terms of quasielastic structure functions, which have a form of peak for $\omega = Q^2/2M$. Due to the absence of enough experimental data this fact is normally used for construction of the peak type approximation. The factors in front of response functions are estimated in peak, and subsequent integration of response functions leads to results in terms of suppression factors $S_{E,M,EM}$ (or of sum rules for electron-nucleus scattering [6]).

To simplify and accelerate the procedure of experimental data processing when rapid analysis is more important than accuracy, it is convenient to have the approximate formulae. In the case of RC calculation one can choose ultrarelativistic approximation

$$m^2, M^2 \ll S, X, Q^2, \tag{6}$$

that allows to calculate exactly first two terms (corrections $\sim \alpha L$ and $\sim \alpha$) of expansion over the leptonic mass m of the lowest order cross section.

Considering τ-dependence of quantities $\Theta_{ij}(\tau)$ in (3) one can see its peaking structure, so called s- and p-peaks [7] (or k_1- and k_2-peaks according [8]): $\Theta_{ij}(\tau) \sim \Theta_{ij}^s(\tau) + \Theta_{ij}^p(\tau)$. Using the identities

$$\begin{aligned} \Theta_{ij}^s(\tau)\Im(x,\tau) &= \Theta_{ij}^s(\tau)(\Im(x,\tau) - \Im(x,\tau_s)) + \Theta_{ij}^s(\tau)\Im(x,\tau_s), \\ \Theta_{ij}^p(\tau)\Im(x,\tau) &= \Theta_{ij}^p(\tau)(\Im(x,\tau) - \Im(x,\tau_p)) + \Theta_{ij}^p(\tau)\Im(x,\tau_p), \end{aligned} \tag{7}$$

one can extract and analytically integrate the terms corresponding to the mass singularities. The first terms in right-hand sides of (7) are free of mass singularities and so they contribute only to $\sim \alpha$ correction. So one can adopt $m^2 = M^2 = 0$ before the integration over τ (or photon radiation angles). SF's do not depend on τ in the rest terms. Hence, the last ones can be integrated analytically using the methods [9] and contribute to the leading correction $\sim \alpha L$.

We also use ultrarelativistic approximation for the cases of elastic and quasielastic radiative tails calculation. As quantities $\Im_i$ are expressed as the combinations of formfactors and therefore decrease with Q^2 as $\sim 1/Q^8$ then one can keep only the Compton peak (t peak) contribution. Here we follow the approach of [10] developed for polarized proton target.

There are no known reasons to consider the $O(\alpha^2)$ corrections to be negligible. We follow the method of SF in [11] for the calculation of the higher order electromagnetic radiative corrections to neutral current unpolarized lepton-proton DIS and generalize it for the case of polarized leptons and polarized nuclear target in the current version of POLRAD. For the case of s- and p-peaks the formulae could be obtained in the terms of the Born cross section and practically coincide with the expressions for unpolarized particles, however the contribution of the t-peak which is extremely important in the cases of elastic and quasielastic radiative tails has to be obtained for polarized DIS.

From the beginning particular emphasis has been placed in POLRAD on the procedure of removing RC from experimental data, which allows to extract Born data sets for cross sections, SF or asymmetries from measured ones. Besides, in POLRAD 2.0 the CERNLIB package MINUIT is used to fit the data with the account of experimental uncertainties and to calculate theoretically the error propagation of statistical uncertainty of fitted experimental data to the value of ΔA.

3. Electroweak radiative correction

The next evident step both from the theoretical and experimental points of view is the treatment of electroweak effects contribution. We include in POLRAD 2.0 the results of Ref. [12] for one-loop electroweak correction within the standard theory and the on-shell renormalization scheme in t'Hooft–Feynman gauge. The result for the correction is obtained as the sum of 1-loop virtual boson (V-contribution) and real photon (R-contribution) emission:

$$\sigma_{ew}^{1-\mathrm{loop}} = \sigma_S^B + \sigma_{Vl} + \sigma_{V\mathrm{q}} + \sigma_{\mathrm{box}} + \frac{\alpha}{\pi}\sum_{\mathrm{q}} \delta_{\mathrm{q}}\sigma_0^{\mathrm{q}} + \sum_{\mathrm{q}} \sigma_R^{\mathrm{q}}, \tag{8}$$

where σ_S^B is the correction to boson propagator, $\sigma_{Vl,\mathrm{q}}$, σ_{box} are infrared free parts of lepton and quark vertex functions and box graphs. The V-contribution is calculated on the basis of Ref. [13]. The quantity σ_R^{q} is an infrared free part of the R-contribution. The correction δ_{q} is obtained after infrared divergence cancellation. It is factorized in front of the born cross section for a quark σ_0^{q} and is an analog of quantity δ_R^{IR} in (2). The quantity

$\sum_q \sigma_R^q$ can be derived in terms of leptonic (σ_l^{ij}, $\hat\sigma_l^{ij}$), hadronic (σ_h^{ij}, $\hat\sigma_h^{ij}$) radiation and their interference (σ_{lh}^{ij}):

$$\sigma_R^q = \sum_{ij=\gamma,Z} \left\{\sigma_l^{ij} + \hat\sigma_l^{ij} + e_q\sigma_{lh}^{ij} + e_q^2\left(\sigma_h^{ij} + \hat\sigma_h^{ij}\right)\right\}. \tag{9}$$

The quantities $\sigma_{l,h,lh}^{ij}$ have the form of one-dimensional integrals over ξ and can be found in Ref. [12]. The hat-quantities in (9) are small corrections arising from the non-leading terms of the expansion of polarization vectors (see Appendix A of Ref. [2]).

An implementation of QCD-improved [14] parton model for the most important case of leptonic current correction requires an additional generalization of (9) which are valid for the simple parton model and cannot be generated directly because an analytical integration over Q_h^2 been already done. As a result, applying methods as in (7), correction takes the form:

$$\sigma_{1\text{lep}}^{\text{in}} + \sigma_{1\text{lep}}^{v} = \frac{\alpha}{\pi}\delta\sigma_0 + \sigma_S^B + \sigma_{Vl} + \sigma_s + \sigma_p + \hat\sigma_l + \sigma_r. \tag{10}$$

Here $\sigma_{s,p,t}$ are the contributions of s- and p-peaks (see Eqs (32) and (39) of Ref. [12]). σ_r is the mass-singularities-free term obtained after the subtraction of SF in the peaks.

4. Semi-inclusive physics

We calculate the radiative corrections to data of semi-inclusive polarized experiments when a hadron is detected in coincidence with the outgoing lepton. In this case the cross section depends additionally on variable z (the amount of virtual photon energy transmitted to measured hadron in lab frame), defined as $z = p_1p_2/p_1q$, where p_1, p_2 and q are 4-momenta of initial nucleus, coincident hadron and virtual photon. The lowest order QED correction was calculated in Ref. [4, 15] and can be written as the sum of factorizing and non-factorizing parts

$$\sigma_{EM} \equiv \frac{d^3\sigma_{EM}}{dx\,dy\,dz} = \sigma_R^F + \frac{\alpha}{\pi}\delta_{VR}\sigma_0, \tag{11}$$

where

$$\sigma_R^F = \frac{\alpha}{\pi}\frac{2\pi\alpha^2}{S}\int\limits_\Omega dt_1dt_2\{\frac{y^2}{x_t^2y_t}[F_R^u\Sigma^+(\tilde x,\tilde z) + P_lP_NF_R^p\Sigma^-(\tilde x,\tilde z)] - \frac{\Theta(t_{1i}-t_1)}{xy}[F_{IR}^u\Sigma^+(x,y) + P_lP_NF_{IR}^p\Sigma^-(x,y)]\} \tag{12}$$

and $F^{u,p}_{R,IR}$ are the combinations of invariants (see Eq. (37) of Ref. [4] for explicit formulae), and $\Sigma^{\pm}$ are the products of parton distributions and fragmentation functions of the considered hadron. The variables $\tilde{x}$ and $\tilde{z}$ differ from the variables x, z by taking into account the dependence of virtual photon 4-momentum q on radiated photon 4-momentum k.

In the experiment the measured values are the longitudinal asymmetry A^H_R, with the target polarization vector parallel and antiparallel to the momentum vector $\vec{k}_1$ of the incident lepton and a ratio $r(x,z)$ of the differences of $\pi^{\pm}$ production off proton and neutron, which is directly accessible to experiment with unpolarized particles:

$$A^H_R = \frac{\sigma\uparrow^H - \sigma\downarrow^H}{\sigma\uparrow^H + \sigma\downarrow^H}, \qquad r(x,z) = \frac{\sigma^{\pi^-}_p - \sigma^{\pi^-}_n}{\sigma^{\pi^+}_p - \sigma^{\pi^+}_n}. \tag{13}$$

POLRAD 2.0 gives the opportunity to calculate radiative corrections to these observables with the possibility of additional integration of cross sections over z or x for the following hadrons H: $\pi^{\pm}$, π^0, $K^{\pm}$, K^0, $\bar{K}^0$, p, $\bar{p}$.

5. Tests and implementation of POLRAD

POLRAD passed a number of both analytical and numerical tests. The basic formulae of POLRAD 2.0 were tested with the help of the algebraic programming system REDUCE 3.5. Also numerical comparisons were done with the results of the programs: TERAD86 and FERRAD35 (spin-independent part); code of Kukhto and Shumeiko [1] and SLAC RC code developed by Stuart on the basis of formulae of [1] (spin-dependent part); HECTOR [16] (in progress).

Now POLRAD is used as the basic and official program for the procedure of radiative corrections in SMC (CERN) and HERMES (DESY) and together with the above mentioned E143 program in SLAC experiments with polarized particles.

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