THE SMALL-x EVOLUTION OF UNPOLARIZED AND POLARIZED STRUCTURE FUNCTIONS*

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A brief overview is presented of recent developments concerning resummed small-x evolution, based upon the renormalization group equation. The non-singlet and singlet structure functions are discussed for both polarized and unpolarized deep-inelastic scattering. Quantitative results are displayed and uncertainties from uncalculated subleading terms are discussed.

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1. Introduction

Many investigations have been performed analyzing the small-x behaviour of deep-inelastic scattering (DIS) structure functions. The reasoning is the evolution kernels of the non-singlet and singlet, polarized and unpolarized parton densities contain large logarithmic contributions in the small-x region and large effects are in principle observable in colliders such as HERA.

The resummation of these small-x terms to all orders in the strong coupling α_s can be completely handled within the framework of perturbative QCD. As collinear and ultraviolet divergences appear in the calculations, the only appropriate method of incorporating the effects of the small-x resummations is to use the renormalization group equations. Evolving in Q^2 is the only way the effects of the resummation can be studied.

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Additionally, the effects of the resummed anomalous dimensions upon the DIS structure functions are strongly dependent upon the parton densities at the initial Q_0^2, which are non-perturbative and must be given as input. As the evolution is based upon the Mellin convolution of the parton densities with the anomalous dimensions, the large and medium-x regions are taken into account as well. The large logarithmic contributions to the anomalous dimensions therefore do not automatically imply a large effect upon the observable DIS structure functions.

In Section 2, the underlying principles of the all-order small-x resummation based upon the renormalization group are recalled. Section 3 presents numerical results[1]. The effect of physically motivated subleading terms is also discussed, as well as additional uncertainties in cases where the input parton densities are not well constrained. Section 4 contains the conclusions.

2. Theoretical background

The evolution of the parton densities is given, for the non-singlet case (and generically for the singlet), by

$$\frac{\partial q(x, Q^2)}{\partial \ln Q^2} = P(x, \alpha_s) \otimes q(x, Q^2), \tag{1}$$

where the $\otimes$ denotes the Mellin convolution. This convolution is not simply limited to the low-x region, rather it is dependent upon the entire possible x-range.

The leading gluonic contributions to the unpolarized singlet anomalous dimension behave according to [2] ($a_s \equiv \alpha_s(Q^2)/4\pi$) as

$$\left(\frac{a_s}{N-1}\right)^k \leftrightarrow \frac{1}{x} a_s^k \ln^{k-1} x \, . \tag{2}$$

The corresponding quark anomalous dimensions, being one power down in $\ln x$ have been calculated in [3]. The leading terms of all anomalous dimensions for the non-singlet [4] and polarized singlet [5] evolutions are given by

$$N\left(\frac{a_s}{N^2}\right)^k \leftrightarrow a_s^k \ln^{2k-2} x \, . \tag{3}$$

The splitting functions $P(x, a_s)$ can be represented by

$$P(x, a_s) = \sum_{l=0}^{\infty} a_s^{l+1} P_l(x) \, . \tag{4}$$

[1] For a much more complete and detailed discussion see [1].

Order by order in a_s, the expansion coefficients $P_l(x)$ are subject to the constraints

$$\int_0^1 dx\, P_l^-(x) = 0\,, \quad \int_0^1 dx\, x \sum_i P_{ij,l}^{\mathrm{unpol.}}(x) = 0\,, \tag{5}$$

where the first equation represents fermion number conservation for the '−' non-singlet expansion coefficients and the second equation four-momentum conservation in the unpolarized singlet case. The resummed anomalous dimensions are subject to the existence of subleading terms and the effects of such terms have been investigated in [1, 6–10]. Physically motivated examples used here will be

$$\begin{aligned}
\mathrm{A}: \Gamma(N,a_s) &\to \Gamma(N,a_s) - \Gamma(1,a_s)\,,\\
\mathrm{B}: \Gamma(N,a_s) &\to \Gamma(N,a_s)(1-N)\,,\\
\mathrm{C}: \Gamma(N,a_s) &\to \Gamma(N,a_s)(1-2N+N^2)\,,\\
\mathrm{D}: \Gamma(N,a_s) &\to \Gamma(N,a_s)(1-2N+N^3)\,,
\end{aligned} \tag{6}$$

where $N \to N-1$ for the unpolarized singlet case.

Beyond leading-order, the parton densities themselves are not observables. The parton densities must be Mellin convoluted with the appropriate coefficient functions to engender the observable structure functions. The accumulated effect upon the structure function determines the real impact of the small-x resummation. The effect of the yet uncalculated subleading terms may be illustrated by the variance of the prescriptions A–D. Only when the variance is small and the results are similar to the original resummation-enhanced structure function can the resummation be considered reliable.

3. Numerical results

3.1. Non-singlet structure functions

The evolution of the '−'-combination

$$\begin{aligned}
xF_3^N(x,Q_0^2) &\equiv \tfrac{1}{2}\left[xF_3^{\nu N}(x,Q_0^2) + xF_3^{\bar\nu N}(x,Q_0^2)\right]\\
&= c_{F_3}^-(x,Q_0^2) \otimes [xu_v + xd_v](x,Q_0^2)
\end{aligned} \tag{7}$$

for an isoscalar target N and the '+'-combination

$$\begin{aligned}
&F_2^{\mathrm{ep}}(x,Q_0^2) - F_2^{\mathrm{en}}(x,Q_0^2) =\\
&c_{F_2}^+(x,Q_0^2) \otimes \tfrac{1}{3}[xu_v - xd_v - 2(x\bar d - x\bar u)](x,Q_0^2)
\end{aligned} \tag{8}$$

have been investigated in [7, 8]. As in all other numerical examples displayed below, the reference scale for the evolution is chosen as $Q_0^2 = 4\ \mathrm{GeV}^2$, and the same input parameters are employed for the next-to-leading-order (NLO) and the resummed calculations. In the present case, the initial parton distributions have been adopted from the MRS(A) global fit [11] together with the value of the QCD scale parameter, $\Lambda_{\overline{MS}}(N_f = 4) = 230$ MeV. From Fig. 1, the resummation effect can be seen to be on the order of one percent and not giving a K-factor on the order of ten as suggested in [12].

The effects of the small-x resummation can also be considered in the context of QED. The effects have been investigated for initial-state QED radiative corrections to deep-inelastic eN scattering. For large y and small x, the effect can reach up to ten percent [13]. The resummation was also investigated for $e^+e^- \to \mu^+\mu^-$. The results can also be found in [13].

The polarized non-singlet case presents interesting features in addition to those observed in the unpolarized non-singlet. In contrast to the unpolarized situation, the shapes of the input parton densities have not been well established yet. An additional freedom is available to adjust the input densities and gauge the impact.

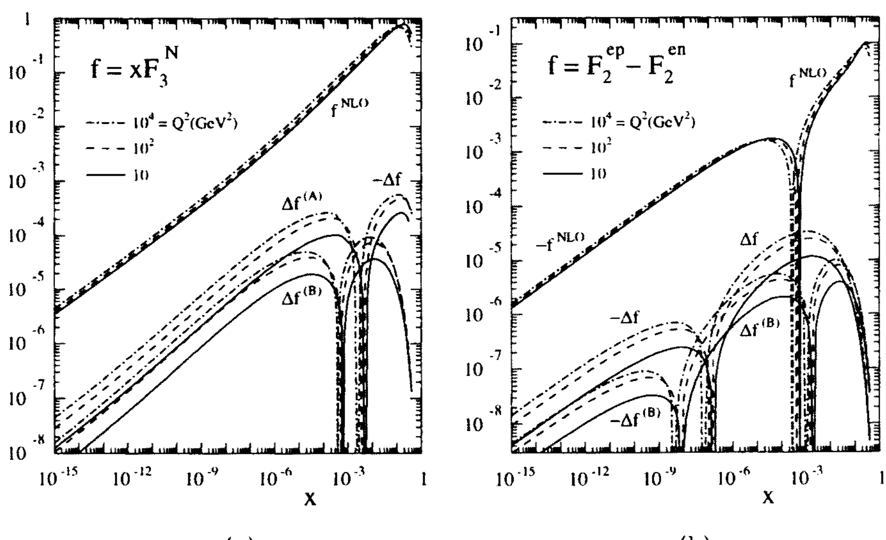

Fig. 1. (a) (left): The small-x Q^2-evolution of xF_3^N in NLO and the corrections to these results due to the resummed kernels. '(A)' and '(B)' denote two prescriptions for implementing fermion number conservation, see Eq. (6). (b) (right): The same as in Figure 1(a), but for the structure function combination $F_2^{ep} - F_2^{en}$. Instead of the prescription '(A)', the result without any subleading terms is shown for this '+'-case.

The effect on similar polarized non-singlet combinations is an enhancement on the order of 15 % at $x \sim 10^{-5}$ using the fermion-number conserving prescription (A) but disappears completely when using (D) for flat input parton densities. For the steep densities, the effect is maximally 1.5 %, and can also be eliminated depending upon the choice of fermion-number conservation prescription.

3.2. Polarized singlet structure functions

Resummation relations for amplitudes [4] related to the singlet anomalous dimensions for polarized DIS have been derived in [5]. Explicit analytic and numerical results for the evolution kernels beyond NLO have been derived using these relations in [6], including an all-order symmetry relation among the elements of the anomalous dimension matrix and a discussion of the supersymmetric case.

Here also we suffer from poorly constrained input parton densities, adding to the uncertainty surrounding the effects of the resummation.

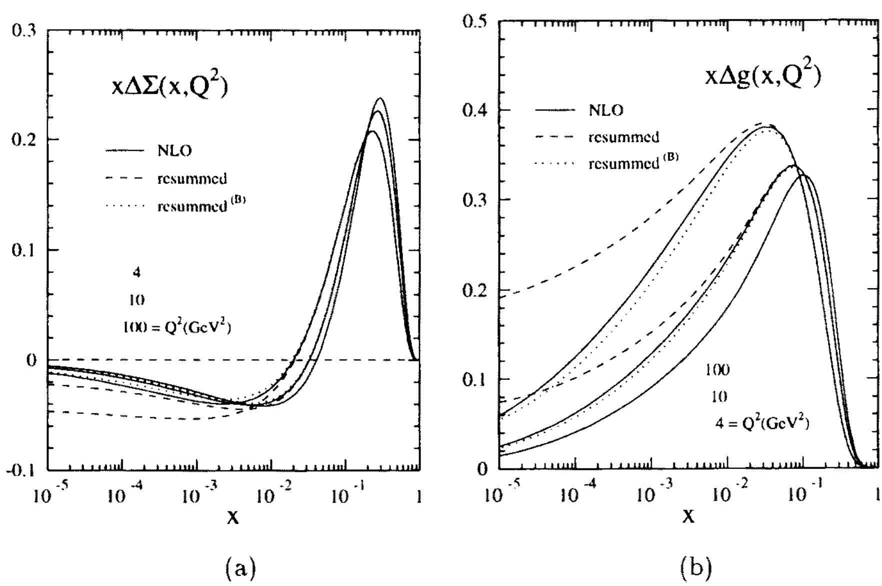

Fig. 2. (a) (left): The evolution of the polarized singlet combination $x\Delta\Sigma$ in NLO and including the resummed kernels. The impact of possible subleading terms is illustrated by the prescription '(B)' in Eq. (6). The input densities are from ref. [14]. (b) (right): As in Figure 2a, but for the polarized gluon momentum distribution $x\Delta g$. As in the previous figure, the Q^2-values in the legend are ordered according to the sequence of the curves at small x.

From figures 2(a) and 2(b), the effects of the resummation are quite evident. The resummation has a considerable impact, in particular upon $x\Delta g(x, Q^2)$. While no sum rule is applicable to the polarized singlet case, we draw upon the knowledge that the coefficients of the terms of the anomalous dimensions subleading by one power in N at LO and NLO are generally of the same magnitude and of the opposite sign [6]. We can therefore take this as an example of what subleading terms as $x \to 0$ might be.

3.3. Unpolarized singlet structure functions

We now turn to the unpolarized singlet case, where the LO small-x resummations have been performed in [2] and the NLO quark sector resummations by [3]. This case is of particular importance as the high-statistics data from HERA are beginning to arrive, testing the physical viability of the resummation as well as the NLO calculations of DIS structure functions. Investigations have been carried out in [1, 9, 10] and an example is displayed in figure 3.

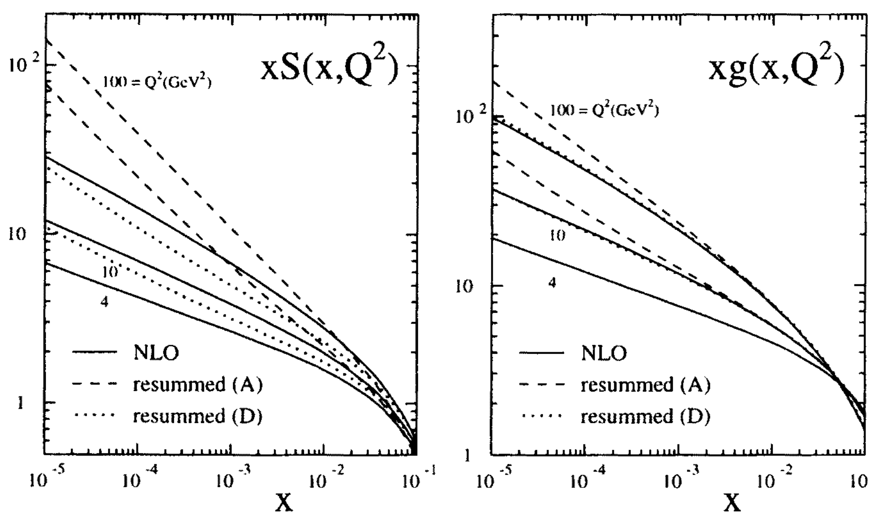

Fig. 3. The evolution of the gluon and total sea densities with the resummed kernels of [2, 3] compared to the NLO results. Prescriptions '(A)' and '(D)' have been implemented (see Eq. (6)).

As in the polarized singlet resummation, the effects of the pure resummations are quite large. The inclusion of the four-momentum conserving subleading terms, however, substantially reduces the effects and the resummed results can even fall below the NLO curve using (D).

4. Conclusions

The current status of small-x resummations of polarized and unpolarized, non-singlet and singlet structure functions has been discussed.

The unpolarized non-singlet structure functions are enhanced by one percent or less. Similar observations have been made for the polarized non-singlet case. The non-singlet QED corrections are found to have an effect on the order of ten percent for $x \sim 10^{-4}$ and $y > 0.9$.

The singlet cases for both the polarized and unpolarized structure functions show large effects stemming from the resummation. Taking into account less singular terms, arising from energy-momentum conservation in the unpolarized situation, or observing the behaviour of the LO and NLO polarized anomalous dimensions, reduces the effect considerably and can even completely eliminate it.

To assess the real effect of the small-x resummations, the subleading terms need to be calculated. As demonstrated here, the subleading terms may be quite important in the evolution of the structure functions. To have an adequate foundation for comparison, the next-to-next-to-leading order results need to be calculated.

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