IMPROVED PREDICTIONS FOR QCD EFFECTS IN e^+e^- ANNIHILATION*

P.A. RĄCZKA**

Institue of Theoretical Physics, Warsaw University
Hoża 69, 00-681 Warsaw, Poland

(Received January 14, 1997)

The three-loop QCD correction to the $R_{e^+e^-}$ ratio is discussed. It is pointed out that the three-loop coefficient is dominated by the term proportional to π^2, which appears as a result of analytic continuation from spacelike to timelike momenta. The possibility of all-order resummation of some of the large contributions of this type is considered. It is demonstrated that the resummed predictions are much less sensitive to the change of the renormalization scheme than the predictions obtained with the conventional expansion. Also the difference between two-loop and three-loop results is significantly reduced. Similar remarks are shown to apply in the case of QCD correction to $\Gamma_Z^{\rm had}$.

PACS numbers: 12.38. Cy

Recently there has been some interest in the precise determination of the QCD correction $\delta_{e^+e^-}$ to the $R_{e^+e^-}$ ratio [1–4]

$$R_{e^+e^-}(s) = \frac{\sigma_{\rm tot}(e^+e^- \to {\rm hadrons})}{\sigma_{\rm tot}(e^+e^- \to \mu^+\mu^-)} = 3\sum_f Q_f^2[1+\delta_{e^+e^-}(s)]\,. \qquad (1)$$

Similar correction dominates the QCD contribution to $\Gamma_Z^{\rm had}$. One of the complications in the evaluation of these corrections comes from the effect of renormalization scheme (RS) dependence. In the next-to-next-to-leading order (NNLO) there are two independent parameters characterizing the choice of RS. The differences in the predictions between various schemes are formally of higher order, but numerically they may be significant. Recently a

* Presented at the Cracow International Symposium on Radiative Corrections to the Standard Model, Cracow, Poland, August 1–5, 1996.

** Address after October 1, 1996: Centre for Particle Theory, Department of Physics, University of Durham, South Road, Durham DH1 3LE, UK.

systematic method has been proposed [5] for estimating the resulting theoretical ambiguities in a quantitative way. This method is based on the notion that one may classify the renormalization schemes according to the degree of cancellation in the expression for the RS invariant combination of the expansion coefficients. When applied to the conventional expansion for QCD effects in the $R_{e^+e^-}$ ratio this method reveals a surprisingly large RS dependence [6], even for relatively large energies.

To analyze in some detail the RS dependence effect of the conventional expansion for $\delta_{e^+e^-}$ let us write the NNLO expression in the form:

$$\delta_{e^+e^-}(s) = a(s)\,[1 + r_1 a(s) + r_2 a^2(s)]\,, \tag{2}$$

where $a(\mu^2) = g^2(\mu^2)/(4\pi^2)$ is the coupling constant, satisfying the renormalization group equation:

$$\mu\frac{da}{d\mu} = -b\,a^2\,(1 + c_1 a + c_2 a^2\,). \tag{3}$$

The coefficients r_i and c_2 depend on the choice of the scheme, but the combination

$$\rho_2 = c_2 + r_2 - c_1 r_1 - r_1^2\,, \tag{4}$$

is independent of the choice of RS. The idea of [5] was to use the function

$$\sigma_2(r_1, r_2, c_2) = |c_2| + |r_2| + c_1|r_1| + r_1^2\,, \tag{5}$$

to select the schemes according to the degree of cancellation in ρ_2, the schemes with unnaturally large expansion coefficients being characterized by large value of σ_2. It was pointed out in [5] that the set of schemes that have the same or smaller degree of cancellation as the principle of minimal sensitivity (PMS) scheme [7] is characterized by the condition

$$\sigma_2(r_1, r_2, c_2) \leq 2\,|\rho_2|. \tag{6}$$

Let us consider for concretness the case of $n_f = 4$, which is at present of considerable experimental interest. For $n_f = 4$ we have in the $\overline{MS}$ scheme $r_1^{\overline{MS}} = 1.52453$, $r_2^{\overline{MS}} = -11.68560$ [2, 1], $c_1 = 1.54$ and $c_2^{\overline{MS}} = 3.04764$, which gives $\rho_2^R = -13.30991$. The region of the scheme parameters that satisfies the constraint (6) falls within the rectangle $r_1 \in (-4.32, 3.76)$, $c_2 \in (-19.96, 6.65)$. The variation of the predictions over this set of scheme parameters is shown in the Fig. 1 as a function of $\sqrt{s}/\Lambda^{(4)}_{\overline{MS}}$. It is clear that this variation is large [6] and would have considerable effect for example on the determination of the $\Lambda^{(4)}_{\overline{MS}}$ parameter from the experimental data.

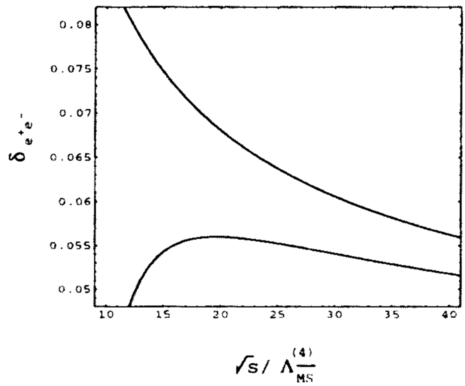

Fig. 1. The RS dependence of the conventional NNLO prediction for $\delta_{e^+e^-}$ as a function of $\sqrt{s}/\Lambda^{(4)}_{\overline{MS}}$ ($n_f = 4$). The curves shown correspond to two sets of RS parameters which satisfy the condition (6): $r_1 = 3.10$ and $c_2 = 6.65$ (upper curve), $r_1 = -4.32$ and $c_2 = 0$ (lower curve).

The origin of this strong RS dependence may be traced back to the large value of the NNLO correction, which manifests itself in a large value of ρ_2^R. However, a closer inspection shows that a dominant contribution to the NNLO correction comes from the term $-b^2\pi^2/12$, which appears as a result of analytic continuation from spacelike to timelike momenta. Indeed, we have

$$R_{e^+e^-}(s) = 12\,\pi\,\mathrm{Im}\,\Pi(s+i\varepsilon) = \frac{6\pi}{i}\left[\Pi(s+i\varepsilon) - \Pi(s-i\varepsilon)\right]. \qquad (7)$$

where $\Pi(q^2)$ is the transverse part of the correlator for the quark electromagnetic currents. The perturbative expression for $\Pi(q^2)$ involves powers of $\ln(-q^2/\mu^2)$, which after taking the discontinuity in (7) give rise to the terms containing powers of π^2. (It is straightforward to predict the structure of these terms beyond the NNLO.) The magnitude of the genuine NNLO QCD correction in e^+e^- annihilation may be seen by considering the so called Adler function [8]

$$D(q^2) = -12\pi^2\,q^2\frac{d}{dq^2}\Pi(q^2) = 3\sum_f Q_f^2\left[1+\delta_D(-q^2)\right], \qquad (8)$$

where δ_D has the form of (2). Assuming $n_f = 4$ we obtain for δ_D the value of RS invariant $\rho_2^D = 0.96903$, which is order of magnitude smaller than ρ_2^R.

One way of exploiting the smallness of the genuine QCD corrections is to express $\delta_{e^+e^-}$ in terms of δ_D using [9]:

$$\delta_{e^+e^-}(s) = -\frac{1}{2\pi i}\int_C d\sigma\,\frac{\delta_D(-\sigma)}{\sigma}, \tag{9}$$

where the contour C runs clockwise from $\sigma = s - i\varepsilon$ to $\sigma = s + i\varepsilon$, around $\sigma = 0$. (The integration contour may be, of course, arbitrarily deformed in the domain of analyticity of the Adler function.) Expanding the δ_D under the contour integral in terms of $a(s)$ and performing trivial contour integration of the logarithmic terms we recover the conventional expansion for $\delta_{e^+e^-}$. However, we may also use under the integral the renormalization group improved expression for δ_D and evaluate the contour integral numerically. In this case we obtain an expression for $\delta_{e^+e^-}$ which is no longer a polynomial in $a(s)$, despite the fact that only the NNLO expression for the Adler function is used. It is easy to convince oneself that in this way we effectively resum all higher order π^2 terms that involve powers of b, c_1 and/or c_2.

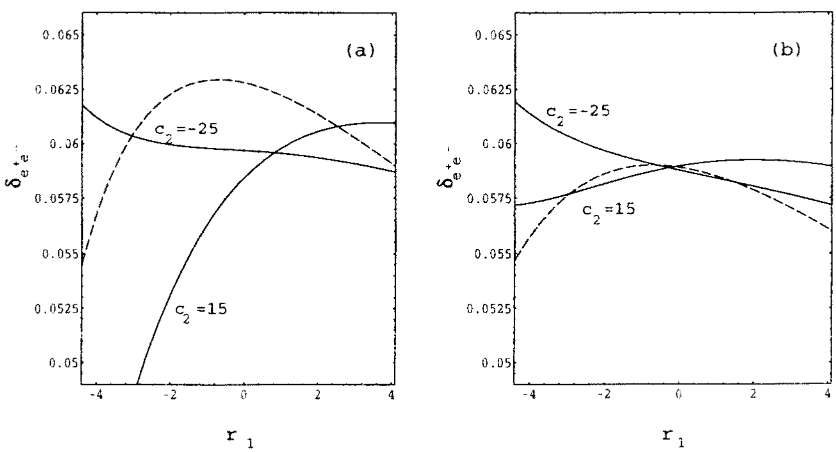

Fig. 2. The NNLO prediction for $\delta_{e^+e^-}$ as a function of r_1, for two values of c_2 and $\sqrt{s}/\Lambda^{(4)}_{\overline{MS}} = 30$, obtained with (a) the conventional expansion and (b) the improved expansion. The dashed line indicates the NLO prediction.

In Fig. 2 we compare the RS dependence of the conventional expression for $\delta_{e^+e^-}$ (Fig. 2(a)) with the RS dependence of the improved expression (Fig. 2(b)), assuming $\sqrt{s}/\Lambda^{(4)}_{\overline{MS}} = 30$. One may clearly see that inclusion of the higher order π^2 terms has stabilizing effect on the NNLO predictions [6] for any considered range of the scheme parameters. (In Fig.2 we parametrize the freedom of choice of RS by r_1 and c_2.) This reduction of RS dependence is even more pronounced if we take into account that the region of scheme parameters satisfying the condition (6) with $\rho_2 = \rho_2^D$ is much smaller compared to the region corresponding to ρ_2^R — it falls within the rectangle $r_1 \in (-0.7, 0.27)$, $c_2 \in (-0.48, 1.45)$. Also, the difference between the next-to-leading (NLO) and NNLO predictions in the PMS scheme is much smaller for the improved approximant, which is just a manifestation of the fact that ρ_2^D is small.

A detailed investigation of $\delta_{e^+e^-}$ performed in [6] shows that the remarks concerning RS dependence and the effect of resummation by means of contour integration apply also for $n_f = 5$ and $n_f = 3$.

The observations made for $\delta_{e^+e^-}$ may be extended to the QCD correction δ_Z to $\Gamma_Z^{\rm had}$:

$$\Gamma_Z^{\rm had} = \frac{G_F M_Z^3}{24\pi\sqrt{2}}\, 3\sum_f (v_f^2 + a_f^2)(1 + \delta_Z)\,. \tag{10}$$

Indeed, taking into account that:

$$\Gamma_Z^{\rm had} = \frac{1}{M_Z} {\rm Im}\, \Pi_1(M_Z^2 + i\varepsilon)\,, \tag{11}$$

where $\Pi_1(q^2)$ is the transverse part of correlator of the neutral weak quark currents, we may express δ_Z as a contour integral (9). The only difference is that we have to include now in the Adler function the contribution from the axial couplings. We may write:

$$\delta_{D,Z} = \delta_{D,Z}^{NS} + \delta_{D,Z}^{S,V} + \delta_{D,Z}^{S,A}\,, \tag{12}$$

where $\delta_{D,Z}^{NS}$ denotes the non-singlet contribution, which is of the form (2) and is the same as the non-singlet contribution relevant for $R_{e^+e^-}$ in the $n_f = 5$ region (*i.e.* $r_1^{\overline{MS}} = 1.40923$, $r_2^{\overline{MS}} = -0.68137$), $\delta_{D,Z}^{S,V}$ denotes the singlet contribution from the vector current correlator

$$\delta_{D,Z}^{S,V}(-q^2) = -1.2395\, a^3(-q^2)\Big(\sum_f v_f\Big)^2 \Big/ \Big(3\sum_f (v_f^2 + a_f^2)\Big) \tag{13}$$

and $\delta_{D,Z}^{S,A}$ is the singlet contribution from the axial current correlator which may be deduced from [10]

$$\begin{aligned}\delta_{D,Z}^{S,A}(-q^2) = {} & \frac{1}{3\sum_f (v_f^2 + a_f^2)} \Bigg\{ \left[-9.25 + 3\ln\left(\frac{-q^2}{m_t^2}\right)\right] a^2(-q^2) \\ & + \Bigg[-66.8799 + 11.1667\ln\left(\frac{-q^2}{m_t^2}\right) + 5.75\ln^2\left(\frac{-q^2}{m_t^2}\right) \\ & + 2\left(-9.25 + 3\ln\left(\frac{-q^2}{m_t^2}\right)\right)(r_1 - r_1^{\overline{MS}})\Bigg] a^3(-q^2)\Bigg\}\,. \end{aligned} \tag{14}$$

(Also, for $n_f = 5$ we have $c_1 = 1.26087$, $c_2^{\overline{MS}} = 1.47479$.) Expanding $a(-q^2)$ in terms of $a(M_Z^2)$ we recover the usual expression [10, 3] for δ_Z with large corrections proportional to π^2 both in the non-singlet and singlet contributions. The improved expression δ_Z is obtained by taking into account the running of the coupling constant in the complex momentum plane and evaluating the contour integral numerically.

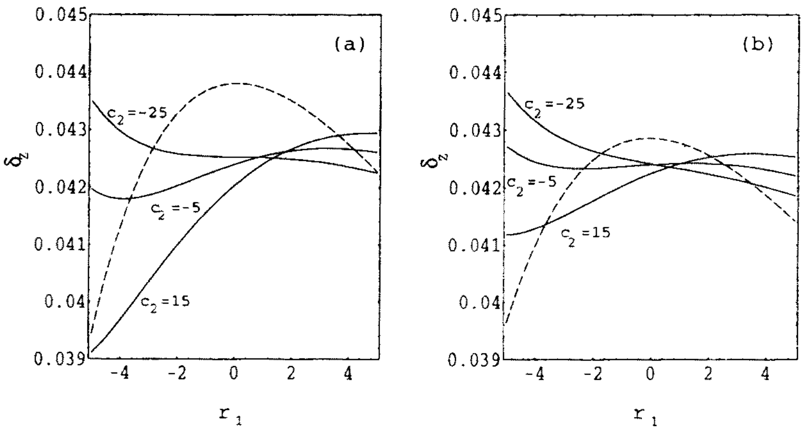

Fig. 3. The NNLO prediction for δ_Z as a function of r_1, for two values of c_2 and $M_Z/\Lambda^{(5)}_{\overline{MS}} = 205$, obtained with (a) the conventional expansion and (b) the improved expansion. The dashed line indicates the NLO prediction. It is assumed, following [11], that $\sin^2\theta_W = 0.2315$ and $m_t = 180$ GeV.

In Fig. 3 we compare the RS dependence of the conventional expression for δ_Z (Fig. 3(a)) with the RS dependence of the improved expression (Fig. 3(b)), assuming $M_Z/\Lambda^{(5)}_{\overline{MS}} = 205$. Again, one may clearly see that the resummation of the higher order π^2 terms with the contour integral technique greatly reduces the RS dependence of the NNLO prediction. It is also evident that the difference between the NLO and NNLO predictions in the PMS scheme is much smaller for the improved expression. This reflects the fact that the genuine NNLO QCD corrections are quite small — the RS invariant combination of the expansion coefficients for $\delta_{D,Z}^{NS}$ is -2.96937. Our analysis indicates that the perturbative QCD contribution to δ_Z may be evaluated with very good precision.

The trick of using the contour integral to resum higher order π^2 corrections is useful in the analysis of other QCD predictions for timelike momenta [12]. It may be extended to treat the strong interaction corrections to the Higgs boson decay into hadrons.

REFERENCES

[1] S.G. Gorishny, A.L. Kataev, S.A. Larin, *Phys. Lett.* **B259**, 144 (1991).

[2] L.R. Surguladze, M.A. Samuel, *Phys. Rev. Lett.* **66**, 560 (1991), Err. **66**, 2416.

[3] For a recent review see K.G. Chetyrkin, J.H. Kühn, *Phys. Lett.* **B342**, 356 (1995), K.G. Chetyrkin, J.H. Hühn, A. Kwiatkowski, report no. TTP-94-32 (hep-ph/9503396), L.R. Surguladze, M.A. Samuel, *Rev. Mod. Phys.* **68**, 259 (1996).

[4] In the Eq. (1) the electroweak corrections and the quark mass effects have been neglected.

[5] P.A. Rączka, *Z. Phys.* **C65**, 481 (1995).

[6] P.A. Rączka, A. Szymacha, *Phys. Rev.***D54**, 3073 (1996).

[7] P.M. Stevenson, *Phys. Lett.* **100B**, 61 (1981), *Phys. Rev.* **D23**, 2916 (1981).

[8] S.L. Adler, *Phys. Rev.* **D10**, 3714 (1974).

[9] A.V. Radyushkin, preprint JINR E2-82-159 (1982), also in Proceedings of 9-th CERN-JINR School of Physics, September 1985, Urbino, Italy, CERN Yellow Report CERN-86-03, vol. 1, p. 35, A.A. Pivovarov, *Nuovo Cim.* **105A**, 813 (1992).

[10] B.A. Kniehl, J.H. Kühn, *Phys. Lett.* **B224**, 229 (1990), *Nucl. Phys.* **B329**, 547 (1990), K.G. Chetyrkin, O. V. Tarasov, *Phys. Lett.* **B327**, 114 (1994), S.A. Larin, T. van Ritbergen, J.A.M. Vermaseren, *Phys. Lett.* **B320**, 159 (1994).

[11] Review of Particle Properties, Particle Data Group, L. Montanet *et al.*, 1995 off-year partial upadate.

[12] A.A. Pivovarov, *Z. Phys.* **C53**, 461 (1992), F. LeDiberder, A. Pich, *Phys. Lett.* **B286**, 147 (1992), P.A. Rączka, A. Szymacha, *Z. Phys.* **C70**, 125 (1996).