RESTORING GAUGE INVARIANCE IN GAUGE-BOSON PRODUCTION PROCESSES*

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A survey is given of the various gauge-invariance-related aspects that play a role when dealing with unstable gauge bosons.

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1. Unstable gauge bosons: lowest order

The physics goals of LEP2 and the next linear collider (NLC) cover a large variety of topics, *e.g.* the determination of the W-boson mass, establishing the Yang–Mills character of the triple gauge-boson couplings, the search for the Higgs boson, the search for supersymmetric particles, a detailed study of the symmetry-breaking mechanism, *etc.* Most of these studies require a careful investigation of processes with photons and/or fermions in the initial and final state.

If complete sets of graphs contributing to such a process are taken into account, the associated matrix elements are in principle gauge-invariant. However, the massive gauge bosons that appear as intermediate particles can give rise to poles $1/(k^2 - M^2)$ if they are treated as stable particles. This can be cured by introducing the finite decay width in one way or another, while at the same time preserving gauge independence and, through a proper high-energy behavior, unitarity. In field theory, such widths arise naturally from the imaginary parts of higher-order diagrams describing the gauge-boson self-energies, resummed to all orders. This procedure has been used with great success in the past: indeed, the Z resonance can be described to very high numerical accuracy. However, in doing a Dyson summation of self-energy graphs, we are singling out only a very limited subset of all the

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possible higher-order diagrams. It is therefore not surprising that one often ends up with a result that retains some gauge dependence.

Till recently two approaches for dealing with unstable gauge bosons were popular in the construction of lowest-order Monte Carlo generators. The first one involves the systematic replacement $1/(k^2 - M^2) \to 1/(k^2 - M^2 + iM\Gamma)$, also for $k^2 < 0$. Here Γ denotes the physical width of the gauge boson with mass M and momentum k. This scheme is called the 'fixed-width scheme'. As in general the resonant diagrams are not gauge-invariant by themselves, this substitution will destroy gauge invariance. Moreover, it has no physical motivation, since in perturbation theory the propagator for space-like momenta does not develop an imaginary part. Consequently, unitarity is violated in this scheme. To improve on the latter another approach can be adopted, involving the use of a running width $iM\Gamma(k^2)$ instead of the constant one $iM\Gamma$ ('running-width scheme'). This, however, still cannot cure the problem with gauge invariance.

At this point one might ask oneself the legitimate question whether the gauge-breaking terms are numerically relevant or not. After all, the gauge breaking is caused by the finite decay width and is, as such, in principle suppressed by powers of Γ/M. From LEP1 we know that gauge breaking can be negligible for all practical purposes. However, the presence of small scales can amplify the gauge-breaking terms. This is for instance the case for almost collinear space-like photons or longitudinal gauge bosons at high energies, involving scales of $\mathcal{O}(p_{\mathrm{B}}^2/E_{\mathrm{B}}^2)$ (with p_{B} the momentum of the involved gauge boson). In these situations the external current coupled to the photon or to the longitudinal gauge boson becomes approximately proportional to p_{B}. In other words, in these regimes sensible theoretical predictions are only possible if the amplitudes with external currents replaced by the corresponding gauge-boson momenta fulfill appropriate Ward identities.

In order to substantiate these statements, a truly gauge-invariant scheme is needed. It should be stressed, however, that any such scheme is arbitrary to a greater or lesser extent: since the Dyson summation must necessarily be taken to all orders of perturbation theory, and we are not able to compute the complete set of *all* Feynman diagrams to *all* orders, the various schemes differ even if they lead to formally gauge-invariant results. Bearing this in mind, we need some physical motivation for choosing a particular scheme. In this context two options can be mentioned, which fulfill the criteria of gauge invariance and physical motivation.

The first option is the so-called 'pole scheme' [1–3]. In this scheme one decomposes the complete amplitude according to the pole structure by expanding around the poles (*e.g.* $f(k^2)/(k^2 - M^2) = f(M^2)/(k^2 - M^2) +$ finite terms). As the physically observable residues of the poles are gauge-invariant, gauge invariance is not broken if the finite width is taken into

account in the pole terms $\propto 1/(k^2 - M^2)$. It should be noted, however, that there exists some controversy in the literature [3, 4] about the 'correct' procedure for doing this and about the range of validity of the pole scheme, especially in the vicinity of thresholds.

The second option is based on the philosophy of trying to determine and include the minimal set of Feynman diagrams that is necessary for compensating the gauge violation caused by the self-energy graphs. This is obviously the theoretically most satisfying solution, but it may cause an increase in the complexity of the matrix elements and a consequent slowing down of the numerical calculations. For the gauge bosons we are guided by the observation that the lowest-order decay widths are exclusively given by the imaginary parts of the fermion loops in the one-loop self-energies. It is therefore natural to perform a Dyson summation of these fermionic one-loop self-energies and to include the other possible one-particle-irreducible fermionic one-loop corrections ('fermion-loop scheme') [5]. For the LEP2 process $e^+e^- \rightarrow 4f$ this amounts to adding the fermionic triple gauge-boson vertex corrections. The complete set of fermionic contributions forms a gauge-independent subset and obeys all Ward identities exactly, even with resummed propagators [6]. As mentioned above, the validity of the Ward identities guarantees a proper behavior of the cross-sections in the presence of collinear photons and at high energies in the presence of longitudinal gauge-boson modes. On top of that, within the fermion-loop scheme the appropriately renormalized matrix elements for the generic LEP2 process $e^+e^- \rightarrow 4f$ can be formulated in terms of effective Born matrix elements, using the familiar language of running couplings [6].

A numerical comparison of the various schemes [5, 6] confirms the importance of not violating the Ward identities. For the LEP2 process $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$, a process that is particularly important for studying triple gauge-boson couplings, the impact of violating the U(1) electromagnetic gauge invariance was demonstrated [5]. Of the above-mentioned schemes only the running-width scheme violates U(1) gauge invariance. The associated gauge-breaking terms are enhanced in a disastrous way by a factor of $\mathcal{O}(s/m_e^2)$, in view of the fact that the electron may emit a virtual (space-like) photon with p_γ^2 as small as m_e^2. A similar observation can be made at high energies when some of the intermediate gauge bosons become effectively longitudinal. There too the running-width scheme renders completely unreliable results [6]. In processes involving more intermediate gauge bosons, *e.g.* $e^+e^- \rightarrow 6f$, also the fixed-width scheme breaks down at high energies as a result of breaking SU(2) gauge invariance.

2. Unstable gauge bosons: radiative corrections

By employing the fermion-loop scheme all one-particle-irreducible fermionic one-loop corrections can be embedded in the tree-level matrix elements. This results in running couplings, propagator functions, vertex functions, *etc.* However, there is still the question about the bosonic corrections. A large part of these bosonic corrections, as *e.g.* the leading QED corrections, factorize and can be treated by means of a convolution, using the fermion-loop-improved cross-sections in the integration kernels. This allows the inclusion of higher-order QED corrections and soft-photon exponentiation. In this way various important effects can be covered. Nevertheless, the remaining bosonic corrections can be large, especially at high energies [7, 8].

In order to include these corrections one might attempt to extend the fermion-loop scheme. In the context of the background-field method a Dyson summation of bosonic self-energies can be performed without violating the Ward identities [9]. However, the resulting matrix elements depend on the quantum gauge parameter at the loop level that is not completely taken into account. As mentioned before, the perturbation series has to be truncated; in that sense the dependence on the quantum gauge parameter could be viewed as a parametrization of the associated ambiguity.

As a more appealing strategy one might adopt a hybrid scheme, adding the remaining bosonic loop corrections by means of the pole scheme. This is gauge-invariant and contains the well-known bosonic corrections for the production of on-shell gauge bosons (in particular W-boson pairs). Moreover, if the quality of the pole scheme were to degrade in certain regions of phase-space, the associated error is reduced by factors of α/π. It should be noted that the application of the pole scheme to photonic corrections requires some special care, because in that case terms proportional to $\log(k^2 - M^2)/(k^2 - M^2)$ complicate the pole expansion [3, 8].

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