TWO-LOOP HEAVY TOP EFFECTS ON PRECISION OBSERVABLES *

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The corrections induced by a heavy top on the main precision observables are now available up to $O(\alpha^2 m_t^2/M_W^2)$. The new results imply a significant reduction of the theoretical uncertainty and can have a sizable impact on the determination of $\sin^2\theta_{\rm eff}^{\rm lept}$.

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The very precise measurements carried out at LEP and SLC in the recent past have made the study of higher order radiative corrections necessary in order to test the Standard Model, and possibly to uncover hints of new physics. The one-loop corrections to all the relevant electroweak observables are by now very well established [1], and two and three-loop effects have been investigated in several cases. The dominance of a heavy top quark in the one-loop electroweak corrections, which depend quadratically on the top mass, has allowed to predict with good approximation the mass of the heaviest quark before its actual discovery.

Among the higher order effects connected with these large non-decoupling contributions, the QCD corrections are now known through $O(\alpha_s^2)$ [2]. As for the purely electroweak effects originated at higher orders by the large Yukawa coupling of the top, reducible contributions have been first studied in [3], while a thorough investigation of leading irreducible two-loop contributions has been initiated in [4], in the limit of a massless Higgs, and later continued by Barbieri *et al.* and others for arbitrary M_H [5]. In this talk I will illustrate some implications of the new calculation of the $O(\alpha^2 m_t^2/M_W^2)$ corrections to the main precision observables [6, 7].

The result of the calculation of the leading $O(G_\mu^2 m_t^4)$ effects on the ρ parameter [5] is shown in Fig. 1 (upper curve). The correction is relatively

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sizable and in the heavy Higgs case reaches the per mille level in the prediction of $\sin^2\theta_{\rm eff}^{\rm lept}$, comparable to the present experimental accuracy [8]. We observe that the correction is extremely small for a small Higgs mass, due to large cancellations. We can naively expect that setting the masses of the vector boson different from zero (and so going beyond the pure Yukawa theory considered in [5]) might spoil the cancellations and lead to relevant deviations from the upper curve of Fig. 1 in the light Higgs region.

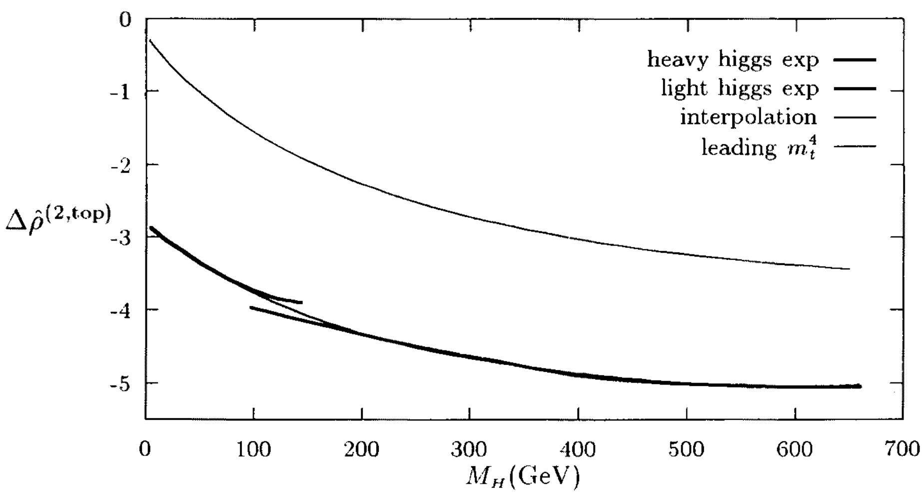

Fig. 1. Two-loop heavy top corrections to $\Delta\hat{\rho}$ in units 10^{-4}. The upper line represents the leading $O(G_\mu^2 m_t^4)$ correction of Ref. [5].

In addition, the theoretical uncertainty coming from unknown higher order effects is dominated by terms $O(\alpha^2 m_t^2/M_W^2)$ [1]. Indeed, the renormalization scheme ambiguities and the different resummation options examined in [1] led to an estimate of the uncertainty of the theoretical predictions which was in a few cases disturbingly sizable, *i.e.* comparable to the present experimental error. In particular, for $\sin^2\theta_{\rm eff}^{\rm lept}$, the estimated uncertainty was $\delta\sin^2\theta_{\rm eff}^{\rm lept}(th)\lesssim 1.4\times 10^{-4}$, while the present experimental average is $\sin^2\theta_{\rm eff}^{\rm lept} = 0.23165\pm 0.00024$ [8]. For M_W the uncertainty, $\delta M_W(\rm th)\lesssim 16$ MeV, was much smaller than the error on the present world average, 125 MeV. A different analysis based on the explicit two-loop calculation of the ρ parameter in low-energy processes has also reached very similar conclusions [9].

Motivated by the previous observations, the complete analytic calculation of the two-loop quadratic top effects has been performed for the relation between the vector boson masses and the muon decay constant G_μ [6] and for the effective leptonic mixing angle, $\sin^2\theta_{\rm eff}^{\rm lept}$ [7, 10].

Corrections to the Z^0 decay width are also under study. All these calculations have been done in the $\overline{\rm MS}$ scheme introduced in [11], *i.e.* using $\overline{\rm MS}$ couplings and on-shell masses, a particularly convenient framework. Besides the leading $O(G_\mu^2 m_t^4)$ result, I have displayed in Fig. 1 the two-loop heavy top correction to $\hat\rho$ up to $O(\alpha^2 m_t^2/M_W^2)$. In the $\overline{\rm MS}$ scheme, $\hat\rho$ is defined as $M_W^2/M_Z^2\ 1/\cos^2\hat\theta_{\overline{\rm MS}}(M_Z)$, and represents the most obvious process-independent analogue of the low-energy ρ parameter. The deviation from the leading $O(G_\mu^2 m_t^4)$ result is striking.

In order to gauge the impact of the new calculation, however, it is best to consider physical observables, and to study the scheme and scale dependence of their predictions from α, G_μ, and M_Z. I will do that for the two most relevant precision quantities: the mass of the W boson and $\sin^2\theta_{\rm eff}^{\rm lept}$. Concerning the scale dependence of the $\overline{\rm MS}$ predictions, the situation is exemplified in Fig. 2. Over a wide range of μ values, the scale dependence of M_W is significantly reduced by the inclusion of the $O(\alpha^2 m_t^2/M_W^2)$ contribution.

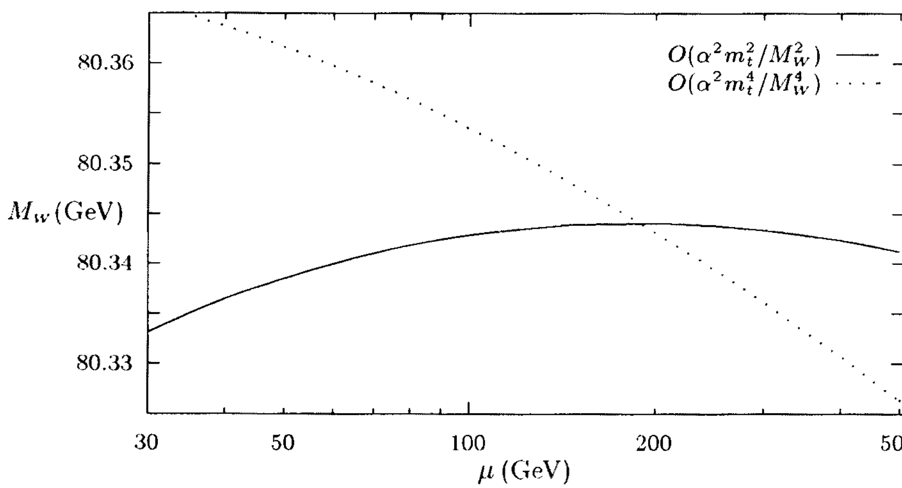

Fig. 2. Scale dependence of the prediction for M_W in the $\overline{\rm MS}$ scheme for m_t = 180GeV, M_H = 300GeV, including only the leading $O(G_\mu^2 m_t^4)$ correction (dotted curve) or all the available two-loop contributions, through $O(\alpha^2 m_t^2/M_W^2)$ (solid curve).

After translating [10] the results of the two-loop calculation into the on-shell (OS) scheme [12], I have compared the predictions for M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$ in the $\overline{\rm MS}$ and OS scheme, before and after the inclusion of the $O(\alpha^2 m_t^2/M_W^2)$ contribution. The results for one particular OS implementation are shown in Table I, where $\Delta s^2_{\rm eff} \equiv \sin^2\theta_{\rm eff}^{\rm lept}(\overline{\rm MS}) - \sin^2\theta_{\rm eff}^{\rm lept}({\rm OS})$ and $\Delta M_W \equiv M_W(\overline{\rm MS}) - M_W({\rm OS})$. As expected, the scheme dependence of the predicted values of $\sin^2\theta_{\rm eff}^{\rm lept}$ and M_W is drastically reduced. After considering alternative OS options, we can safely conclude that the inclusion of the $O(\alpha^2 m_t^2/M_W^2)$ correction reduces the scheme dependence in the two cases considered by at least a factor corresponding to the expansion parameter $M_W^2/m_t^2 \approx 0.2$.

TABLE I

Scheme dependence of the prediction of $\sin^2\theta_{\rm eff}^{\rm lept}$ before and after the inclusion of the new $O(\alpha^2 m_t^2/M_W^2)$ correction for $m_t = 175$GeV. $\Delta s^2_{\rm eff}$ is in units 10^{-4}, ΔM_W in MeV and M_H in GeV.

M_H	$\Delta\, s^{2\,\rm lead}_{\rm eff}$	$\Delta\, s^2{}_{\rm eff}$	$\Delta M_W^{\rm lead}$	ΔM_W
65	-0.90	-0.14	8.4	1.5
100	-0.90	-0.12	8.2	1.3
300	-0.87	-0.08	7.6	0.5
600	-0.83	-0.05	7.0	0.1
1000	-0.79	-0.03	6.5	-0.3

Finally, let me consider the impact of the new calculations on the precise predictions of $\sin^2\theta_{\rm eff}^{\rm lept}$ and M_W. It is clear that the shifts induced depend very strongly on the actual implementations of OS and $\overline{\rm MS}$ scheme. The results shown in Table II refer to a typical $\overline{\rm MS}$ implementation [7, 11] and to two different OS implementations, OSI and OSII. Using $\delta M_W \equiv M_W - M_W^{lead}$ and $\delta\, s^2_{\rm eff} \equiv \delta s^2_{\rm eff} - \delta s^{2\,lead}_{\rm eff}$ for the shifts induced by the $O(\alpha^2 m_t^2/M_W^2)$ corrections on M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$, we see that δM_W and $\delta\, s^2_{\rm eff}$ tend to be larger in the light Higgs region, and that $\delta\, s^2_{\rm eff}$ can be quite sizable, more than 1×10^{-4}.

TABLE II

Shifts induced by the $O(\alpha^2 m_t^2/M_W^2)$ corrections in the OS and $\overline{\rm MS}$ scheme for $m_t = 175$GeV. $\delta s^2_{\rm eff}$ is in units 10^{-4}, δM_W in MeV.

M_H	$\delta s^{2\,\rm OSI}_{\rm eff}$	$\delta s^{2\,\rm OSII}_{\rm eff}$	$\delta s^{2\,\overline{\rm MS}}_{\rm eff}$	$\delta M_W^{\rm OSI}$	$\delta M_W^{\rm OSII}$	$\delta M_W^{\overline{\rm MS}}$
65	0.04	1.56	0.80	−6.5	−14.5	−13.4
100	−0.02	1.27	0.76	−5.9	−12.7	−12.9
300	−0.14	0.35	0.65	−4.1	−6.7	−11.1
600	−0.20	−0.33	0.58	−2.6	−1.9	−9.5
1000	−0.30	−0.93	0.46	−0.5	2.9	−7.3

Because of the sign of the shifts, in general the $O(\alpha^2 m_t^2/M_W^2)$ correction further enhances the screening of the top quark contribution by higher order effects.

In summary, two-loop electroweak m_t^2 effects are now available in analytic form for the main precision observables in *both* $\overline{\rm MS}$ and OS frameworks. The new contributions consistently reduce the scheme and scale dependence of the predictions by *at least* a factor $M_W^2/m_t^2 \approx 0.2$, suggesting a relevant improvement in the theoretical accuracy. The impact on the value of the effective sine can be sizable, up to 1.5×10^{-4}, but it is highly sensitive to the scheme adopted.

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